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Analytical Approach for Base Flow in Watershed Model

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ABSTRACT. A relatively simple procedure to predict the base flow component of stream flow in a watershed is presented. This base flow depends upon the aquifer recharge, the initial state of water table elevations and the fluctuations in river stage. The methodology consists of deriving analytical solutions to the linearized one-dimensional Boussinesq equation for groundwater flow. The developed methodology was verified using a selected watershed and a good agreement was obtained between observed and computed results. An illustrative numerical example is given.

In one dimension system linear Boussinesq equation governing groundwater flow was shown by Morel-Seytoux (1979) as:

$$\frac{\partial h(x, t)}{\partial t} - \alpha \frac{\partial^2 h(x, t)}{\partial x^2} = 0 \qquad (1)$$

where h(x, t) is the water table elevation measured from certain datum, x is horizontal abscissa with origin at river bank and running perpendicular to river mean course, t is time, and α is aquifer diffusivity (*i.e.* ratio of transmissivity, T, over effective aquifer porosity, ϕ).

In linear systems and under the conditions of eq.(1) the general time response o(t) to an arbitrary excitation pattern, e(τ), is given by the following convolution integral (Schwarz and Friedland 1965):

$$o(t) = \int_{0}^{t} k(t-\tau)\ e(\tau)d\tau \qquad (2)$$

where the function k(.) is the unit impulse kernel of the system which completely characterizes the system. Variables τ and t are response and excitation times. Naturally for different types of excitation (*e.g.* recharge and river stage) and response (*e.g.* water-table elevation, return flow to stream, etc.) different kernels apply which are not independent but can be deduced from each other Several researchers have used the convolution integral equation to determine return flows analytically in stream-aquifer system. An example of these researchers are: Hall and Moench (1972), Higgins (1980), Neuman (1981), and Gill (1985).

Problem Definition and Analysis

The problem is to determine the aquifer return flow discharge $Q_r(t)$ to a river in response to three different excitations. These excitations are mean aquifer recharge rate q(t) expressed as depth per unit time, deviation of river stage y(t) from initial stage y_0 (selected to coincide with initial position of water table at river bank), and initial average water table elevations over grids of a distance (a) between river bank and opposite aquifer boundary (assumed to be no-flow boundary). System components are shown in Fig. (1).

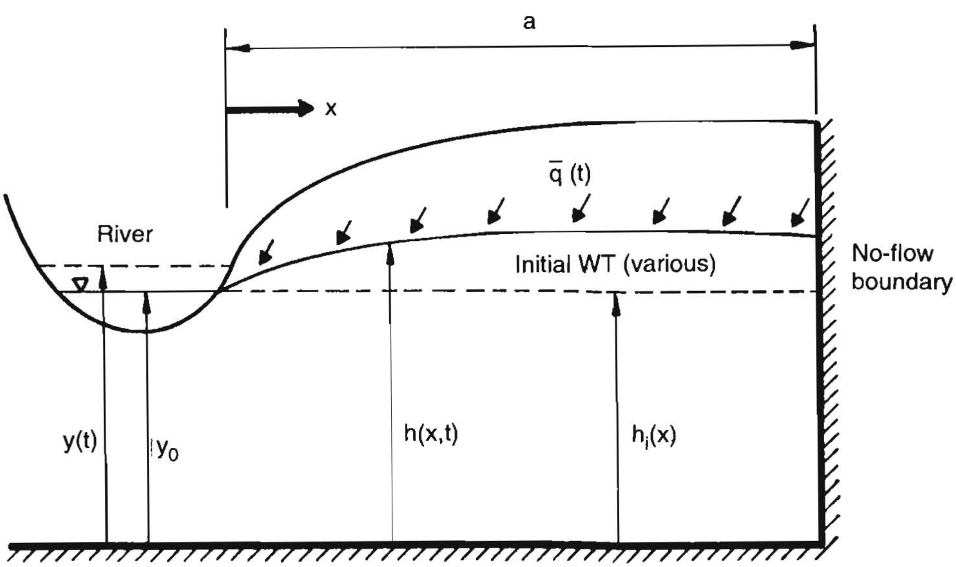

Fig. 1. Components river-aquifer system.

The solution to the general problem is obtained as the superposition of the solutions of the three following problems: (1) variable river stage y(t) with constant initial horizontal water table $h_i(x)$ and with no recharge, q(t) = o, (2) nonuniform initial water table with constant head at river bank equals to initial river stage and with no rechage and (3) uniform recharge q(t) with constant river stage and with constant initial horizontal water table.

Variable River Stage Solution

The problem is to solve eq. (1) given that river stage fluctuation is the only boundary condition to be considered. To obtain general solution to this problem, it is easier to find head response to a unit step rise in stage at the river. According to Carslaw and Jaeger (1959), this unit step response is given by:

$$K_{h,y}(x, t) = 1 - \frac{4}{\pi}\sum_{m=1}^{\infty} \frac{\sin\left[(2m-1)\frac{\pi}{2}\frac{x}{a}\right]}{(2m-1)} \quad e^{-\alpha\left[\frac{(2m-1)\pi}{2a}\right]^2 t} \tag{3}$$

where $k_{h,y}$ (x, t) is the unit step response in head (h) due to stage fluctuation (y) and t is the response time. Let

$$C_m = \frac{(2m-1)\,\pi}{2a} \tag{4}$$

It can be verified readily that K = 1 at x = 0 for all times. Thus the boundary condition at x = 0 is satisfied. The derivative of $K_{h,y}(x, t)$ with respect to x is proportional to cos (c_m x) which is zero for all m values when x = a; thus no-flow boundary condition is satisfied at x = a. For t = o, $K_{h,y}$ (x, o) must be zero for all x values.

The return flow, $Q_{r,y}(t)$, can be obtained by the application of Darcy's law at river bank. The solution is given by:

$$Q_{r,y}(t) = TL\,\frac{\partial h}{\partial x}\Big|_{x=0} \tag{5}$$

where L is the reach length, and T is aquifer transmissivity. By replacing the head term in eq. (5) with the unit step kernel of head given in eq. (3), the unit step kernel of return flow due to stage fluctuation, $K_{Qr,y}$, is given by:

$$K_{Qr,y}(t) = -\frac{2TL}{a}\sum_{m=1}^{\infty} e^{-\alpha[C_m]^2 t} \tag{6}$$

since this kernel is singular at time zero, it is necessary to consider an integrated response, Morel-Seytoux (1979). The cumulative return flow W_r can be defined as:

$$Wr(t) = \int_o^t Q_r(\tau)\, d\tau \tag{7}$$

Thus after substituting into eq. 6, simplifying and integrating, unit step of cumulative return flow is:

$$K_{W_{r,y}}(t) = \frac{8La\,\phi}{\pi^2} \sum_{m=1}^{\infty} \left[\frac{1-e^{-\alpha C_m^2 t}}{(2m-1)^2}\right] \tag{8}$$

Discrete kernels of cumulative return flow for each individual m-term can be deduced from eq. (8) as:

$$\delta_{W_{r,y}}(n,m) = K_{W_{r,y}}(n, m) - K_{W_{r,y}}(n-1, m) \tag{9}$$

During the first period the mean return flow rate is the same as the cumulative return flow. Thus discrete kernels are the same which means that:

$$\delta_{Q_{r,y}}(1) = \frac{-8\,La\,\phi}{\pi^2} \sum_{m=1}^{\infty} \left(\frac{e^{\alpha C_m^2}-1}{(2m-1)^2}\right) \tag{10a}$$

After first period, the return flow discrete kernels during given period (n = 2, 3, 4, ...) are obtained as :

$$\delta_{Q_{r,y}}(n) = \frac{-8\,La\,\phi}{\pi^2} \sum_{m=1}^{\infty} \frac{(e^{\alpha C_m^2}-1)^2}{(2m-1)^2}\, e^{-\alpha C_m^2 n},\ n = 2, 3, \ldots \tag{10b}$$

Physically, one expects that the sum of all discrete kernels of mean return flow adds up to zero since the bank storage created by the rise in stage during the first period will gradually dissipate back into the stream.

To determine the mean return flow to the stream during period n due to succession of mean stages (y(υ) during period υ = 1, 2, ..., n) one needs to apply the usual unit hydrograph theory which yields the well-known formula:

$$Q_r(n) = \sum_{\upsilon=1}^{n} \delta_{Q_{r,y}}(n-\upsilon+1)\,[\,y(\upsilon)-y_o] \tag{11}$$

Non-Equilibrium Initial Water Table Elevation Solution

For the solution of head, the initial condition has an instantaneous character The solution involves the various m-terms of the unit impulse kernel defined in a linear combination such that at t = 0 the head matches the initial head for all x values. Clearly the solution is of the general form:

$$h(x, t) = \sum_{m=1}^{\infty} A_m \sin(c_m x)\, e^{-\alpha c_m^2 t} \qquad (12)$$

In this equation, the expression of h(x,t) satisfies the boundary conditions that h = 0 at x = 0 and $\partial h/\partial x$ = o at x = a for all times. For Fourier sine serie A_m is given by the expression:

$$A_m = \frac{2}{a} \int_0^A h_i(x) \sin(c_m x)\, dx$$

If the initial condition excitation $h_i(x)$ is of a uniform unit value in the range $(\lambda-1)\Delta x \leq \xi \leq \Delta x$ and zero everywhere else (λ is an integer running index, $\Delta x = a/G$ and G is the number of grids) then A_m takes a special form which after superposition leads to the solution:

$$h(x, t) = \frac{4}{\pi} \sum_{m=1}^{\infty} \frac{1}{2m-1} \sum_{\lambda=1}^{G} h_i(\lambda)\, B_m \sin C_m x\, e^{-\alpha C_m^2 t} \qquad (13)$$

where $B_m = [\cos C_m (\lambda-1)\Delta x - \cos C_m \lambda\Delta x]$ and $h_i(\lambda)$ is mean initial head over the grid. The unit impulse kernel of head due to initial head change can be deduced from eq. (13), Morel-Seytoux and Al-Hassoun (1987). The unit impulse and return flow is then obtained by applying Darcy's law at river bank which yields:

$$F_{Q_r,h_i}(t;m,\lambda) = \frac{2\,LT}{a} B_m\, e^{-\alpha C_m^2 t} \qquad (14)$$

The average return flow over a given period n is then given by:

$$F_{Qr,hi}(n;m,\lambda) = \frac{8\,aL\phi\, B_m}{\pi^2 (2m-1)^2} (e^{-\alpha C_m^2} - 1) e^{-\alpha C_m^2 t} \qquad (15)$$

One should notice that as the aquifer storage is depleted as time proceeds the return flow decreases exponentially up to zero for large times. By superposition, the return flow response to any pattern of initial mean heads over the grid system can be determined as:

$$Q_r(n) = \frac{8\,aL\phi}{\pi^2} \sum_{\lambda=1}^{G} \left[\sum_{m=1}^{\infty} \frac{B_m}{(2m-1)^2} (e^{\alpha C_m^2}-1)e^{-\alpha C_m^2 n} \right] h_i(\lambda) \tag{16}$$

Uniform Recharge Solution

For the solution of return flow a unit impulse of recharge which is applied instantaneously over a grid length is equivalent to an instantaneous change in initial head of magnitude $1/\phi$ followed by redistribution of that head. Consequently eq. (14) provides the solution except for the factor $1/\phi$. Therefore, the unit impulse kernel of return flow due to recharge over one grid length is given by:

$$k_{Q_{r,q}}(t;m,\lambda) = \frac{2LT}{a\phi} [\cos C_m(\lambda-1)\Delta x - \cos(C_m \lambda \Delta x)]\, e^{-\alpha C_m^2 t} \tag{14}$$

However, since the recharge is uniform along all grids, the total contribution from all grids after simplification is obtained as:

$$K_{Q_{r,q}}(t;m) = \frac{2LT}{a\phi} e^{-\alpha C_m^2 t}$$

By integration with respect to time the return flow response to unit step of uniform recharge can be obtained. By integrating the resulting equation with respect to time one can obtain unit step kernel of cumulative return flow which when combined with eq. (9) the discrete kernels of cumulative return flow can be deduced as:

$$\delta_{w_{r,q}}(n;m) = \frac{8\,La}{\pi^2 (2m-1)^2} \left[1 - \frac{(e^{\alpha C_m^2}-1)\, e^{-\alpha C_m^2 n}}{\alpha C_m^2} \right] \tag{17}$$

The discrete kernels of mean return flow can be obtained using eqs. (17) and (10). Kernel for n = 1 is obtained as:

$$\delta_{Q_{r,q}}(1) = \frac{32a^3 L\phi}{\pi^4 T} \sum_{m=1}^{\infty} \frac{[e^{-\alpha C_m^2} + \alpha C_m^2 - 1]}{(2m-1)^4} \tag{18a}$$

and for n = 2, 3, 4, ... is given as:

$$\delta_{Q_{r,q}}(n) = \frac{32a^3 L\phi}{\pi^4 T} \sum_{m=1}^{\infty} \frac{(e^{\alpha C_m^2}-1)^2}{(2m-1)^4} e^{-\alpha C_m^2 n} \qquad (18b)$$

Note that all discrete kernels are positive as they should be. The contribution of all m-terms is included in the above equations. A pulse of recharge will induce return flow then after recharge stops the mound, created by the recharge, will continue to sustain return flow to the river.

The mean return flow during period n due to succession of mean recharge rates q(υ) in various υ periods is given by the usual unit hydrograph theory as:

$$Q_r(n) = \sum_{\upsilon=1}^{n} \delta_{Q_{r,q}}(n-\upsilon+1)\, q(\upsilon) \qquad (19)$$

Mean Return Flow Due to All Excitations Together

The mean return flow in a given period (n) as a result of river stage fluctuations, initial aquifer levels, and aquifer recharge is given by the summation of return flows given in eqs. (11), (16) and (19) due to each excitation, respectively.

Practical Truncation of Series

It is not practical in eqs. (10), (16) and (18) to carry out the series summations from m = 1 to large number close to infinity.

A value of M = 5 was found to be a reasonable truncation value for all the series since the dropped terms (*i.e.* M > 5) added only about 3% of the series value.

In case of non-equilibrium initial head solution, it can be recognized that regardless of grid index the sum of discrete kernels for various n-periods should add up to a value equals to ϕ/G. Thus it is much easier, for practical purposes, to first calculate $\delta(2)$, $\delta(3)$, ... until $\delta(M_e+1) = 0$, where M_e is the memory of the system. The kernel $\delta(1)$ is then estimated as the complement of all other $\delta(.)$ to ϕ/G, namely:

$$\delta_{Q_{r,h_i}}(1, \lambda) = \frac{\phi}{G} - \sum_{n=2}^{M_e} \delta_{Q_{r,h_i}}(n, \lambda) \qquad (20)$$

In case of aquifer recharge solution, sum of discrete kernels (per unit area) should add up to 1. Thus the most efficient computation procedure for discrete kernel of first period is to be estimated by mass balance as:

$$\delta_{Q_{r,q}}(1) = 1 - \sum_{n=2}^{M_e} \delta_{Q_{r,q}}(n) \tag{21}$$

Where $\delta_{Q_{r,q}}(n)$ for n = 2, 3, ..., M_e can be estimated using eq. (18b).

Verification of Base Flow Component

In order to test the proposed analytical formulation of baseflow component in stream-aquifer system, this formulation was used in a distributed hydrological watershed model named SWATCH. Details of this watershed model can be obtained from Morel-Seytoux and Al-Hassoun, 1987. The model was applied to Turner basin in Georgia, USA which has an area equal to 16km^2 and the subsurface flow is the dominant component to outflow.

Two events were used for calibrating and verifying the model. Fig. 2 shows event 1 used for calibration with observed and computed hydrographs. The calibrated value of K for the upper unsaturated zone is high (0.70 cm/hr), which is the reason for the small (about 20%) contribution of overland flow. The effective rainfall is computed by subtracting the flow abstractions (mainly infiltration) from total rainfall. A different event was used to verify the model and a good agreement was found between the predictions and the observations as shown in Fig. 3. Hydrographs of lateral contribution to the stream from overland and subsurface flows can be seen in Fig. 4. These two hydrographs of the computed outflow for event 2 have distinct shapes where the surface hydrograph did not start until the overland flow contribution has almost vanished. The peak of subsurface hydrograph is much smaller (about half) though its volume is about four times that under the overland flow hydrograph.

Illustrative Numerical Example

A block of aquifer influenced by three excitations was simulated. The aquifer block has a rectangular shape with a length (a) perpendicular to river axis equals to 200 m while the length along the river is 1000 m. The aquifer has properties as: T = 5000

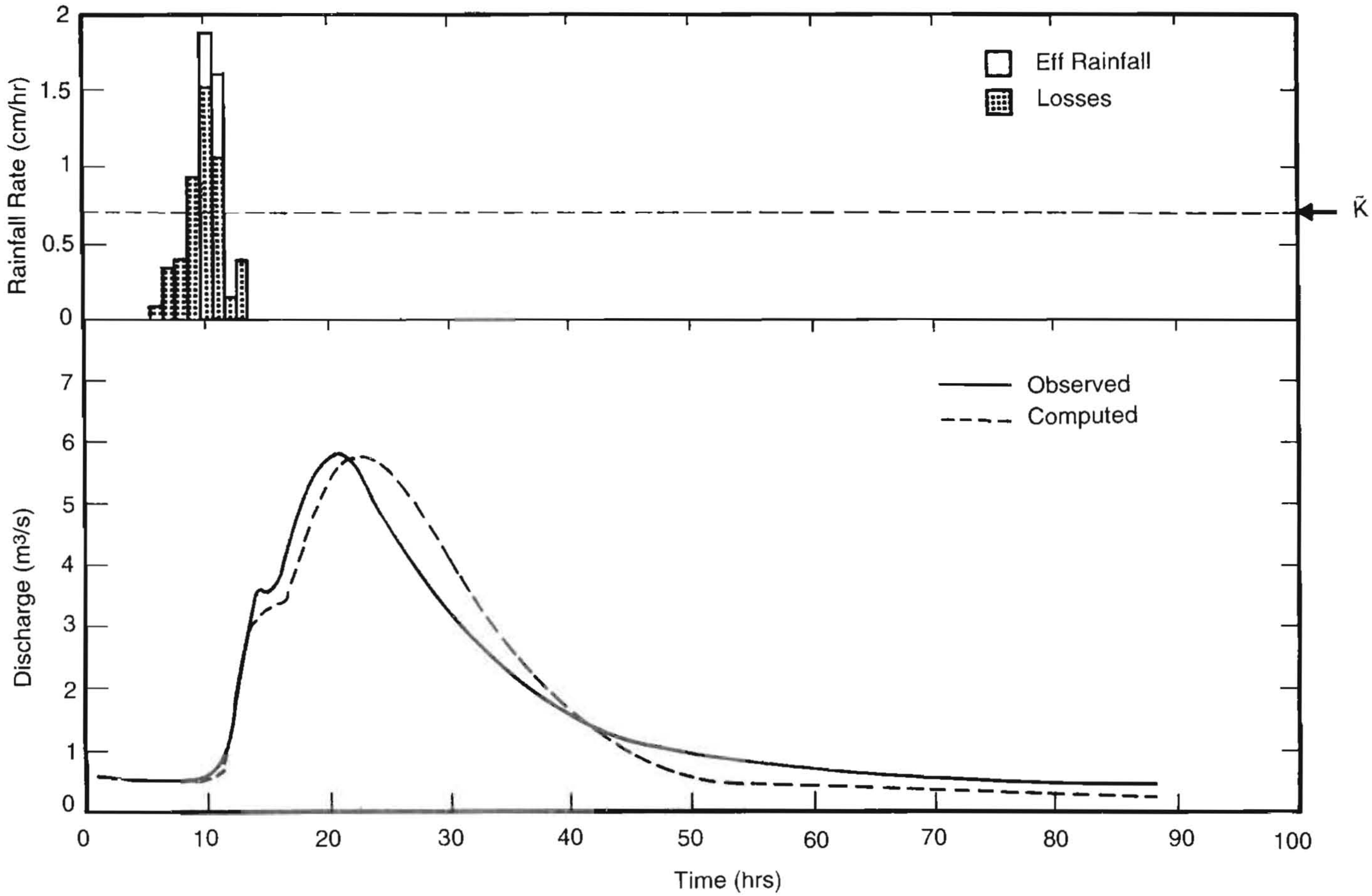

Fig. 2. Event 1 used for model calibration.

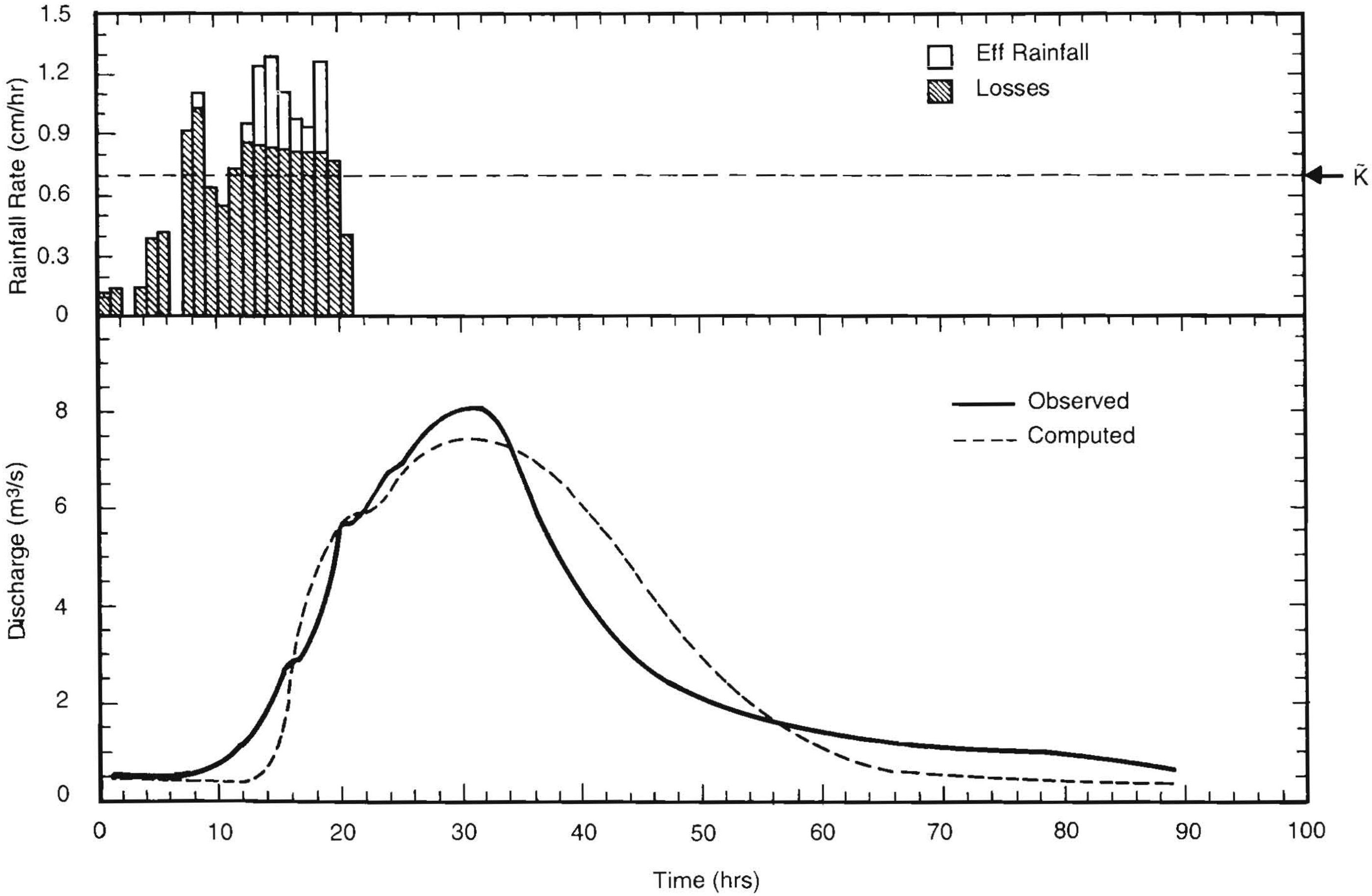

Fig. 3. Observed and computed hydrographs for event 2 of calibration run.

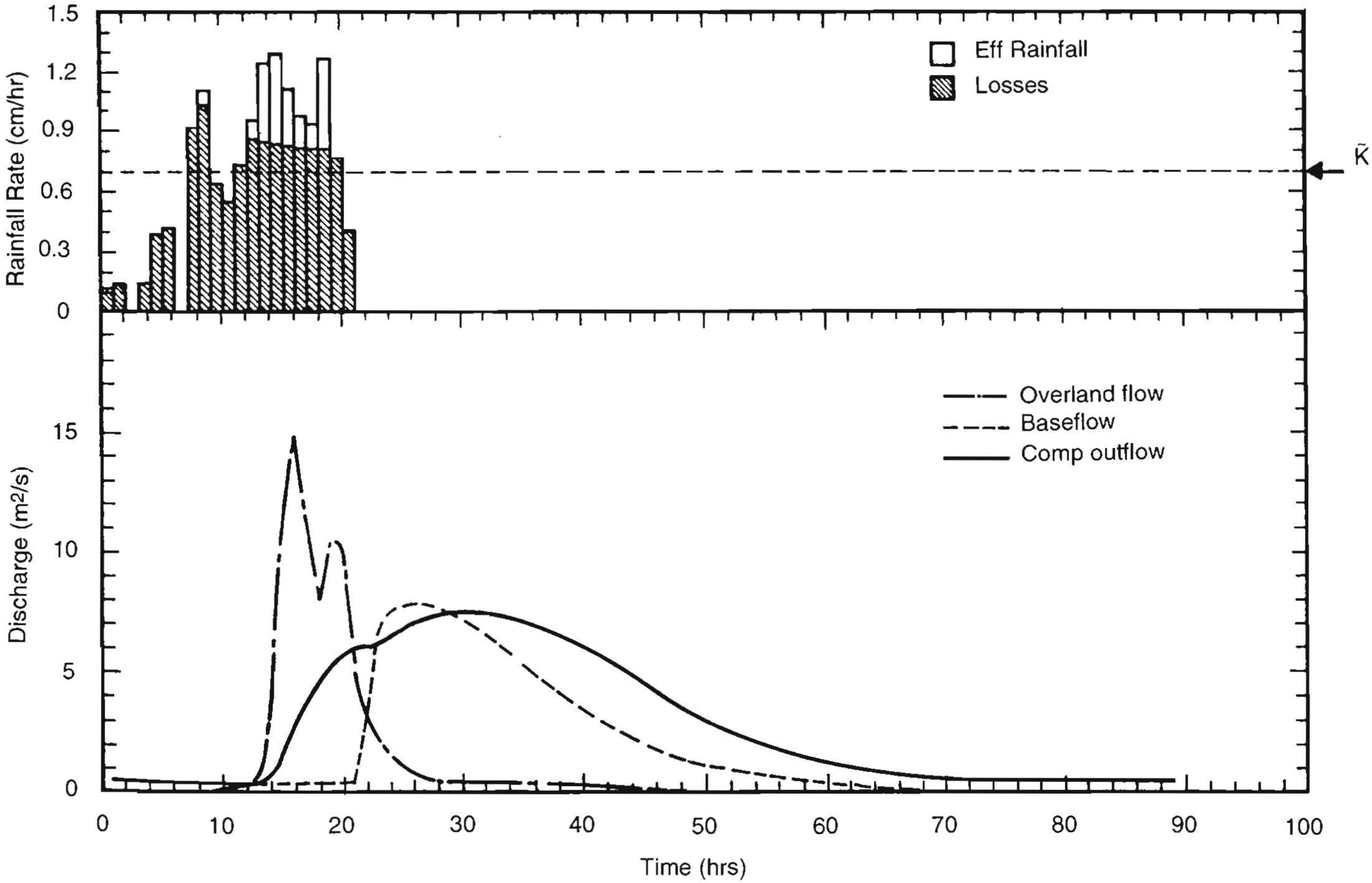

Fig. 4. Lateral Hydrographs of Computed Outflow for Event 2.

Table 1. Pattern of variation in river stage

Period υ (week)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
y (υ) mean river stage (meter)	5.30	5.80	6.20	5.40	5.20	5.00	4.80	4.75

Table 2. Pattern of variation in aquifer recharge

Period υ (week)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
q (υ) mean recharge rate (m/wk)	0.01	0.03	0.09	0.19	0.30	0.25	0.16	0.10

Table 3. Initial head distribution along aquifer length

Grid index	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
mean initial head (meter)	0.01	0.20	0.40	0.30	0.26	0.22	0.15	0.20	0.17	0.17

Table 4. Discrete kernels of mean return flow (m/week) due to river stage excitation

Period (υ)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
$\delta_{Qr,y}$ (υ)	–0.1571	0.1296	0.0217	0.0047	0.0010	0.0002	0	0

Table 5. Discrete kernels of mean return flow (m/week) due to aquifer recharge excitation

Period (υ)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
$\delta_{Q_{r,q}}$ (υ)	0.5800	0.3315	0.0695	0.0149	0.0032	0.0007	0.0001	0.0001

m^2/week, $\phi = 0.2$. The initial river stage (y_o) is assumed to be 5 m and aquifer length (a) is divided into 10 grids (*i.e.* $\Delta x = 20$ m). The excitations are presented in Tables 1, 2 and 3.

Values of river stage discrete kernels (per unit area) were obtained using equation (10) and truncating the series by M = 5. These values are listed in Table 4. One notice that the sum of all these discrete kernels is equal to zero as it should be.

Values of aquifer recharge discrete kernels (per unit area) were obtained using equations (18b) and (21). The values of $\delta_{Q_{r,q}}$ (.) for n = 2, 3, 4, ..., 8 ($M_e = 7$) were first calculated using equation (18b) then the first kernel is estimated from equation (21). Table 5 presents values of these kernels. As it should be, the sum of all kernels is equal to one.

Table 6 provides the calculated values of return flow discrete kernels (per unit area) due to initial head excitation. These values were first obtained using kernels in equation (16) for n = 2, 3, ..., 8, then using equation (20) to obtain the discrete kernel for n = 1.

Given the pattern of excitations in Tables 1, 2, and 3 and by using discrete kernels in Tables 4, 5, and 6, it is straightforward to evaluate the return flow contributions due to river stage fluctuations, initial head excitations, and aquifer recharge using equations (11), (16) and (19), respectively. These values are presented in Table 7. As an example, the contribution to return flow due to stage fluctuation during the 5th period (per unit area) can be computed as:

$$Q_{r,y}(5) = \sum_{\upsilon=1}^{5} \delta_{Q_{r,y}}(5-\upsilon+1)\,[y(\upsilon)-y_o]$$

$$= \delta(5)\,[y(1) - y_0) + \ldots\ldots + \delta(1)\,\{y(5) - y_o]$$

$$= 0.001(5.30-5) + 0.0047\,(5.80-5) + 0.0217(6.20-5) + 0.1296(5.40-5) - 0.1571(5.20-5)$$

$$Q_{r,y}(5) = 0.0505 \text{ m/week}$$

So the volumetric return flow for that week due to stage fluctuations is equal to 0.0505 (200) (1000) = 10104 m^3/week.

Table 6. Discrete kernels of mean return flow (m/week) due to initial head excitation

Period υ	Grid index									
	$\lambda = 1$	$\lambda = 2$	$\lambda = 3$	$\lambda = 4$	$\lambda = 5$	$\lambda = 6$	$\lambda = 7$	$\lambda = 8$	$\lambda = 9$	$\lambda = 10$
2	0.0003	0.0010	0.0016	0.0022	0.0026	0.0032	0.0036	0.0039	0.0042	0.0043
3	0.0001	0.0002	0.0003	0.0005	0.0006	0.0007	0.0008	0.0008	0.0009	0.0009
4	0	0.0005	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002
1	0.0196	0.0188	0.0180	0.0172	0.0167	0.0160	0.0154	0.0151	0.0147	0.0146

Table 7. Return flow contributions and total return flow

Period (υ) (week)	Contribution of river stage fluctuation		Contribution of aquifer recharge		Contribution of initial head		Total return flow
	(m/week)	(m^3/week)	(m/week)	(m^3/week)	(m/week)	(m^3/week)	(m^3/week)
1	–0.0471	– 9426	0.0058	1160	0.0363	7250	1016
2	–0.0868	–17360	0.0207	4143	0.0057	1170	–12077
3	–0.0783	–15666	0.0628	12568	0.0012	243	–2855
4	0.1115	22290	0.1423	28454	0.0002	43	50787
5	0.0505	10104	0.2437	48744	0	0	58848
6	0.0411	8220	0.2591	51820	0	0	60040
7	0.0390	7800	0.1997	39933	0	0	47733
8	0.0149	2887	0.1336	27610	0	0	29697

It is interesting to notice that during the first week the bank storage effect (–9426 m^3) can not compensate the combined recharge (1160 m^3) and initial head effects (7250 m^3). During the second and third periods there is a net seepage loss to the aquifer Later on, even though the river is still higher than initially (5.40 m at n = 4), the high bank storage due to earlier higher stages (6.20 m during period 3) and high aquifer recharge rate (0.30 m/week) combine to give very high return volumes during weeks 4, 5, 6 and 7. During period 8 the return flow is almost solely due to aquifer recharge as the bank has essentially released all that went into storage.

Conclusions

An analytical approach was developed to predict base flow in a river in response to the physical causes of change in base flow which mainly are: river stage fluctuations, initial water table elevations and aquifer recharge. The method is physically based and requires only simple and few algebraic calculations. The base flow component, forming a part of an existing watershed model (SWATCH), was verified using actual data. Good agreement between observed and computed hydrographs was obtained. A given illustrative example summarizes the procedure to use that approach.

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التمثيل الرياضي التحليلي للمياه الجوفية
مطبق على نموذج حوض مائي

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تعتمد كمية المياه الجوفية المتدفقة إلى الانهار الواقعة في تكوين مائي على عوامل مختلفة من أهمها : التغذية الطبيعية من مياه الأمطار للتكوينات المائية الجوفية عن طريق الرشح ، والحالة الابتدائية لمستوى سطح المياه الجوفية داخل التكوينات المائية ، وكذلك تذبذب منسوب المياه في الأنهار . لذا فإنه من المهم عند دراسة حركة المياه الجوفية بصورة واقعية اخذ العوامل السابقة بالاعتبار . وتحقيقًا لذلك تم استنتاج طريقة رياضية لتقدير كمية المياه الجوفية المتدفقة إلى الانهار الواقعة في حوض مائي معين باعتبار تلك العوامل السابقة .

وتشتمل طريقة الحل على اشتقاق حلول تحليلية لمعادلة (بوسنسك) الخطية الاحادية الواصفة لحركة المياه الجوفية حيث تعطي مقدار التغير في منسوب المياه الجوفية بدلالة الزمن والمسافة . وقد تم تقسيم الحل إلى ثلاثة حلول فرعية حيث يتم في كل مرة استنتاج حل لمعادلة (بوسنسك) بفرض تأثرها فقط

بأحد العوامل الثلاثة أعلاه ، ومن ثم تم تجميع الحلول الفرعية الثلاثة لتكوّن الحل النهائي لتقدير كمية المياه الجوفية المتدفقة إلى الانهار بطريقة رياضية تحليلية . هذا وقد عُرض مثال حسابي إفتراضي لبيان كيفية إيجاد الحلول الفرعية الثلاثة ثم إيجاد الحل النهائي وذلك بغرض توضيح كيفية تطبيق تلك الطريقة المستنتجة لتقدير كمية المياه الجوفية .

ومن مميزات هذه الطريقة أنها بنيت على أسس فيزيائية واقعية ولاتحتاج إلا لحسابات جبرية معدودة . وقد تم استخدام هذه الطريقة المشتقة كاحد عناصر نموذج هيدرولوجي مبني على أسس فيزيائية وقد تم التأكد من صحة تحقق تلك الطريقة وذلك بتطبيقها في دراسة كمية المياه الجوفية المتدفقة إلى أحد الانهار الواقعة في حوض (تيرنر) المائي في ولاية جورجيا بالولايات المتحدة الأمريكية ، ومن خلال ذلك التحقق وجد أن هناك تطابقاً جيداً بين قيم كمية المياه الجوفية الحقيقية المقاسة في الحقل وبين القيم المحسوبة بالطريقة الرياضية المستنتجة في هذا البحث .