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Anharmonic Solution of the Problem of Diffraction of Acoustic Waves

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ABSTRACT. The diffraction of an acoustic wave from a point source of sound by an absorbing half-plane and a nearly half plane ($y = \varepsilon g(x,z)$; $x \leq 0$ and $0 \leq \varepsilon << 1$) in the presence of a moving fluid is studied. This is done by using a recently developed method. It is assumed throughout the paper that the point source sets in at time $t = 0$. When $\varepsilon = 0$, it is shown that the far field of the diffracted wave is anharmonic. It behaves merely as the inverse of the distance from the point source but as if the point source moves at the fluid speed far from the barrier. On the other hand it is found that acoustic noise reduction by a nearly half-plane barriers $y = \varepsilon g(x,z)$; $0 \leq g(x,z) \leq 1$; $x \leq 0$ and $-\infty < z < \infty$ is better than reduction by a half-plane one.

Noise reduction by using barriers in heavily built-up areas has received the attention of several workers (Butler 1974, Kurze 1974, Jones 1952, 1972, Rawlins 1974, 1975 and Asghar *et al.* 1991). It has been found that good barriers are those having absorbing lining on the surface. In these works, the sound waves are taken generated by a permanent harmonic point source and the solution of the problem is a priori assumed to be harmonic in time. A mathematical model for reduction of noise by means of barrier with absorbing lining on one face has been proposed (Rawlins 1974, 1975). In this context the problem of diffraction of an acoustic wave by an absorbing half-plane has been studied (Asghar 1991). Here, by using a recently developed method (Abdel-Gawad 1991), this problem is reconsidered for a half-plane and a nearly half-plane: $y = \varepsilon g(x,z)$, $0 < \varepsilon << 1$; $x \leq 0$ and $0 \leq g(x,z) \leq 1$ for $x \leq 0$ and $-\infty < z < \infty$. This method introduces the notion of exponential operator which, we think, simplify calculations. In the present work, the Weiner-Hopf technique is also used.

Here, the point source is setting in at time t = 0. The anharmonic far field is then evaluated when $\varepsilon = 0$ and when $0 < \varepsilon << 1$ separately. Finally the obtained results in this paper are disscused.

Mathematical Formulation

Here, we adopt the mathematical model considered by Rawlins (1975). The point source is considered to be located at (x_0, y_0, z_0) and is taken to be harmonic in time. We suppose that the sound wave propagates in a fluid moving with velocity U parallel to the x-axis. First the barrier is taken to be the semi-infinite plane $y = 0$, $x \leq 0$ and is of negligible thickness. The geometry of the problem is shown in Fig. (1). The absorbing boundary condition on the two sides of the barrier is

$$P - u_n Z = o, \qquad (2.1)$$

here P is the pressure on the surface, u_n is the normal derivative of the perturbation velocity of the irrotational sound filed; Z is the acoustic impedance of the surface and $\hat{n}$ is the normal to the surface in the inward direction with respect to the barrier The perturbation velocity $\vec{u}$ is given in terms of the velocity potential ϕ as $\vec{u}$ = grade ϕ. The resulting pressure in the sound field is given by:

$$P = -\rho_0 (\partial_t + U\partial_x)\, \phi \qquad (2.2)$$

where U is defined before and ρ_0 is the mass density in the sound stream. We assume that the excitation of the source sets on at time t = o. In this case, the equation satisfied by ρ in the presence of a time harmonic source is

$$[\nabla^2 - (\frac{1}{c}\partial_t + M\partial_x)^2]\, \phi = \{e^{-i\omega t}\, \delta(y-y_0)_0\, \delta(z-z_0);\ t > 0;\ t < 0 \qquad (2.3)$$

The problem of diffraction of acoustic waves by an absorbing half-plane $y = 0$, $x \leq 0$ is described by the boundary conditions (Rawlins 1974, Jones, 1952).

$$(\partial_y \pm \beta M \partial_x \pm \frac{\beta}{c}\partial_t)\, \phi\, (x, 0^{\pm}, z, t) = 0,\ x < 0 \qquad (2.4)$$

$$\phi\,(x, 0^{+}, z) = \phi\,(x, 0^{-}, z, t)\ ;$$

$$\partial_y \phi(x, 0^{+}, z) = {}_y\, \phi\,(x, 0^{-}, z, t) \quad , x > 0, \qquad (2.5)$$

where $\beta = \rho_0\, c/Z$, $M = U/c$ is the Mach number and for a subsonic flow $|M| < 1$ and Re $\beta > 0$.

Now, we confine ourselves to find the general solution of (2.3). As the boundary conditions (2.4-5) are given at $y = 0^{\pm}$, it is convenient to rewrite (2.3) in the form

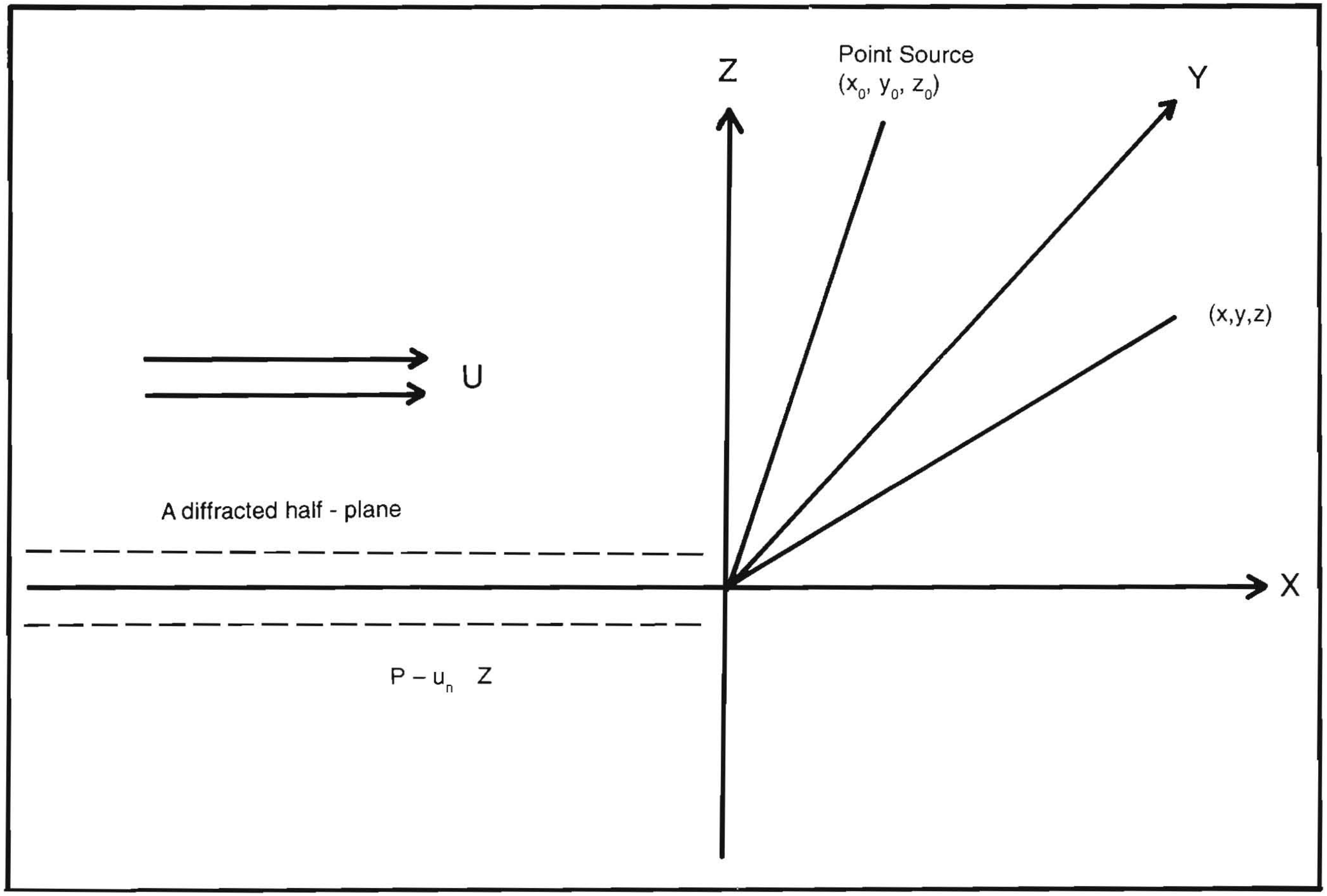

Fig. 1. The geometry of the problem of diffraction of a point source by a half-plane in the presence of a moving fluid. The half-plane is represented by $y = 0$ and $x \leq 0$. The fluid is assumed moving with velocity U parrallel to the x-axis.

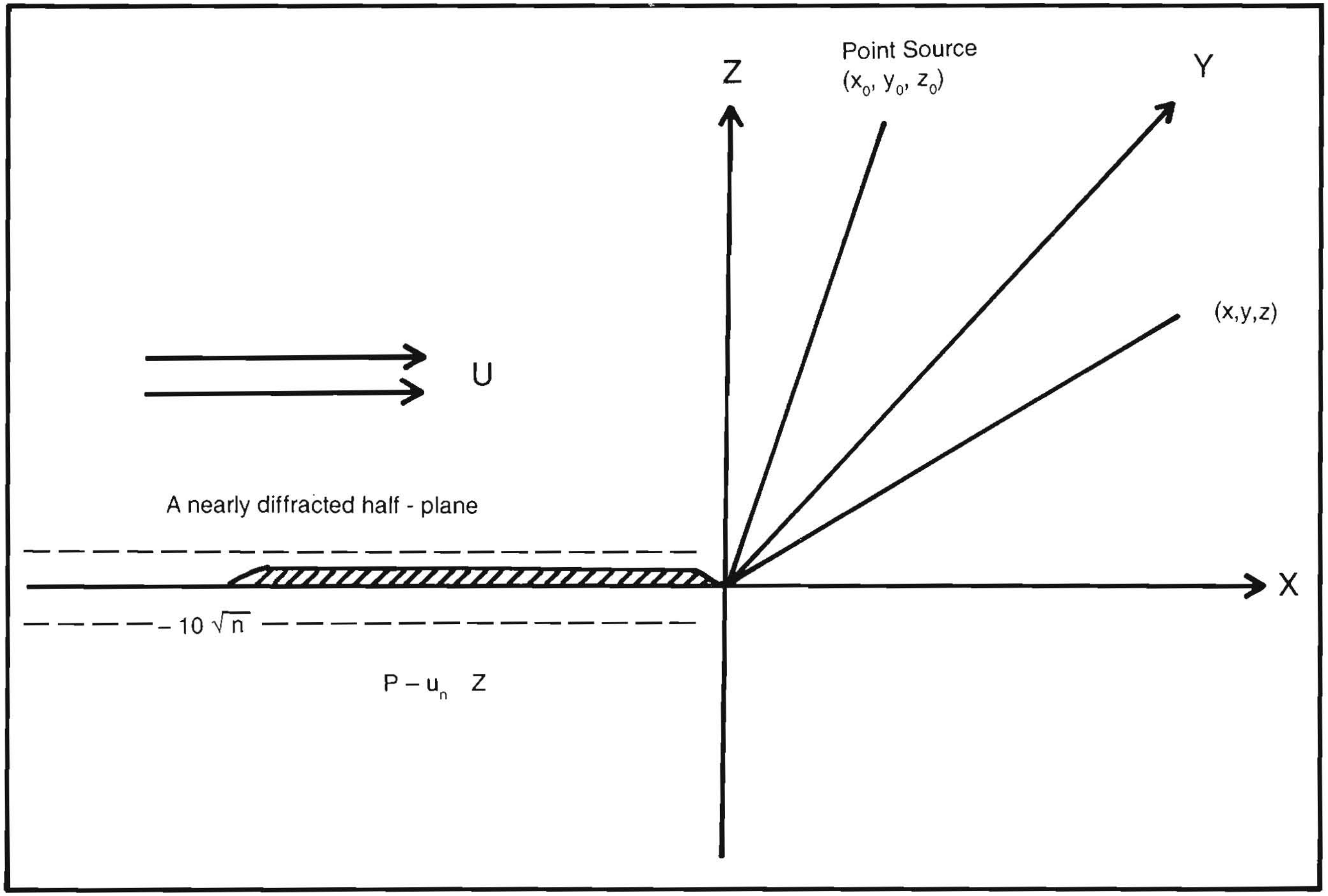

Fig. 2. The geometry of the problem of diffraction of a point source by a nearly half-plane in the presence of a moving fluid. The nearly half-plane is modeled by $y = 0.1 \sin(x^2/100) \cos(z^2/100)$; $-10\sqrt{\pi} \le x \le 0$, $|z| \le 10\sqrt{\pi}$ and $y = 0$ elsewhere. The fluid is assumed moving with velocity U parrallel to the x-axis.

$$(\partial^2 y - \hat{K}^2)\,\phi = f_s \tag{2.6a}$$

$$\hat{K}^2 = (\frac{1}{c}\,\partial_t + M\,\partial_x)^2 - (\partial_x^2 + \partial_z^2) \tag{2.6b}$$

Where f_s stands for the right hand side of (2.3). In view of theorem (3)of Abdel Gawad (1991), the solution of (2.4) is

$$\phi = \phi_{Bv} + \phi_S, \tag{2.7}$$

where ϕ_{Bv} is the solution of the homogeneous equation of (2.4) and is given byAbdel-Gawad (1991)

$$\phi_{Bv} = e^{y\hat{K}}\,\psi_0\,(x,z,t) + e^{-y\hat{K}}\;\Psi_1\,(x,z,t). \tag{2.8}$$

The second term in (2.7), namely ϕ_S, is the part of the solution which is due to the source term and is given by

$$\phi_S = \frac{1}{2}\int^{y} dy_1 \int^{y_1} dy_2\,[e^{(y-2y_1+y_2)}\,\hat{K} + e^{-}\,(y-2y_1+y_2)\,\hat{K}]f_S \tag{2.9}$$

the lower bounds in the integrals in (2.9) are arbitrarily taken. Equations (2.7-9) are the general solution of (2.6). There remain two points to illustrate in (2.7-9). The first point is about the justification of validity of the fraction power operator $\hat{K}$. The second one is about how to evaluate $\hat{K}\psi_{0,1}$. In this respect, we require that the boundary conditions are in $C^\infty \cap L_2$ over an appropriate domain. If this is the case, then so will be the function ψ_0 and ψ_1. Now as the operator $\hat{K}^2$ is a linear differential operator, acting on $C^\infty \cap L_2$, then it is L_2 – bounded. We define a fraction power of any linear bounded operator $\hat{M}$ as

$$\hat{M}^{-B} = \frac{1}{\Gamma(B)}\int_0^\infty e^{-\lambda\hat{M}}\,\lambda^{B-1}\,d\lambda\;,\; B > 0, \tag{2.10}$$

where we bear in mind that the integral in (2.10) is convergent. Also we have

$$\hat{M}^{B} = (\hat{M}^{-B})^{-1} \tag{2.11}$$

The exponential operator in (2.10) is defined by

$$e^{\lambda\hat{M}} = \frac{1}{2\pi i}\int_\Gamma e^{\lambda p}\,(\hat{M} - p\hat{I})^{-1}\,dp, \tag{2.12}$$

where $(\hat{M}-P\hat{I})^{-1}$ is the resolvant operator and Γ is a Cauchy contour surrounding a subset of the spectral set of the operator $\hat{M}$ if the spectrum is discrete. If the spectrum is continuous and, for example, is positive real valued, then Γ is taken with a branch cut along the positive real axis. The definitions (2.10-12) are adopted by Bemelmans (1980).

An estimate for the L_2- norm of the operator $\hat{K}$ is given by:

$$\|\hat{K}\,\psi_1\|\,L_2 < K\,\|\psi_1\|\,L_2,$$

where $k = \omega/c$ is the wave number. Thus the L_2– norm of the operator $\hat{K}$ is bounded and the validity of this operator is justified. Now, we evaluate $\hat{K}\,\psi_{0,1}$. By using (2.10), we have

$$(\hat{K})^{-1}\,\psi_{0,1} = (\hat{K}^2)^{-1/2}\,\psi_{0,1} = \frac{1}{\Gamma(1/2)}\int_0^{\infty} e^{-\lambda\hat{K}^2}\,\lambda^{-1/2}\,d\lambda\,.\,\psi_{0,1}, \tag{2.13}$$

and use $\hat{K}\,\psi_{0,1} = (\hat{K}^{-1})^{-1}\,\psi_{0,1}$. It remains to evaluate $\exp{-\lambda\hat{K}^2}.\ \psi_{0,1}$. To this end, we introduce the Fourier-Laplace transform of ψ_0 and ψ_1. Thus we have

$$\exp{-\lambda\hat{K}^2}\ \psi_{0,1} = \int_{a-i\infty}^{a+i\infty} ds \iint_{-\infty}^{\infty} \frac{dpdq}{i(2\pi)^3}\, e^{-\lambda\hat{K}^2}\, e^{st+i(px+qz)}\,\tilde{\psi}_{0,1}. \tag{2.14}$$

Now we assume that Abdel-Gawad (1990)

$$\exp-\lambda\hat{K}^2\ e^{st+i\,(px+qz)} = e^{st+i\,(px+qz)} \sum_{n=1}^{\infty} .e\ \frac{\lambda^n g_n}{n_1} \tag{2.15}$$

We evaluate $g_1, g_2, \ldots, g_n, \ldots$ by differentiating both sides of (2.15) with respect to λ and setting $\lambda = 0$ in both sides at each time of differentiation. Thus we have

$$g_1 = p^2 + q^2 + \left(\frac{s}{c} + iMp\right)^2 \tag{2.16}$$

$$g_n = 0 \qquad , \quad n \geq 2$$

By substituting from (2.16) into (2.15), (2.14) and in (2.13), we find

$$(\hat{K})^{-1}\,\psi_{0,1} = \int \frac{ds}{D} \iint \frac{dpdq}{i(2\pi)^3}\, K^{-1}\, e^{st+i(pz+qz)}\,\tilde{\psi}_{0,1}. \tag{2.17}$$

where $K = [p2 + q2 + (s/c + iMP)^2]^{1/2}$. The condition of convergence of the integral in (2.13) requires to restrict the domain of the integrals in (2.14) to the domian $D = \{(p,q,s) : \mathrm{Re}\, K^2 > 0\}$. After (2.17), we have

$$\hat{K}^{-1}\, e^{st+i(px+qz)} = K^{-1}\, e^{st+i(px+qz)} \tag{2.18}$$

Acting by the operator $\hat{K}$ on both sides of (2.18) we obtain

$$\hat{K}\; e^{st+i(px+zq)} = K\, e^{st+i(px+qz)} \tag{2.19}$$

Thus, we obtain a result similar to (2.17) but $\hat{K}$ and K replace $(\hat{K})^{-1}$ and K^{-1} respectively.

Diffraction of an acoustic wave by an absorbing half-plane

The boundary conditions (2.4-5) suggest to rewrite the solution of (2.3), namely (2.6-9) in terms of $|y|$ instead of y. Thus, we have

$$\phi = \phi_{Bv} + \phi_S \tag{3.1a}$$

$$\phi_{Bv} = e^{|y|\hat{K}}\ \psi_0(x,z,t) + e^{-|y|\hat{K}}\ \psi_1(x,z,t). \tag{3.1b}$$

For the reason of finiteness, we set $\psi_0(x,z,t) = 0$ and (3.1) becomes

$$\phi_{Bv} = e^{-|y|\hat{K}}\ \psi_1(x,z,t). \tag{3.2}$$

Also, for the reason of finiteness, we rewrite ϕ_S as

$$\phi_S = \int^{y} dy_1 \int^{y_1} dy_2\ e^{-||y|-2|y_1|+|y_2||\hat{K}_{f_s}} \tag{3.3}$$

One can verify that ϕ_S satisfies (2.3).

Now, we work with the Fourier - Laplace transform of the functions ϕ, ϕ_{BV}, ϕ_S and ψ, thus we have

$$\tilde{\phi} = \tilde{\phi}_{Bv} + \tilde{\phi}_S, \tag{3.4}$$

$$\tilde{\phi}_{Bv} = e^{-|y|K}\ \tilde{\psi}_1, \tag{3.5}$$

$$\tilde{\phi}_S = \frac{\overset{+}{e}{}^{ipx_0 + iqz_0}}{(s + i\omega)} \int^{y} dy_1 \int^{y_1} dy_2\ e^{-||y| - 2|y_1| + |y_2||K}\ \delta(y_2 - y_0) \tag{3.6}$$

$$(\tilde{\phi}, \tilde{\psi}_1) = \int_0^\infty dt \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dz e^{-st-i(qz=px)} (\phi, \psi_1)(x,z,t) \quad (3.7)$$

We evaluate the integrals in (3.6). For this we assume that $y_0 > 0$. If the lower bounds in (3.6) are taken $+\infty$, then we find

$$\tilde{\phi}_S^+ = \frac{1}{K(s+i\omega)} e^{i(px_0+qz_0)} [1 - e^{-(|y_0| - |y|)K}], |y| < |y_0 \quad (3.8)$$

But if the lower bounds are taken $-\infty$, then we find

$$\tilde{\phi}_S^- = \frac{1}{K(s+i\omega)} e^{i(px_0+qz_0)} [1 - e^{-(|y| - |y_0|)K}], |y| > y_0 \quad (3.9)$$

Thus it is convenient to take $\tilde{\phi}_S = 1/2\ (\tilde{\phi}_S^+ + \tilde{\phi}_S^-)$, where $\pm$ corresponds to the choiceof the lower limits as $\pm\infty$ in (3.6). Consequently we have

$$\tilde{\phi}_S = \frac{e^{i(px_0+qz_0)}}{K(s+i\omega)} [1 - e^{-|\,|y| - |y_0||K}], \quad (3.10)$$

and this result is valid for all values of y.

We write $\tilde{\phi} = \tilde{\phi}^+ + \tilde{\phi}^-$, where $\tilde{\phi}^+$ and $\tilde{\phi}^-$ correspond to the integral over x on $(0, \infty)$ and $(-\infty, 0)$ respectively (cf.– (3.7)). We use the Wiener - Hopf technique and have for the boundary conditions (2.4-5)

$$\tilde{\phi}^+ (0^+) = \tilde{\phi}^+ (0^-) = \tilde{\phi}^+ (0) \quad (3.11)$$

$$\partial_y \phi^+ (0^+) = \partial_y \tilde{\phi}^+ (0^-) = \partial_y \tilde{\phi}^+ (0) \quad (3.12)$$

$$\partial_y \tilde{\phi}^- (0^+) + \partial_y \tilde{\phi}^+ (0^+) - \beta \left(\frac{s}{c} + iMp\right) (\tilde{\phi}^- (0^+) + \tilde{\phi}_S (0^+)) = 0 \quad (3.13)$$

$$\partial_y \tilde{\phi}^- (0^-) + \partial_y \tilde{\phi}_S (0^-) + \beta \left(\frac{s}{c} + iMp\right) (\tilde{\phi}^- (0^-) + \tilde{\phi}_S (0^-)) = 0 \quad (3.14)$$

Now, we use the boundary conditions (2.5) into (3.4-5), and have

$$\tilde{\phi}^+ (0^+) + \tilde{\phi}^- (0^+) = \tilde{\psi}_1 + \tilde{\phi}_S (0^+) \quad (3.15)$$

$$\tilde{\phi}^{+}(0^{-}) + \tilde{\phi}^{-}(0^{-}) = \tilde{\psi}_{I} + \tilde{\phi}_{S}(0^{-}) \tag{3.16}$$

$$\partial_y \tilde{\phi}^{+}(0^{+}) + \partial_y \tilde{\phi}^{-}(0^{+}) = -K\,\tilde{\psi}_{I} + \partial_y \tilde{\phi}_{S}(0^{+}) \tag{3.17}$$

$$\partial_y \tilde{\phi}^{+}(0^{-}) + \partial_y \tilde{\phi}^{-}(0^{-}) = K\,\tilde{\psi}_{I} + \partial_y \tilde{\phi}_{S}(0^{-}) \tag{3.18}$$

As $\tilde{\phi}_S(0^+) = \tilde{\phi}_S(0^-)$, then from (3.15) and (3.16), we obtain

$$\tilde{\phi}^{-}(0^{+}) = \tilde{\phi}^{-}(0^{-}) \tag{3.19}$$

Also, as $\partial_y \tilde{\phi}_S(0^+) = -\partial_y \tilde{\phi}_S(0^-)$, then from (3.17) and (3.18), we have

$$\partial_y \tilde{\phi}^{-}(0^{+}) + \partial_y \tilde{\phi}^{-}(0^{-}) + 2\,\partial_y \tilde{\phi}^{+}(0) = 0 \tag{3.20}$$

From (3.13) and (3.14), we obtain

$$\partial_y \tilde{\phi}^{-}(0^{+}) = -\,\partial_y \tilde{\phi}^{-}(0^{-}) \tag{3.21}$$

Combining (3.20) and (3.21), we have

$$\partial_y \tilde{\phi}^{+}(0) = 0 \tag{3.22}$$

Now, we consider (3.13), (3.15) and find

$$\partial_y \tilde{\phi}^{-} - (0^{+}) + \partial_y \tilde{\phi}_S(0^{+}) - \beta\left(\frac{s}{c} + iMp\right)[\tilde{\psi}_I + \tilde{\phi}_S(0)] = 0 \tag{3.23}$$

Eliminating $\partial_y \tilde{\phi}^{-}(0^+)$ between (3.23) and (3.17), we find

$$-\partial_y \tilde{\phi}_S(0^{+}) + \beta\left(\frac{s}{c} + iMp\right)(\tilde{\psi}_I + \tilde{\phi}_S(0)) = -\,K\tilde{\psi}_I + \partial_y \tilde{\phi}(0^{+}). \tag{3.24}$$

Solving for $\tilde{\psi}_I$, we find

$$\tilde{\psi}_I = \frac{1}{K^{+}}\left[2\partial_y \tilde{\phi}_S(0^{+}) - \beta\left(\frac{s}{c} + iMp\right)\tilde{\phi}_S(0^{+})\right]. \tag{3.25}$$

Or by using (3.10), we obtain

$$\tilde{\psi}_I = \frac{e^{i(px_0+qz_0)}}{K^{+}(s+i\omega)}\left[2e^{-|y_0|K} - \frac{\beta}{K}\left(\frac{s}{c} + iMp\right)(1-e^{-|y_0|K})\right], \tag{3.26}$$

where $K^{+} = K + \beta\,(\frac{s}{c} + iMp)$

By substituting from (3.26) into (3.5) and (3.14), we find

$$\tilde{\phi} = (s+i\omega)^{-1}\, e^{i(px_0 + qz_0)}\, \Lambda, \tag{3.27a}$$

$$\Lambda = (K\,K^{+})^{-1}\,[(K+K^{+})\, e^{-A_0K} - \beta(\frac{s}{c} + iMp)\, e^{-A_1K} + K^{+}\,(1-e^{-A_1K})], \tag{3.27b}$$

where $A_0 = |y_0| + |y|,\ A_1 = |\,|y_0| - |y|\,|,\ A_2 = |y|,$

We remark that the last term in (3.27b) gives the field of the incident wave. While the remaining terms give the field of the diffracted wave

The inverse Fourier-Laplace transform of $\tilde{\phi}$ is

$$\phi = \int_D ds \int\int \frac{dpdq}{i\,(2\pi)^3}\,(s+i\omega)^{-1}\, e^{+ip(x-x_0) + q(z-z_0) + st}\,\Lambda, \tag{3.28}$$

where $D = \{(s,p,q) : \mathrm{Re}k^2 = \mathrm{Re}\,[p^2 + q^2 + (\frac{s}{c} + iMp)^2] > 0\}$.

In fact as $|M| < 1$, then $\mathrm{Re}\,K^2 > 0$ holds every where when $-\infty < p,\ q < \infty$ and $\mathrm{Re}\ s > 0$. Now by making the transformations $s = s_1 - iMcP$, $P = R\cos\theta$, $q = R\sin\theta$ and carrying out the integral over R by using the steepest descent method, we find

$$\phi = \int_{\varepsilon-i\infty}^{\varepsilon+i\infty} \frac{ds_1\,\sqrt{s_1/c}}{(2\pi)^{5/2}} \int_0^{2\pi} d\,\theta\, R_0^{*} \sum_{j=0}^{2} \Lambda_j^{*}\,(1 - \frac{M\cos\theta R_0^{*}}{R_{0j}^{*}})^{-1} \tag{3.29}$$

$$\Lambda_0^{*} = -\frac{(\beta+2A_0/R_{00})\,(s_1+i\omega_0^{*})^{-1}}{R_{00}^{*3/2}\,(\beta+A_0/R_{00}^{*})}\, e^{-S_1R_{00}^{*}/c}, \tag{3.30}$$

$$\Lambda_2^{*} = \frac{\beta\,(S_1+i\omega_0^{*})^{-1}}{R_{00}^{*3/2}\,(\beta+A_2/R_{02}^{*})}\, e^{-S_1R_{02}^{*}/c}, \tag{3.31}$$

$$\Lambda_1^{*} = \frac{(1+M\cos\theta R_0^{*}/R_{01}^{*})}{(s_1/c)^{3/2}\,(s+i\omega)R_0^{*3}} + \frac{(S_1+i\omega_1^{*})^{-1}}{R_{01}^{*\,3/2}}\, e^{-S_1R_{01}^{*}/c}, \tag{3.32}$$

where $R^*_0 = x^* \cos\theta + (z-z_0)\sin\theta$, $R^{*2}_0; = R^*_0 + Aj^2$, $\omega^* j = \omega/$

$(1-M\ R^*_0 \cos\theta / R^*_{0j})$, $j=0,1,2$, $x^* = x-x_0-Mct$.

We evaluate the integral in the s_1– plane and obtain for the field of the incident and diffracted wave, ϕ_I and ϕ_D respectively:

$$\phi_I \sim \frac{ie^{-i\omega t}}{2\pi^{3/2}} \int_0^{2\pi} d\theta\ R^{*-2}_0 + \frac{1}{\sqrt{c}\,(2\pi)^{3/2}} \int_0^{2\pi} \frac{d\theta \sqrt{\omega^*_1} e^{-i\omega^*_1 t_1} F(\omega^*_1 t_1) \cdot R^*_0}{R^{*3/2}_{01}(1-M\,R^*_0\cos\theta/R^*_{01})} \quad (3.33)$$

$$\phi_D \sim \int_0^{2\pi} d\theta \left[\frac{-\sqrt{\omega^*_0}(\beta+2A_0/R^*_{00})\, e^{-iw^*_0 t_0} F(\omega^*_0 t_0)}{R^{*3/2}_{00}(\beta+A_0/R^*_{00})(1-MR^*_0\cos\theta/R^*_{00})} \right.$$

$$\left. \frac{+\beta\sqrt{\omega^*_2}\, e^{-i\omega_2 t_2} F(\omega^*_2 t_2)}{R^{*3/2}_{02}(\beta+A_2/R^*_{02})\left(1-\frac{MR^*_0\cos\theta}{R^*_{02}}\right)} \right] \frac{R^*_0}{(2\pi)^{3/2}\sqrt{c}} \quad (3.34)$$

where $t_j = t - R^*_{0j}/c$. Finally, when evaluating the integral over θ by the method of stationary phase, we obtain

$$\phi_I \sim + \frac{e^{-i\omega_1 t_1 + \pi i/4} F(\omega_1 t_1)}{2\pi R_{01}(1-Mx^*/R_{01})} \quad (3.35)$$

$$\phi_D \sim \frac{e^{-\pi i/4}}{2\pi} \left\{ - \frac{(\beta+2A_0/R_{00})\, e^{-i\omega_0 t_0} F(\omega_0 t_0)}{R_{00}(\beta+A_0/R_{00})(1-Mx^*/R_{00})} \right.$$

$$\left. \frac{+\beta\ e^{-i\omega_2 t_2} F(\omega_2 t_2)}{R_{02}(\beta+A_2/R_{02})(1-Mx^*/R_{02})} \right\} \quad (3.36)$$

where $\omega_j = \omega/(1-\frac{x^*M}{R_{0j}})$, $R^2_{0j} = x^{*2} + (z-z_0)^2 + A^2_j$, and $F(\lambda) = \int_0^{\lambda^{1/2}} \frac{2e^{i\mu^2}}{\sqrt{\pi}} d\mu$.

The results (3.35.36) for the far field ϕ reveal some important physical interpretations:

(i) The field ϕ_D behaves merely as the inverse of the distance from the point source as if it moves with speed Mc (=U) far from the barrier.

(ii) The frequency shift of the sound wave due to diffraction by the barrier is given by $\Delta\omega/\omega = Mx^*/(R_{0j} - Mx^*)$ which is spacetime dependent.

(iii) The phase of the wave has a strong nonlinear time dependence in contrast to the result of the previous work (Asghar 1991), where it was found that it is linear in time.

(iv) In view of (3.36), one finds that the principle of limiting amplitude holds (Morawetz 1962) while the principle of limiting absorption (Eidus 1962) does not hold. This is in contrast to the results found previously (Rawlins 1974, 1975), where the two principles hold. This disagreement results from the fact that in the previous works, the source was assumed permanently harmonic. Consequently, the filed of the diffracted wave is also permanently harmonic. Here, the source is taken as setting in at time t = 0. Hence, it is necessary that the solution would be anharmonic.

In the next section we study diffraction of acoustic waves by a nearly half-plane.

Diffraction of acoustic waves by a nearly half-plane

We suppose that the boundaries of the barrier are given by $y = \pm\varepsilon g(x,z)$, $x<0$; where $g(x,z) \in L_1\, [(-\infty, O) \times (-\infty, \infty)]$ and is a positive definite function which is bounded by unity. Here, we rewrite the solution of (2.3) as

$$\phi^* = \int_{a-i\infty}^{a+i\infty} ds \iint_{-\infty}^{+\infty} \frac{dpdq}{i(2\pi)^3}\, e^{st+ip(x-x_0)+iq(z-z_0)} \Bigg[e^{-|y|K}\, \tilde{\psi}_1^* + \frac{[1-e^{-|\,|y_0|-|y|\,|K}]}{(s+i\omega)K} \Bigg] \tag{4.1}$$

We notice that, for solving this problem, the boundary conditions (2.4-3) have to be modified by terms of order ε. When evaluating the far filed of the diffracted wave, these terms would produce corrections of order ε^2. So that, we confine ourselves with the conditions (2.4-5) as we shall conserve only terms of order ε.

Now, we evaluate ϕ^* $(y = \varepsilon g\,(x,z))$ by substituting for $y = \varepsilon g\,(x,z)$ into (4.1) and expanding $e^{-\varepsilon g(x,z)K}$ up to first order in ε. Finally, we find the Fourier-Laplace transform of both sides of (4.1). We write $\tilde{\psi}_1^* = \tilde{\psi}_1 + \varepsilon\tilde{\psi}$, where $\tilde{\psi}_1$ is given by (3.25) and have

$$\tilde{\phi}^*\,(y = \varepsilon g\,(x,z)) = \tilde{\phi}\,(y = 0^+) + \varepsilon\tilde{\psi}_E - \varepsilon\tilde{H}_0, \tag{4.2a}$$

$$\tilde{H}_0 = \int\int_{-\infty}^{\infty} \frac{dp_1 dq_1 E_1}{(2\pi)^2} \; [e^{-|y_0|K_1} \; \tilde{\phi}_S(0) + K_1 \psi_1(p_1, q_1, s)] g\,(p-p_1, q-q_1) \tag{4.2b}$$

where E_1, $= \exp -i(p_1, x_0+q_1, z_0)$. Also, we have

$$\partial_y \tilde{\phi}^* (y = \varepsilon g(x,z)) = -K\tilde{\phi}(y = 0^+) - \varepsilon E K \tilde{\psi} + \varepsilon \tilde{H}_1 , \tag{4.3a}$$

$$\tilde{H}_1 = \int\int_{-\infty}^{\infty} \frac{dp_1 dq_1 K_1 E_1}{(2\pi)^2} \; [- e^{-|y_0| K_1} \; \phi_S(0) + \psi_1(p_1, q_1, s)] g\,(p-p_1, q-q_1) \tag{4.3b}$$

where $K_1 = K\,(p \to p_1 , q \to q_1)$ and K is given as before.

Combining (3.13) (evaluated at $y = \varepsilon g(x,z)$ and (4.2), we obtain an equation similar to (3.23) but the right hand side of (4.2) replaces the terms in the last brackets. Finally we have

$$-K\tilde{\phi}(0^+) - \varepsilon E\, K\, \tilde{\Psi} + \varepsilon H_1 - \beta\,(\frac{s}{c} + iMp)\,(\tilde{\phi}(0^+) + \varepsilon\tilde{\psi}E - \varepsilon H_0) = 0. \tag{4.4}$$

Solving (4.4) for $\tilde{\psi}$, we find

$$EK^+\tilde{\psi} = \beta\,(\frac{s}{c} + iMp)\,\tilde{H}_0 + \tilde{H}_1, \tag{4.5}$$

$$\tilde{\psi} = \frac{1}{K^+} \int\int_{-\infty}^{\infty} \frac{dp_1 dq_1 E^*}{(2\pi)^2} \; [- K^-_1 e^{-|y_0|K_1} \; \phi_S(0) + K_1 K^+_1 \tilde{\psi}_1(p_1, q_1, s)] g\,(p-p_1, q-q_1) \tag{4.6}$$

where $K^{\pm}_1 = K_1 \pm \beta\,$ (s + iMp) and $E^* = \exp i\,((p-p_1)x_0 = (q-q_1)z_0)$.

By substituting from (4.6) into (4.1), bearing in mind the results (3.35-36), we find

$$\phi^* \sim \phi_1 \,[\, 1 - \frac{\varepsilon\omega_1 A_1}{cR_{01}} \; g(x^{(1)}, z^{(1)}] = \phi_D^{(2)} \;\; [1 - \frac{\varepsilon\omega_0 A_0}{cR_{01}}\, g(x^{(0)}, z^{(0)}]$$

$$+ \;\; \phi_D^{(2)} \;\; [1 - \frac{\varepsilon\omega_2 A_2}{cR_{02}}\, g(x^{(2)}, z^{(2)}] \tag{4.7}$$

where $\phi_D^{(1)}$ and $\phi_D^{(2)}$ stand for the first and second parts of (3.36) respectively while ϕ_I is given by (3.35). Also $x^{(j)} = (x - x^*c/\omega_j R_{0j})$ and $z^{(j)} = (z - (z-z_0)c/\omega_j R_{0j})$.

After the result (4.7), we find that whenever $g(x,z) > 0$, acoustic noise reduction by a nearly half-plane barrier is better than reduction a half-plane one. In fact we have $| \phi^*_D | < | \phi_D|$ if $g(x,z) > 0$. Also, waves diffracted by a nearly half-plane and whose far field are vanishingly small, are coming from a point source with high frequency; namely when $\omega = O(c/\varepsilon)$.

Finally, the result (4.7) is valid for a barrier in the form of a half-plane but with thickness 2ε when $g(x,z) = 1 \leq 0, -\infty < z < \infty$.

Conclusion

By using a recently developed method, we studied the problem of reduction of acoustic noise by barriers in the shape of a half-plane and a nearly half-plane.

The anharmonic far field has been found in both two cases. It is shown that the far field behaves mainly as the inverse of the distance from the point source as if it moves at the fluid speed far from the barrier. Consequently, the field of the diffracted wave tends to zero as $t \rightarrow \infty$. This result was not found in the literature. This disagreement results from the fact that in these works, the source was assumed permanently harmonic. It is worth noteing that the problem studied in this paper is different from that studied there. Indeed, here, the source is taken as setting in at time $t = 0$. Furthermore, we have found that acoustic noise reduction by a nearly half-plane is better than reduction by a half-plane one. In particular, when acoustic waves are generated by a point source of high frequency. Finally, our method based on the use of fractional power operators facilitates the study of the problem that considered here. We think that this problem can not be easiling the known techniques.

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حل غير توافقي لمسئلة إنكسار الموجات الصوتية

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درس إنكسار الموجات الصوتية المنبعثة من مصدر للصوت بواسطة نصف مستوى ماص أو شبه نصف مستوى معادلته $0\leq\varepsilon<<1,\ y=\varepsilon g(x,z);\ x\leq 0$ في وجود وسط مائع متحرك . وقد تم هذا بإستخدام طريقة مطورة حديثا بواسطة الباحث ، قد أفترض خلال هذه المقالة أن منبع الصوت قد بدأ عندما $t=0$. عندما $\varepsilon=0$ قد أوضحنا أن المجال البعيد للموجة المنكسرة غير توافقي ، أنه يتناسب عكسياً مع المسافة من مصدر الصوت لكن كما لو كان مصدر الصوت - يتحرك بعيدا عن الحاجز الماص للصوت من ناحية أخرى قد وجد أن تخفيض الضوضاء الصوتية بواسطة حواجز على شكل شبه نصف مستوى $0\leq g(x,z)\leq 1,\ \ y=\varepsilon g\ (x,z)$ أفضل من التخفيض بإستخدام حواجز على شكل نصف مستوى ، قد أثبتنا أيضا أن شدة الموجة الصوتية المنكسرة تؤول إلى الصفر عندما $t\to\infty$ وذلك على خلاف ماهو موجود في الأبحاث ، ذلك يوضح أن مبدأ محدودية الشدة الموجية يتحقق بينما لايتحقق مبدأ محدودية قدرة الأمتصاص .

لإجراء هذا البحث قد إستخدمنا طريقة جديدة مبتكرة بواسطة الباحث

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وذلك بإدخال مفهوم المؤثرات التفاضلية ذات الرتب الكسرية وقد قمنا بتفصيل الحاسابات حتى يسهل للباحثين إعادة إستخدامها في أبحاث أخرى نظراً لعدم إتصال الشروط الحدية . قمنا بإستخدام طريقة Hopf-Wiener لتعيين الدوال الإختيارية في الحل العام