Arab Gulf J. Scient. Res. **13**(3), pp.481-497 (1995)

A Computation Method for Calculating Outflow Drainage From Initially-Saturated Stratified Soil

S. A. Awadalla[1] and S. A. Gutub[2]

[1]Department of Hydrology and Water Resources Management, Faculty of Meterology Environment and Arid Land Agriculture,
[2]Civil Engineering Department, King Abdulaziz University, P.O. Box 9027, Jeddah 21413, Saudi Arabia

ABSTRACT. Land drainage is a common practice in agricultural areas where excess water is to be drained to maintain the necessary wetting state for healthy plant roots and to prevent rotting. This requires an understanding of the infiltration process. Several researchers have dealt with the situation analytically and numerically. Their methods of solution varied according to their representation of the physical process of water percolation through soil. In this paper, an attempt is made to demonstrate the capability of a numerical model to describe the stratified drainage process. The output from the numerical analysis is compared with the analytical solution utilizing the Green and Ampt approach. The comparison shows good agreement especially during the early-stage of drainage.

Watson and Awadalla (1985) described a comparative study of the Green and Ampt analysis for the gravity drainage of a homogeneous profile of porous material under falling water table conditions. Appropriate equations of the Green and Ampt were developed for a regime where the water table fell suddenly at zero time and then continued to fall at a uniform rate thereafter. The direct application of such analysis is made for the case of an unconfined aquifer, where, as soon as pumping commences, there is an instantaneous drop in the water table followed by a continuing gradual decline. The case presented here is based on the drainage from an initially saturated profile to a fixed water table. The main application can be noted in the land drainage context.

Watson and Whisler (1976) applied the sharp front analysis for stratified materials to a coarse over fine sequence and included different ratios of layer thicknesses.

Aggellides and Youngs (1978) found that the estimated soil water pressure at the wetting front were generally more negative than the corresponding values obtained by direct measurements of parameters.

Brakensiek (1977) describes an alternative method for determining the average wetting front capillary pressure parameter. Only a moisture characteristics would be required.

In this paper, the general approach, given by Watson and Awadalla, (1985) outlined above, is again applied to fixed water table conditions for the case of a stratified profile. Two surface boundary conditions can be considered in the analysis, namely the zero surface flux and the applied constant surface flux. The zero flux condition has been applied in this analysis.

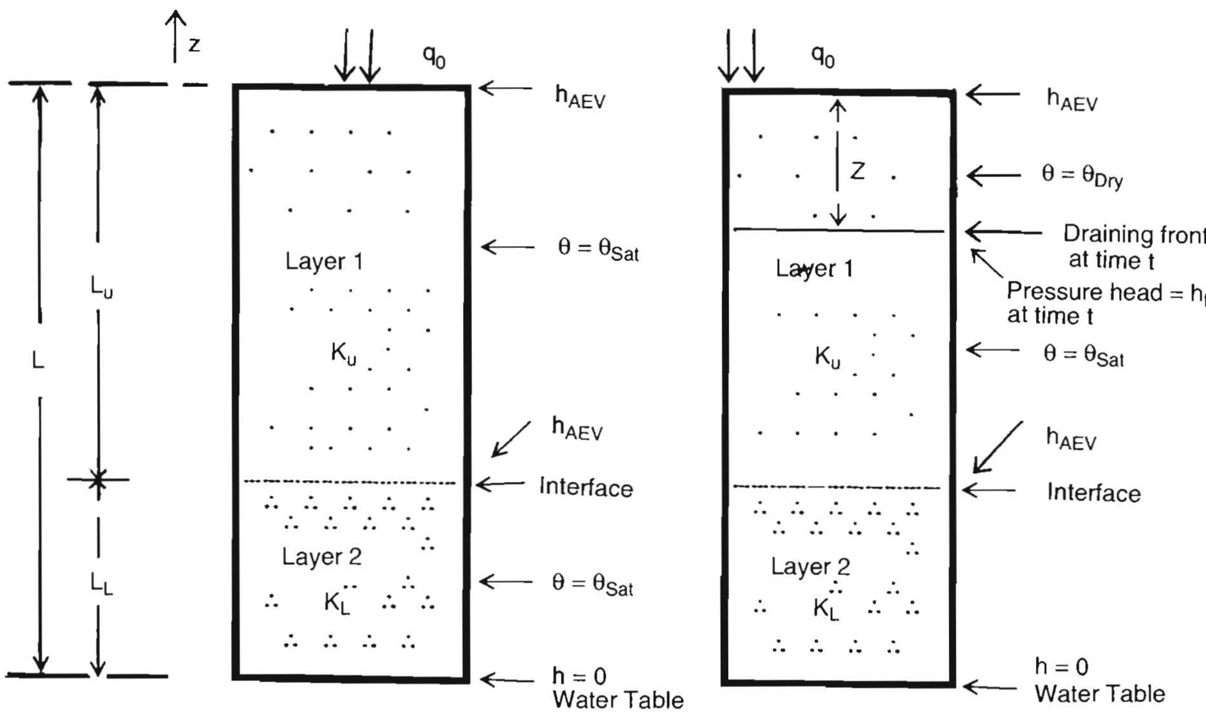
Analytical Solution

In applying the Green and Ampt equation to the drainage regime, the assumption is made that saturated material yet to drain and fully drained material above it are separated by a sharp front where the soil water pressure head is h_f. Let K_u and K_L represent the saturated hydraulic conductivities of the upper and lower layers, respectively, and L_u and L_L the thicknesses of these layers as shown in Fig. 1. The vertical ordinate z is measured positively upwards from the water table and h=0 at z=0. At time t=0 when the surface is about to drain, the pressure head at the interface between the layers can be determined from Darcy's law as follows:

$$h_{io} = \frac{h_f K_u L_L + L_L L_u (K_u - K_L)}{K_u L_L + K_u L_u} \qquad \text{.......... (1)}$$

At any time t when the front is at height z and $Z_L > L$, the pressure head at the interface is,

$$h_i = \frac{h_f K_u L_L + L_L (z - L_L)(K_u - K_L)}{z K_L + L_L + L_L (K_u - K_L)} \qquad \text{.......... (2)}$$



(a) Initial setting of the soil column at t = 0.

(b) Setting of the column at a later time t > 0.

Fig. 1. Definition of parameters used in the Green and Ampt analysis falling water table in stratfied profile.

In Equation (1) and (2) h_f will be negative in value; h_{io} will be when L_u $(1 - K_L / K_u)$ $> |h_f|$, and h_i will be positive when $(z-L_L)(1-K_L / K_u) = |h_f|$. In this study $K_u > K_L$ hence $(1 - K_L / K_u) > 0$.

If the difference in water content across the well-defined front is $\Delta\theta$, then by equating the flux resulting from the fall of the drainage front with the flux through the impeding layer, the following can be written,

$$\Delta\theta\,(dz/dt) = -K_L\,(L_L + h_f) / L_L \quad \text{.......... (3)}$$

By substituting for h_f from equation (2)

$$\Delta\theta\, dz/t = \frac{-K_L\,[h_f\,K_u + z\,K_u]}{z\,K_L + L_L\,(K_u - K_L)} \quad \text{.......... (4)}$$

then,

$$-dt = \frac{\Delta\theta}{K_L}\;\frac{K_L\,(z-L_L) + K_u\,K_L\,dz}{K_u\,(h_f + z)} \quad \text{.......... (5)}$$

Equation (5) can be rewritten as,

$$\int_t^o dt = \frac{-\Delta\theta}{K_u}\int_z^L \frac{[z - L_L]}{h_f + z}\,dz + \frac{K_u\,L_L}{K_L}\int_z^L \frac{1}{h_f + z}\,dz \quad \text{.......... (6)}$$

Based on integration tables the following relations can be applied:

$$\int \frac{x}{a+x}\,dx = a + x - a\ln(a+x) + \text{constant}$$

$$\int \frac{1}{a+x}\,dx = \ln(a+x) + \text{constant}$$

The integration equation (6) yields,

$$t = \frac{\Delta\theta}{K_u}\left[(L-z) + \left\{h_f + L_L\,(1-K_u/K_L)\right\}\ln\left[\frac{z + h_f}{L_f + h}\right]\right] \quad \text{.......... (7)}$$

where $L = L_u + L_L$

For a uniform column where $K_u = K_L = K$ and $L_L\ (1 - K_u/K_L) = 0$, it can be stated that,

$$t = \left[(L-z) = h_f \quad \ln\left[\frac{z + h_f}{L + h_f} \right] \right] \qquad \text{.......... (8)}$$

Equation (7) is normally solved to obtain the time when the drainage front reaches any prescribed value of z.

Numerical Solution

The moisture content distribution was simulated in a drainage stratified column of soil consisting of two layers with variable thicknesses and with a water table located at the bottom of the lower layer and under the influence of a recharging boundary specified at the ground surface, *i.e.*, with zero flux. A one-dimensional, transient, unsaturated flow model was developed to study the infiltration process into the soil column. The model is based on the Galerkin finite element approximation and uses elements with linear shape functions. The model solves the modified Richard's equation in one dimension for the change of pressure. The nonlinearity of the solution was treated using the Newton-Raphson method. The model uses upstream weighting and mass lumping procedures in an effort to maintain accuracy, smoothness, and stability of the solution.

Formulation of the Model

The equation describing the flow in the vertical direction in a slightly compressible soil medium is presented by Huyakorn and Pinder (1983) as follows,

$$\frac{\partial}{\partial Z} [K_{zz}\, k_r\, (\frac{\partial h}{\partial z} + 1)] = \eta \frac{\partial h}{\partial t} \qquad \text{.......... (9)}$$

where,

- h = pressure head
- k_r = relative permeability
- $\eta = C + Ss\ S$
- C = specific moisture capacity
- S_s = specific storage
- S = saturation
- K_{zz} = hydraulic conductivity in z-direction

Using the Galerkin Technique in which the weighting function equals the basis function, and applying Green's theorem and the product rule, the following typical finite element equation is obtained,

$$A_{IJ}\ h_j + B_{IJ}\ \frac{d\,h_j}{d\,t} - F_I = 0 \qquad \text{(10)}$$

which is a set of nonlinear, time-independent integro-differential equations, with the matrix coefficients defined as,

$$A_{IJ} = \sum_e A^e_{IJ} = \sum_e \iint_{R^e} K_{ij}\ k_{rw}\ \frac{\partial N_I}{\partial x_i}\ \frac{\partial N_J}{\partial x_j}\ dR \qquad \text{(11)}$$

$$B_{IJ} = \sum_e B_{IJ}{}^e = \sum_e \iint_{R^e} \eta\, N_I\ N_J\ \ dR \qquad \text{(12)}$$

$$F_I = \sum_e F_I{}^e = \sum_e \iint_{R^e} - K_{ij}\ k_{rw}\ \frac{\partial N_I}{\partial x_i}\ e_j\ dR - \sum_e \int_B q_n\ N_I\ dL \pm$$

$$\sum_e N_I\ Q_s\ dL \qquad \text{(13)}$$

Applying the Newton-Raphson technique, the following Finite Element governing equation in the unsaturated zone of the porous media is obtained,

$$\{\ A^e_{IJ} + A^{e*}_{IJ} + B^{e*}_{IJ} + B^{e***}_{IJ} - B^{e***k}_{IJ} + F^{e*}_{IJ}\}^r\ \Delta\, h^{r+1}_J$$

$$= \{\ A^e_{IJ}\ h_J + B^{e*}_{IJ} h_J\ - B^{e*}_{IJ}\ h^k{}_J - F^e{}_I\ \}^r \qquad \text{(14)}$$

where r is the iterative level at the current time.

The matrix coefficients are defined as follows,

$$A^e_{IJ} = \frac{K_{zz}}{l_e}\ k^w_r\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad \text{(15)}$$

$$B^e = \frac{l_e}{6}\ \bar{\eta}\ \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \qquad \text{(16)}$$

$$F^e_I = \begin{bmatrix} Q \\ -Q \end{bmatrix} - \begin{bmatrix} -K_{zz}\ k^w_r \\ K_{zz}\ \ k^w_r \end{bmatrix} \qquad \text{(17)}$$

where k^w_r is the weighted relative permeability.

See Appendix for the definitions of A^{e*}_{IJ}, B^{e*}_{IJ}, B^{e***}_{IJ}, B^{e***k}_{IJ}, and F^{e*}_{IJ} .

The boundary conditions used for simulating gravity drainage to a fixed water table in the numerical analysis were as follows:

(1) upper boundary zero surface flux

$$K(h)\,(\partial h / \partial z + 1) = 0, \quad t > 0 \text{ and} \qquad (18)$$

(2) lower water table boundary

$$h(z,t)_{z=L} = 0, \quad t > 0 \qquad (19)$$

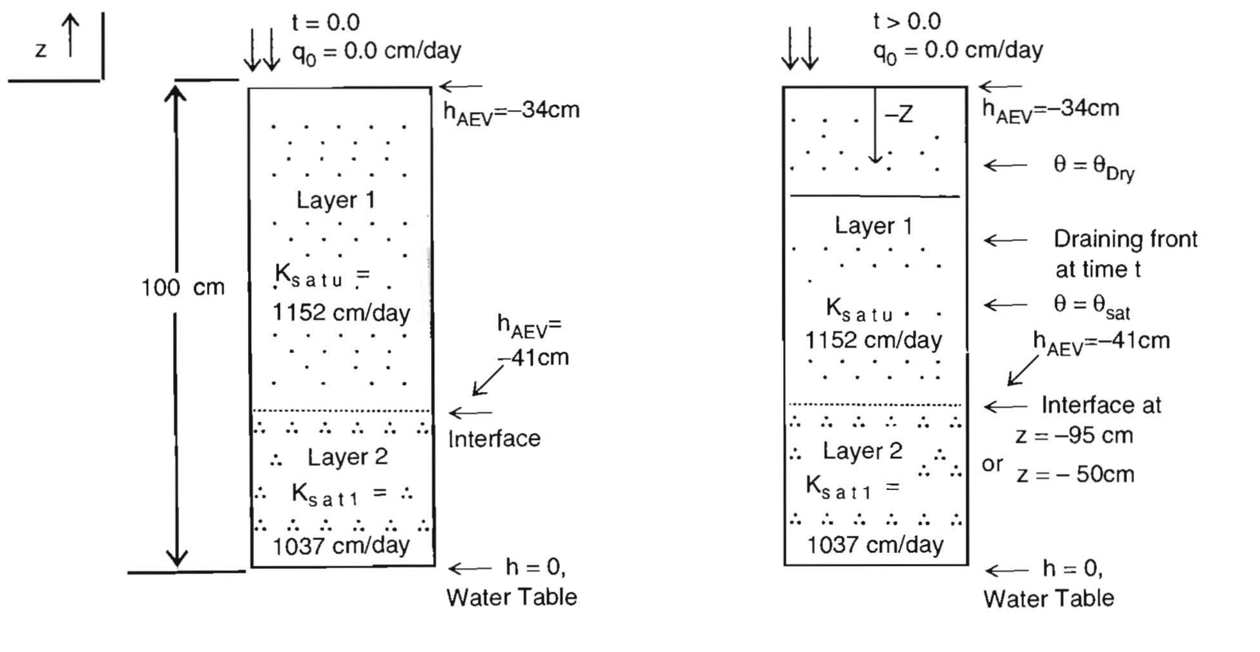
The form of the initial pressure head distribution used in the analysis was assumed to vary linearly from zero at the water table to the air entry pressure at the soil surface. A linear characteristic curve describing the soil properties has been implemented in the solution and is presented as,

$$S(h) = 1 - (1 - S_r)\, h / h_r \qquad (20)$$

where h_r is the pressure head at residual saturation.
and S_r is the residual saturation.

Problem Description

A vertical column of soil 100 cm high is shown in Fig. 2, with an entry pressure head specified at the ground surface and an initial linear pressure head distribution from the ground surface to the water table, located at the bottom boundary (*i.e.* a specified pressure head, h=0.0). A flux of 0.0 $cm^3/day/cm^2$ is applied at the inlet and zero pressure head at outlet. The soil properties and some parameters used in the model are presented in Table 1.



(a) Initial setting of the soil column at t = 0.

(b) Setting of the column at a later time t > 0.

Fig. 2. Problem illustration.

Table 1. Simulation Parameters and Soil Properties

Parameters	Values
Ksatu	1152 cm/day
ksat l	1037 cm/day
Hr u	–100 cm
Hr l	–110 cm
Sr u	0.01
Sr l	0.02
øu	0.292
øl	0.310
Hd u	–34 cm
Hd l	–41 cm
λu	2.00
λl	1.80

Results and Discussion

For stratified profiles the Green and Ampt analysis must be considered in two parts. The first part is defined as the time required for the drainage front to reach the interface. The second part occurs as soon as the drainage front reaches the interface. The Green and Ampt assumptions postulate, first, an instantaneous completion of the drainage of the upper layer, and, second, an immediate commencement of the drainage of the lower layer. In addition, the drainage of the lower layer is purely of homogenous material type, and as such, the equations given by Watson and Awadalla (1985) for homogenous material may be used.

The approach adopted in latter paper in relation to the inclusion of the parameter L, namely, the instantaneous fall of the water table at t=0, is particularly significant in facilitating the application of the Green and Ampt equations to the lower layer.

The sand used for the profile above the impeding layer is Botany Sand. The saturated conductivity Ku is 0.8 cm/min with air entry pressure of –34.0 cm. The case considered in the numerical analysis is zero flux at the soil surface. The results of the numerical analysis in the form of pressure head versus depth, and water content versus depth is presented in Figures 3,4,5 and 6. The question of interest concern the applicability of the approximate equation (7) to the drainage of stratified profiles.

Figures 7 and 8 show the relation between the cumulative drainage outflow and time for the No. 17 sand/R8A sequence for q=0 for both the Green and Ampt and numerical results at z = –95 and –50 cm respectively.

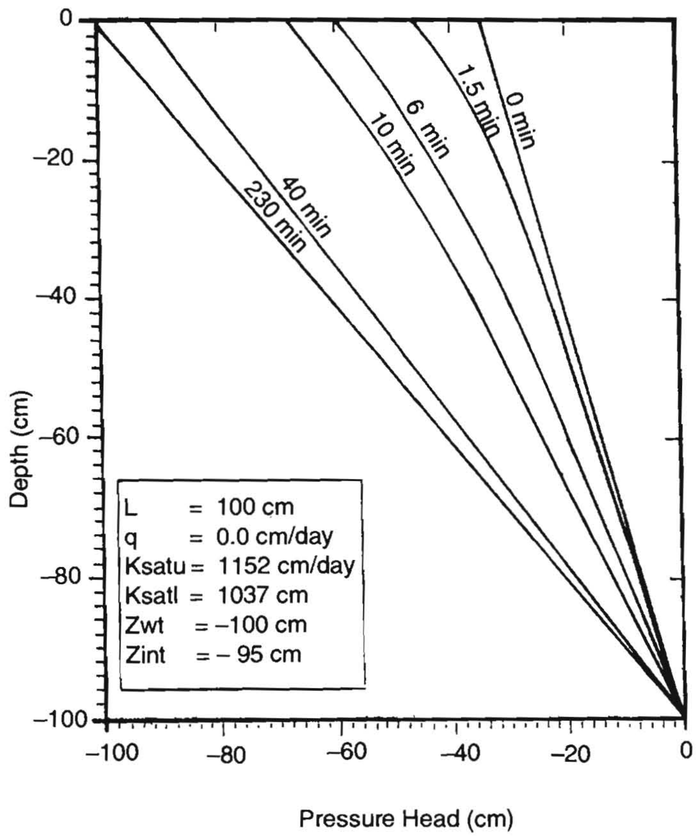


Fig. 3. Pressure Head Profile in a Stratified Column.

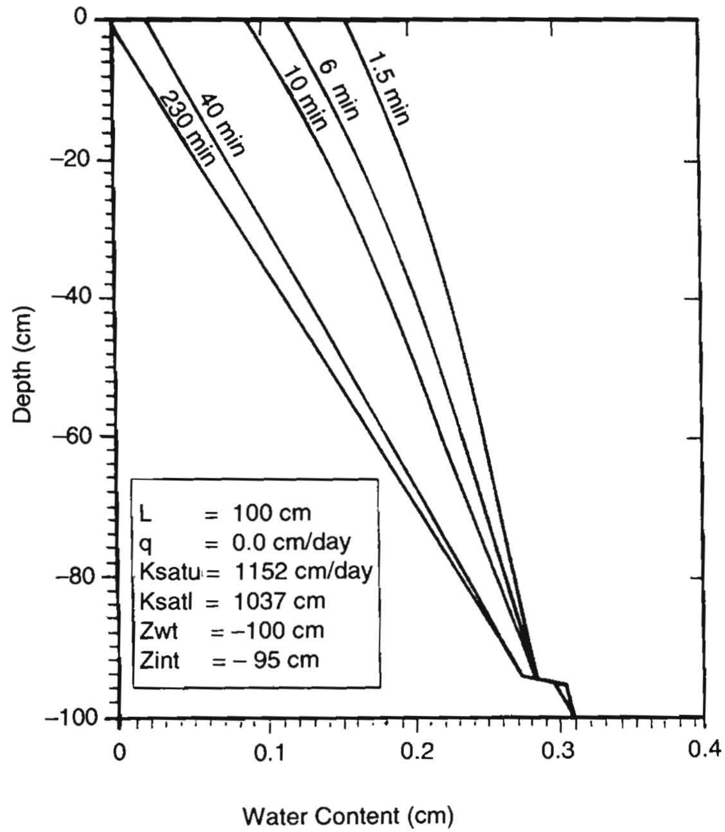


Fig. 4. Water Content Profile in a Stratified Column.

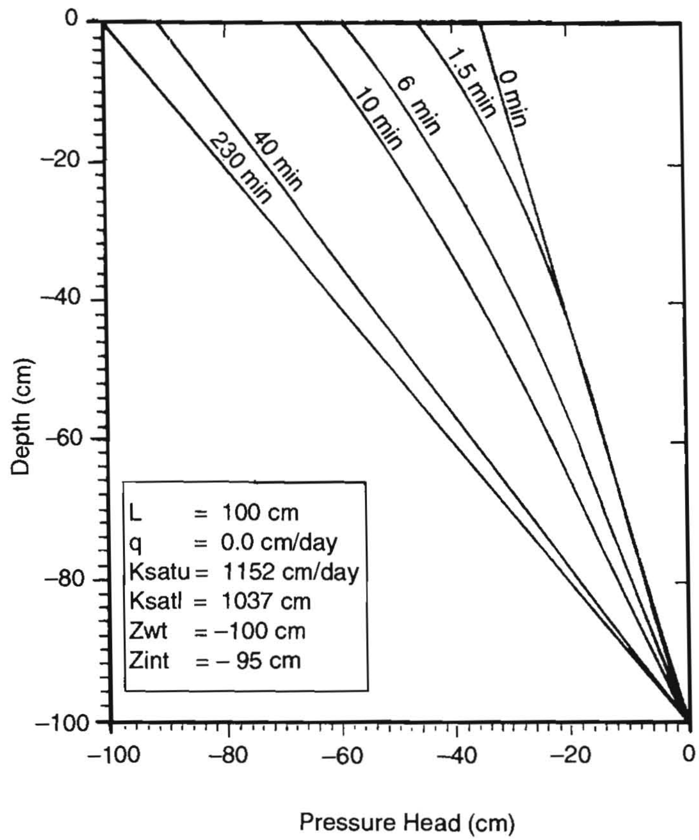


Fig. 5. Pressure Head Profile in a Stratified Column.

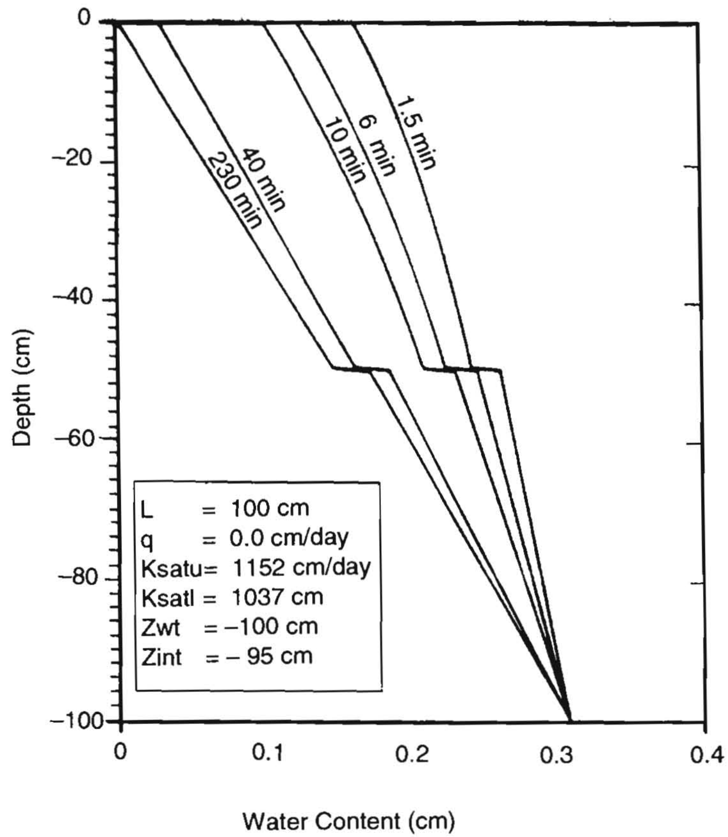


Fig. 6. Water Content Profile in a Stratified Column.

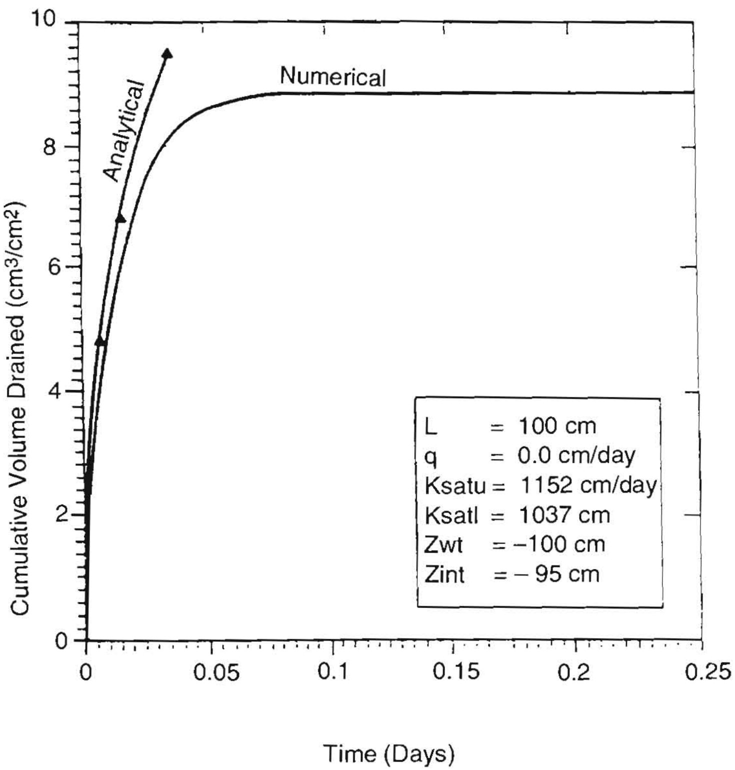


Fig. 7. Cumulative Volume Drained in a Stratified Column.

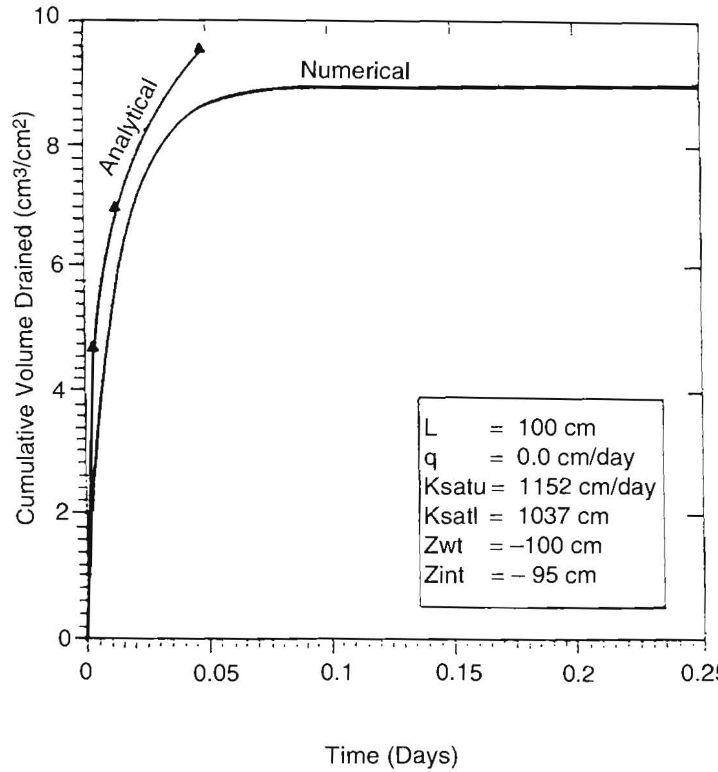


Fig. 8. Cumulative Volume Drained in a Stratified Column.

The basic Green and Ampt analysis used $\Delta\theta$ value of 0.238 cc/cc for the draining of the No. 17 sand and 0.263 cc/cc for the draining of R8A sand. These values estimated from the respective $h(\theta)$ curves as shown in Figurs 4 and 6.

These large $\Delta\theta$ values results in the over prediction of the cumulative outflow as shown in Figures 7 and 8. More realistic values can be obtained through experience for selecting an appropriate value of $\Delta\theta$.

The predicted outflow by the Green and Ampt method is significantly higher than that given by the numerical solution. This is a direct result of the overestimation of the degree of profile drainage, and this in turn is due to the fact that the residual value of "0.054 cc/cc" that has been used as the lower bound in calculating $\Delta\theta$ only occurs in the vicinity of the soil surface, rather than through the entire drained profile as the sharp-front analysis assume. Additional work is needed on developing general guidelines for estimating the magnitude of reduction in $\Delta\theta$ for different materials which is directly related to the cumulative outflow drainage.

Conclusion

The conclusion to be drawn from the study is that the Green and Ampt equation is very limited in presenting the drainage process in a stratified profile. This is expected due to the strong nonlinearity of the drainage process. However, the general observation can be made that the approximate results are in agreement with the numerical analysis, especially during the early stages of drainage. The inability of the Green and Ampt to model the physical condition that occurs in the interface region is significant particularly for the No. 17 sand/R8A.

Appendix

The matrix coefficients are,

For $h_L > h_R$: $k_r^w = \varepsilon\ k_{rL} + (1-\varepsilon)\ k_{rR}$

$$A^{e*} = \frac{K_{zz}}{l_e}(h_L - h_R) \begin{bmatrix} \varepsilon \dfrac{\partial k_{rL}}{\partial h_L} & (1-\varepsilon)\dfrac{\partial k_{rR}}{\partial h_R} \\ -\varepsilon \dfrac{\partial k_{rL}}{\partial h_L} & -(1-\varepsilon)\dfrac{\partial k_{rR}}{\partial h_R} \end{bmatrix}$$

where h_L = pressure head at the left node of an element,
h_R = pressure head at the right node of an element.

For $h_L < h_R$: $k_r^w = (1-\varepsilon)\ k_{rL} + \varepsilon\ k_{rR}$

$$A^{e*} = \frac{K_{zz}}{l_e}(h_L - h_R) \begin{bmatrix} (1-\varepsilon)\dfrac{\partial k_{rL}}{\partial h_L} & \varepsilon \dfrac{\partial k_{rR}}{\partial h_R} \\ -(1-\varepsilon)\dfrac{\partial k_{rL}}{\partial h_L} & -\varepsilon \dfrac{\partial k_{rR}}{\partial h_R} \end{bmatrix}$$

$$B^{e*} = \frac{l_e}{6\,\Delta t}\,\eta^{-} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$B^{e***} = \frac{l_e}{6\,\Delta t} \begin{bmatrix} (2h_L + h_R)\,\eta_L & (2h_L' + h_R)\ \eta_R' \\ (h_L + 2h_R')\,\eta_L & (h_L + 2h_R)\ \eta_R' \end{bmatrix}$$

where $\eta'_L = \dfrac{\partial \eta_L}{\partial h_L}$ and $\eta'_R \dfrac{\partial \eta_R}{\partial h_R}$

similarly,

$$B^{e***k} = \frac{l_e}{6\,\Delta t} \begin{bmatrix} (2h_L^K + h_R^K)\, h'_L & (2h_L^K + h_R^K)\, h'_R \\ (h_L^K + 2h_R^K)\, h_L & (h_L^K + 2h_R^K)\, h'_R \end{bmatrix}$$

For $h_L > h_R$:

$$F_{IJ}^{e*} = K_{zz} \begin{bmatrix} \varepsilon \dfrac{\partial k_{rL}}{\partial h_L} & (1-\varepsilon) \dfrac{\partial k_{rR}}{\partial h_R} \\ -\varepsilon \dfrac{\partial k_{rL}}{\partial h_L} & -(1-\varepsilon) \dfrac{\partial k_{rR}}{\partial h_R} \end{bmatrix}$$

For $h_L > h_R$:

$$F_{IJ}^{e*} = K_{zz} \begin{bmatrix} (1-\varepsilon) \dfrac{\partial k_{rL}}{\partial h_L} & \varepsilon \dfrac{\partial k_{rR}}{\partial h_R} \\ -(1-\varepsilon) \dfrac{\partial k_{rL}}{\partial h_L} & -\varepsilon \dfrac{\partial k_{rR}}{\partial h_R} \end{bmatrix}$$

Note: The sign convention used in the above formulation is that the Z-Coordinate is positive upward, and the flux is positive upward and negative downward.

References

Aggelides, S. and **Youngs, G.** (1978) The dependence of the parameters in the Green and Ampt infiltration equation on the initial water content in draining and wetting states, *Water Resources Research,* **14:** 857-862.

Brakensiek, D.L. (1977) Estimating the effective capillary pressure in the Green and Ampt infiltration equation, *Water Resources Research,* **13**: 680-682.

Green, W.H. and **Ampt, G.A.** (1911) Studies of Soil Physics; 1 Flow of air and water through soils, *J. Agr. Sci.* **4:** 1-24.

Huyakorn, P.S. and **Pinder, G.F.** (1983) *Computational Methods in Subsurface Flow,* Academic Press, New York.

Watson, K.K. and **Awadalla, S.A.** (1985) Comparative study of Green and Ampt analysis for a falling water table in a homogeneous sand profile, *Water Resources Research,* **21:** 1157-1164.

Watson, K.K. and **Whisler, F.D.** (1976) Comparison of drainage equations for the gravity drainage of stratified profile, *Soil Sci. Soc., Am., J.,* **40:** 631-635.

(Received 29/04/1994;
in revised form 19/11/1994)

طريقة حسابية لحساب الصرف في التربة المشبعة ذات الطبقات المتعددة

صلاح الدين أحمد عوض الله [١] و سعود عبد العزيز قطب [٢]

[١] قسم الهيدرولوجيا وإدارة مصادر المياه-كلية الأرصاد والبيئة وزراعة المناطق الجافة
جامعة الملك عبد العزيز - ص .ب ٩٠٢٧ - جدة ٢١٤١٣
[٢] قسم الهندسة المدنية - كلية الهندسة - جامعة الملك عبد العزيز
ص .ب ٩٠٢٧ - جدة ٢١٤١٣ - المملكة العربية السعودية

إنه من الشائع في المناطق الزراعية التخلص من المياه الفائضة عن حاجة النبات في منطقة الجذور النباتية حفاظاً عليها من التلف . وذلك أوجد الضرورة لدراسة واستيعاب عملية تخلخل وسريان الماء الراشح في المنطقة شبه المشبعة والواقعة أعلى خط الماء الأرضي . ولقد قام نخبة من الباحثين بدراسة هذه الظاهرة حسابياً وعددياً باستخدام المعادلات والنماذج الرياضية . ولقد تفاوتت طرقهم في درجة الدقة في وصف ومعالجة هذه الظاهرة . وجاء هذا البحث كمحاولة لعرض كفاءة وقدرة النماذج الرياضية لتمثيل ووصف حركة المياه في التربة ذات الطبقات المتعددة مقارنة بطريقة (Green-Ampt approach) . وأثبتت الدراسة التطابق في النتائج المقارنة خاصة في المراحل الأولى من عملية الصرف .