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Some Conventional Identification Procedures for ARMA(1,0) with Small Parameter Values: A Simulation Study

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ABSTRACT. The behaviour of an ARMA process for parameter values closer to unity has been of paramount interest in the literature. Various studies have been undertaken for this case. The present study undertakes the problem of a small parameter value when it is closer to zero and therefore the process being on the boundary of a white noise process. Some conventional procedures for identification of ARMA (1,0) process with small parameter values are evaluated using simulation.

Let $\{Z_t, t \geq 1\}$ be a time series generated by stochastic process ARMA(1,0)

$$Z_t = \phi Z_{t-1} + \epsilon_t$$

where $Z_t = 0$ for $t \leq 0$, and $\{\epsilon_t, t \geq 1\}$ are independent identically distributed random variables with mean zero and variance $\sigma_a^2 > 0$, commonly known as white noise and ϕ is an unknown parameter. Given a series Z_t, the basic conventional tools for the identification of an ARMA(1,0) process are autocorrelation and partial autocorrelation functions. The use of Akaike information criteria (AIC) and Schwartz Bayesian Criteria (SBC) is also helpful to recognize the correct order of the underlying ARMA process which has given rise to the observed series Z_t. These procedures for the identification of Autoregressive Moving Average (ARMA)

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process are to be tested in special cases. See Priestly (1981).

Various studies are undertaken for the case when the parameter is closer to unity and therefore process being on the boundary of non-stationarity. The examples in the literature are Dickey and Fuller (1979), Hasza (1979), Rao (1978), and Evans (1981). The present study focuses on problem when parameter values are small. We investigate the effectiveness of conventional identification procedures when the parameter is closer to zero and therefore the process is on the boundary of a white noise. The study adopts the Monte Carlo method to achieve this aim.

Procedure

It is observed that for smaller parameter values and sample size less than 200, the usual procedure of employing autocorrelation and partial autocorrelation functions (ACF and PACF) as identification tools fail as the values of ACF and PACF are not significant and therefore do not give a clear picture of the order of autoregressive and moving average parameters. Therefore a simulation procedure is used like Barry and Khan (1995), Dent and Min (1978), Nelson and Schwert (1982) to generate realizations from ARMA(1,0) process for small samples. The disturbances are generated as mutually independent and uncorrelated random normal variates. The IMSL subroutine GGNML is used to generate these numbers for different seed values. The algorithm employed has been rigorously tested by Learmonth and Lewis (1973). The first 200 values of each series were discarded to get rid of the transient effect.

The series length selected as 50, 100 and 150. The values of parameter ø are taken 0.10, 0.15, 0.20, 0.25 and 0.30 for each series length.

For each of these series length, first we use the conventional tools of ACF and PACF to identify the process. We find that in case of series length 50 the ACF and PACF are all insignificant for all selected parameter values. In case of series length 100 and series length 150 the same pattern repeats for parameter values 0.10, 0.15, 0.20. For parameter value 0.25, ACF and PACF are approaching to significant values but their patterns do not help for the identification of the process. For parameter value 0.30, ACF and PACF are significant but again fail to identify the process. The next thing is to simulate ARMA(1,0) process as mentioned above and to study various tentative models and to find their suitability by using AIC and SBC criteria. The results of this study are recorded in Tables 1-3. The discussion of the results follow in the next section.

Discussion of results for tentative models fitted to ARMA(1,0) process

The ARMA(1,0) process $Z_t = \phi Z_{t-1} + \epsilon_t$ is generated with sample size 50, 100 and 150 and for varying parameter values. We attempt to identify these processes by traditional method of using autocorrelation and partial autocorrelation functions. We find that for smaller values of parameters ACF and PACF are not helpful. We also attempt the use of Akaike Information Criteria AIC and Schwartz Bayesian Criteria SBC to recognize the correct order of each generated process (see Wei 1990). The values of AIC and SBC are computed in each case for the tentative models fitted to realizations from ARMA(1,0). The estimated standard deviation of residual series, $\hat{\sigma}_a$, are also obtained. The values of AIC, SBC and $\hat{\sigma}_a$ are recorded in Tables 1, 2 and 3 for sample size 50, 100 and 150 respectively, the abbreviation ERSD is used for $\hat{\sigma}_a$. The discussion of the results is given as follows.

According to both AIC and SBC, the best fitted model to the simulated series $Z_t = \phi Z_{t-1} + \epsilon_t$ with $\phi = 0.10$ and sample size 50 is ARMA(0,0) *i.e.* white noise and the next best model is ARMA(1,0). The minimum value of $\hat{\sigma}_a$ is also for ARMA(0,0) followed by ARMA(1,0). The parameter values for all the tentative models are not significant. The autocorrelation function and partial autocorrelation function do not help in the identification of the process.

For the simulated series with $\phi = 0.15$, AIC and SBC show that best fitted model for this series is also ARMA(0,0) and the next best model is ARMA(1,0). The standard deviations of residual series $\hat{\sigma}_a$, are also in accordance with these results. But the parameter estimate of ARMA(1,0) is not significant. The only significant estimates are for the model ARMA(2,2) in which the value of $\hat{\sigma}_a$ is also comparatively small as shown in Table 1. For this series the autocorrelation function and partial autocorrelation function are not helpful in identifying the tentative models. The parameter values for ARMA(2,2) are given as follows:

Model	Parameter value	Standard error
ARMA(2,2)	$\hat{\theta}_1 = 0.830$	0.157
	$\hat{\theta}_2 = 0.833$	0.140
	$\hat{\phi}_1 = 0.911$	0.200
	$\hat{\phi}_2 = -0.796$	0.191

Table 1. Fitted Models to ARMA(1,0) for series length 50

p / q	0 AIC	0 SBC	0 ERSD	1 AIC	1 SBC	1 ERSD	2 AIC	2 SBC	2 ERSD	3 AIC	3 SBC	3 ERSD
	ø = 0.10											
0	134.81	134.81	0.932	136.69	138.60	0.940	137.98	141.80	0.942	139.93	145.67	0.952
1	136.71	138.62	0.940	138.51	142.33	0.948	139.94	145.67	0.953	141.47	149.11	0.458
2	137.95	141.77	0.943	139.92	145.66	0.952	141.92	149.57	0.962	141.61	151.17	0.451
3	139.91	145.64	0.952	141.91	149.91	0.963	141.61	151.17	0.951	143.57	155.05	0.962
	ø = 0.15											
0	135.26	135.26	0.876	136.71	138.62	0.940	138.00	141.82	0.943	139.91	145.66	0.952
1	136.82	138.73	0.941	138.44	142.26	0.947	139.93	145.67	0.952	141.43	149.08	0.958
2	137.96	141.78	0.943	139.91	145.66	0.952	139.67	147.26	0.941	141.60	151.16	0.951
3	139.90	145.64	0.952	141.90	149.95	0.962	141.60	151.16	0.951	144.55	156.02	0.971
	ø = 0.20											
0	136.04	136.04	0.943	136.74	138.65	0.940	138.04	141.86	0.943	139.41	145.65	0.952
1	137.01	138.92	0.943	138.40	142.23	0.947	140.56	141.30	0.958	141.92	149.57	0.963
2	137.99	141.81	0.943	139.92	145.66	0.952	141.92	149.57	0.963	141.60	151.16	0.951
3	139.90	145.65	0.952	141.89	149.59	0.962	140.36	149.92	0.939	143.69	155.17	0.963
	ø = 0.25											
0	137.12	137.12	0.953	136.76	138.67	0.941	138.08	141.90	0.944	139.89	145.63	0.952
1	137.28	139.19	0.946	138.39	142.22	0.947	139.93	145.67	0.952	141.92	149.57	0.963
2	138.02	141.85	0.943	139.92	145.66	0.952	141.93	149.57	0.963	141.60	151.16	0.951
3	139.90	145.63	0.952	141.88	149.53	0.962	143.80	153.36	0.972	144.47	155.94	0.970
	ø = 0.30											
0	138.55	138.55	0.967	136.76	138.68	0.941	138.12	141.94	0.942	139.87	145.61	0.952
1	137.65	139.56	0.949	138.39	142.22	0.947	139.93	145.87	0.952	141.35	148.99	0.957
2	138.07	141.90	0.944	139.91	145.65	0.952	141.93	149.58	0.963	141.59	151.15	0.950
3	139.89	145.63	0.952	141.85	149.50	0.962	134.57	153.13	0.970	144.70	156.17	0.972

For the series simulated for $\phi = 0.20$ the best fitted model is again ARMA(0,0) and the next best model is ARMA(1,0) as indicated by AIC and SBC. The minimum value of $\hat{\sigma}_a$ is for ARMA(1,0) model. But the parameter estimate for ARMA(1,0) is not significant. The significant parameters are only for ARMA(2,1), but the model is found non-stationary and non-invertible and therefore is not given any consideration. The autocorrelation and partial autocorrelation are again insignificant for this series also and therefore are not authentic to select the tentative models.

The estimated parameter values are again not significant for all tentative models fitted to the simulated series with $\phi = 0.25$. The AIC values, SBC values and estimated standard deviations of the residuals indicate that the best among the tentative models is ARMA(1,0). The parameter value for ARMA(1,0) and its corresponding standard error are 0.216 and 0.139 respectively which shows that parameter value is approaching to be significant. According to AIC values still the next best model to be fitted is ARMA(0,0), the white noise. But the value of estimated standard deviations of the residuals is among the largest for ARMA(0,0) and eliminates the possibility for the selection of this model. The behaviour of autocorrelations and partial autocorrelations is still not clear for the selection of the tentative model.

For the simulated series with $\phi = 0.30$, the parameter values for all tentative models are insignificant. But for ARMA(1,0) model the estimated parameter is 0.27 with corresponding standard error equal to 0.138 and is very close to significant value. The AIC value is minimum for this model and the estimated standard deviation of residual series is also minimum. The SBC value is smaller than other models except ARMA(0,0) but the difference is not much.

The result for the tentative models fitted to ARMA(1,0) process with series length 100 are recorded in Table 2.

According to AIC and SBC computed for tentative models fitted to series with $\phi = 0.10$, the best fit is ARMA(0,0) and estimated standard deviation of residuals is also minimum for this model indicating that generating process is white noise. The next best fit is ARMA(1,0) followed by ARMA(1,1). The only models with significant parameter values are ARMA(1,1), ARMA(1,2) and ARMA(1,3). In all these models significant parameter values are only $\hat{\phi}_1$ and $\hat{\theta}_1$. Still autocorrelations and partial autocorrelations do not help in identifying tentative models. The parameter values for these models are as follows:

Model	Parameter estimate	Standard error
ARMA(1,1)	$\hat{\theta}_1 = 0.880$	0.188
	$\hat{\phi}_1 = 0.939$	0.150
ARMA(1,2)	$\hat{\theta}_1 = 0.896$	0.186
	$\hat{\theta}_2 = -0.019$	0.110
	$\hat{\phi}_1 = 0.938$	0.159
ARMA(1,3)	$\hat{\theta}_1 = 0.896$	0.200
	$\hat{\theta}_2 = -0.022$	0.137
	$\hat{\theta}_3 = 0.033$	0.114
	$\hat{\phi}_1 = 0.938$	0.175

In case of series Z_t with $\phi = 0.15$, we observe that the minimum values of AIC and SBC are for ARMA(0,0) followed by ARMA(1,0). The minimum of estimated standard deviation of residuals is for ARMA(1,0). But the only models with significant parameter values are ARMA(1,2), ARMA(2,1) and ARMA(3,1). In these models the significant parameter estimates are only for ϕ_1 and θ_1. The autocorrelation and the partial autocorrelations do not help in identifying the tentative models. The parameter values for the models are as follows.

Model	Parameter estimate	Standard error
ARMA(1,2)	$\hat{\theta}_1 = 0.846$	0.184
	$\hat{\theta}_2 = 0.022$	0.113
	$\hat{\phi}_1 = 0.937$	0.155
ARMA(2,1)	$\hat{\theta}_1 = 0.871$	0.214
	$\hat{\phi}_1 = 0.963$	0.240
	$\hat{\phi}_2 = -0.025$	0.123
ARMA(3,1)	$\hat{\theta}_1 = 0.875$	0.254
	$\hat{\phi}_1 = 0.967$	0.277
	$\hat{\phi}_2 = -0.018$	0.144
	$\hat{\phi}_3 = -0.009$	0.123

Table 2. Fitted Models to ARMA (1,0) for series length 100

p q	0 AIC	 SBC	 ERSD	1 AIC	 SBC	 ERSD	2 AIC	 SBC	 ERSD	3 AIC	 SBC	 ERSD
	ø = 0.10											
0	276.90	276.90	0.966	278.57	281.17	0.969	280.02	285.25	0.972	282.04	289.85	0.978
1	278.61	281.22	0.970	278.79	284.01	0.966	280.92	288.74	0.971	282.91	293.33	0.976
2	280.16	285.37	0.972	280.76	288.57	0.971	281.83	292.25	0.971	283.64	296.66	0.975
3	282.16	289.97	0.977	282.76	293.18	0.976	283.58	296.61	0.975	283.91	299.54	0.972
	ø = 0.15											
0	277.83	277.83	0.971	278.59	281.20	0.967	282.02	285.23	0.972	282.02	289.84	0.977
1	278.77	281.37	0.970	278.80	284.01	0.966	280.75	288.57	0.970	282.74	93.16	0.975
2	280.17	285.38	0.972	280.75	288.57	0.970	282.19	292.61	0.973	283.61	296.63	0.975
3	282.17	289.98	0.977	282.74	293.16	0.975	283.45	296.48	0.974	283.31	298.94	0.969
	ø = 0.20											
0	279.36	279.36	0.978	278.63	281.23	0.970	280.01	285.22	0.972	282.01	289.83	0.977
1	279.05	281.66	0.972	279.14	284.35	0.967	280.76	288.57	0.971	282.75	293.17	0.976
2	280.20	285.41	0.973	280.77	288.59	0.971	282.75	293.17	0.976	283.59	296.62	0.975
3	282.20	290.01	0.978	282.72	293.14	0.975	283.30	296.32	0.974	283.14	298.77	0.968
	ø = 0.25											
0	281.54	281.54	0.989	278.66	281.27	0.970	279.99	285.21	0.972	281.99	289.81	0.977
1	279.51	282.11	0.974	279.63	284.84	0.970	280.92	288.73	0.971	282.75	293.17	0.976
2	280.70	285.47	0.973	280.83	288.64	0.971				290.23	303.25	1.008
3	282.25	290.07	0.978	282.67	293.09	0.975	283.12	296.15	0.973	283.16	298.79	0.968
	ø = 0.30											
0	284.38	284.38	1.003	278.68	280.38	0.970	278.96	285.17	0.971	281.95	289.77	0.976
1	280.18	282.78	0.977	279.56	284.97	0.970	285.75	288.57	0.970	282.73	293.15	0.975
2	280.34	285.55	0.973	280.91	280.73	0.971	282.90	293.32	0.976	283.54	296.57	0.975
3	282.34	290.15	0.978	282.62	293.02	0.975	282.94	295.97	0.972	283.32	298.95	0.969

For the fitted models to series Z_t with $\phi = 0.20$, AIC values show that the best fit is ARMA(1,0) followed by ARMA(1,1). The minimum $\hat{\sigma}_a$ is for ARMA(1,1) followed by ARMA(1,0). The only models with significant parameters are ARMA(1,1), ARMA(1,2), ARMA(1,3) and ARMA(3,1) of which ARMA(3,1) is non-stationary. In all these models the significant estimates are for ϕ_1 and θ_1. The parameter values for these models are as follows:

Model	Parameter estimate	Standard error
ARMA(1,1)	$\hat{\theta}_1 = 0.827$	0.181
	$\hat{\phi}_1 = 0.918$	0.137
ARMA(1,2)	$\hat{\theta}_1 = 0.796$	0.182
	$\hat{\theta}_2 = 0.060$	0.116
	$\hat{\phi}_1 = 0.936$	0.150
ARMA(1,3)	$\hat{\theta}_1 = 0.794$	0.197
	$\hat{\theta}_2 = 0.046$	0.115
	$\hat{\theta}_3 = 0.023$	0.133
	$\hat{\phi}_1 = 0.939$	0.171

For series Z_t with $\phi = 0.25$ we find that according to AIC and SBC values the best fit is ARMA(1,0) followed by ARMA(0,1). The minimum estimated standard deviation of residuals is for the model ARMA(1,0). The models with significant parameter estimates are ARMA(1,0), ARMA(2,0) and ARMA(1,3). In each ARMA(1,0) and ARMA(2,0), the significant parameter estimate is $\hat{\phi}_1$ and in ARMA(1,3) the significant estimates are $\hat{\phi}_1$ and $\hat{\theta}_1$.

The models having significant parameters are listed below with estimated parameter values and corresponding standard error.

Lastly for series Z_t with $\phi = 0.30$, both AIC and SBC values are minimum for ARMA(1,0) model. The estimated standard deviation of residuals is also minimum for this model. The parameter estimate for the model is significant as well.

Model	Parameter value	Standard error
ARMA(1,0)	$\hat{\phi}_1 = 0.221$	0.100
ARMA(2,0)	$\hat{\phi}_1 = 0.205$	0.102
	$\hat{\phi}_2 = 0.083$	0.103
ARMA(1,3)	$\hat{\theta}_1 = 0.742$	0.195
	$\hat{\theta}_2 = 0.074$	0.132
	$\hat{\theta}_3 = 0.037$	0.117
	$\hat{\phi}_1 = 0.938$	0.167

Finally we consider the case when samples size is 150. The results for the tentative models fitted to ARMA(1,0) process for this case are recorded in Table 3. According to AIC and SBC values the best fit to series with $\phi = 0.10$ is ARMA(0,0) followed by ARMA(1,1) and ARMA(1,0) in that order. The minimum value of $\hat{\sigma}_a$ is also for ARMA(0,0) and the same value repeats for ARMA(2,1) and ARMA(1,2). The models with significant estimates for parameter values are ARMA(1,1), ARMA(1,2), ARMA(2,1) and ARMA(3,1). In all these models the significant estimates are for ϕ_1 and θ_1. Still autocorrelations and partial autocorrelations do not help in identifying tentative models.

The parameter values and corresponding standard errors for the models mentioned above are as follows:

Model	Parameter value	Standard error
ARMA(1,1)	$\hat{\theta}_1 = -0.937$	0.091
	$\hat{\phi}_1 = -0.969$	0.067
ARMA(1,2)	$\hat{\theta}_1 = -0.985$	0.110
	$\hat{\theta}_2 = -0.069$	0.085
	$\hat{\phi}_1 = -0.956$	0.077
ARMA(2,1)	$\hat{\theta}_1 = -0.915$	0.106
	$\hat{\phi}_1 = -0.893$	0.133
	$\hat{\phi}_2 = 0.064$	0.088
ARMA(3,1)	$\hat{\theta}_1 = -0.869$	0.150
	$\hat{\phi}_1 = -0.843$	0.170
	$\hat{\phi}_2 = 0.028$	0.109
	$\hat{\phi}_3 = -0.056$	0.093

Table 3. Fitted Models to ARMA(1,0) for series length 150

p / q	0 AIC	0 SBC	0 ERSD	1 AIC	1 SBC	1 ERSD	2 AIC	2 SBC	2 ERSD	3 AIC	3 SBC	3 ERSD
	ø = 0.10											
0	358.23	358.23	0.799	360.22	363.23	0.801	362.16	368.18	0.804	363.72	372.75	0.805
1	360.22	363.23	0.801	360.04	366.06	0.798	361.49	370.53	0.799	363.17	375.21	0.801
2	362.17	368.19	0.804	361.44	370.47	0.799	363.31	375.35	0.802			
3	363.76	372.79	0.805									
	ø = 0.15											
0	358.75	358.75	0.800	360.24	363.25	0.801	362.17	368.19	0.804	363.80	372.83	
1	360.27	368.28	0.801	362.21	368.23	0.804	361.47	370.50	0.799	366.54	378.58	0.806
2	362.14	368.16	0.804	361.33	370.36	0.799						
3	363.67	372.71	0.805	363.21	375.25	0.801						
	ø = 0.20											
0	360.07	360.07	0.804	360.27	363.28	0.801	362.17	268.19	0.804	363.87	372.90	0.806
1	360.40	363.41	0.802	362.21	368.23	0.804				365.61	377.66	0.808
2	362.10	368.12	0.804	361.34	370.27	0.799						
3	363.60	373.63	0.805	363.21	375.25	0.801						
	ø = 0.25											
0	362.23	362.23	0.809	360.29	363.60	0.801	362.17	368.19	0.804	363.91	372.95	0.806
1	360.64	363.66		362.20	368.22	0.804						
2	362.03	368.05	0.803	361.20	370.23	0.799	365.52	377.01	0.808			
3	363.52	372.56	0.805	363.22	375.25	0.801						
	ø = 0.30											
0	365.32	365.32	0.818	360.34	363.35	0.802	362.18	368.26	0.804	363.98	373.01	0.806
1	361.13	364.14	0.804	362.21	368.23	0.804						
2	361.98	368.00	0.803									
3	363.52	372.55	0.805	363.29	375.34	0.802						

The minimum value in each of AIC and SBC in the fitted model to series with ø = 0.15 is for ARMA(0,0) followed by ARMA(1,0). The minimum $\hat{\sigma}_a$ is for ARMA(1,2) and ARMA(2,1). The next minimum value is for ARMA(0,0) The modles with some estimates having significant values are ARMA(1,2), ARMA(2,1), and ARMA(1,3). In All these models the only significant estimates are $\hat{\phi}_1$ and $\hat{\theta}_1$. The parameter values and corresponding standard errors for these models are listed below:

Model	Parameter value	Standard error
ARMA(1,2)	$\hat{\theta}_1 = -1.030$	0.111
	$\hat{\theta}_2 = -0.118$	0.084
	$\hat{\phi}_1 = -0.952$	0.081
ARMA(2,1)	$\hat{\theta}_1 = -0.915$	0.105
	$\hat{\phi}_1 = -0.844$	0.131
	$\hat{\phi}_2 = 0.111$	0.086
ARMA(1,3)	$\hat{\theta}_1 = -0.852$	0.265
	$\hat{\theta}_2 = -0.080$	0.112
	$\hat{\theta}_3 = 0.075$	0.055
	$\hat{\phi}_1 = -0.722$	0.057

The AIC and SBC values for tentative models fitted to generated series with ϕ = 0.20 show that the best fit is ARMA(0,0) followed by ARMA(1,0). The minimum value for $\hat{\sigma}_a$ is for ARMA(1,0). Among the fitted models ARMA(1,2), ARMA(1,3) and ARMA(3,1) have significant parameter estimates. In all these models the significant parameter values are for ϕ_1 and θ_1. The parameter values and corresponding standard errors for these models are as follows:

Model	Parameter value	Standard error
ARMA(1,2)	$\hat{\theta}_1 = -1.073$	0.114
	$\hat{\theta}_2 = -1.166$	0.083
	$\hat{\phi}_1 = -1.945$	0.187
ARMA(1,3)	$\hat{\theta}_1 = -1.885$	0.280
	$\hat{\theta}_2 = -1.128$	0.119
	$\hat{\theta}_3 = 0.065$	0.097
	$\hat{\phi}_1 = -0.755$	0.272
ARMA(3,1)	$\hat{\theta}_1 = 0.836$	0.418
	$\hat{\phi}_1 = 0.938$	0.426
	$\hat{\phi}_2 = -0.067$	0.122
	$\hat{\phi}_3 = -0.013$	0.097

For generated series with $\phi = 0.25$ AIC values for tentative models show that the best fit is ARMA(1,0) followed by ARMA(0,1), ARMA(1,2), ARMA(2,0) and ARMA(0,2) in that order. The SBC value is minimum for ARMA(0,0) followed by ARMA(1,0). The minimum value of $\hat{\sigma}_a$ is for ARMA(1,2). The next minimum value is for ARMA(1,0) and ARMA(3,1). The models with significant parameter values are ARMA(0,3), ARMA(1,2) and ARMA(1,3). In these models the significant parameter values are only for $\hat{\phi}_1$ and $\hat{\theta}_1$. The parameter values and corresponding standard errors for these models are listed below:

Model	Parameter value	Standard error
ARMA(0,3)	$\hat{\theta}_1 = -0.177$	0.083
	$\hat{\theta}_2 = -0.044$	0.085
	$\hat{\theta}_3 = -0.063$	0.184
ARMA(1,2)	$\hat{\theta}_1 = -1.106$	0.123
	$\hat{\theta}_2 = -0.214$	0.082
	$\hat{\phi}_1 = -0.930$	0.101
ARMA(1,3)	$\hat{\theta}_1 = -0.929$	0.289
	$\hat{\theta}_2 = -0.182$	0.128
	$\hat{\theta}_3 = -0.050$	0.101
	$\hat{\phi}_1 = -0.749$	0.280

Among tentative models fitted to series with $\phi = 0.30$, the models with significant parameter values are ARMA(3,0), ARMA(0,3) ARMA(2,0), ARMA(0,1), ARMA(1,0) and ARMA(1,3). The best fit is ARMA(1,0) considering AIC and SBC values and the value of $\hat{\sigma}_a$. The parameter values and corresponding standard errors for these models are given below. The significant parameters in all these models are $\hat{\theta}_1$ and $\hat{\phi}_1$.

Model	Parameter value	Standard error
ARMA(1,0)	$\hat{\phi}_1 = 0.214$	0.080
ARMA(0,1)	$\hat{\theta}_1 = -0.189$	0.081
ARMA(3,0)	$\hat{\phi}_1 = 0.208$	0.083
	$\hat{\phi}_2 = 0.041$	0.086
	$\hat{\phi}_3 = -0.037$	0.084
ARMA(0,3)	$\hat{\theta}_1 = 0.288$	0.083
	$\hat{\theta}_2 = -0.065$	0.085
	$\hat{\theta}_3 = 0.060$	0.084
ARMA(2,0)	$\hat{\phi}_1 = 0.207$	0.083
	$\hat{\phi}_2 = 0.033$	0.083
ARMA(0,2)	$\hat{\theta}_1 = -0.215$	0.083
	$\hat{\theta}_2 = -0.080$	0.083
ARMA(1,3)	$\hat{\theta}_1 = -0.985$	0.295
	$\hat{\theta}_2 = -0.241$	0.139
	$\hat{\theta}_3 = 0.027$	0.105
	$\hat{\phi}_1 = -0.775$	0.282

Concluding Remarks

For sample size 50 and small parameter values, the conventional methods of identification for ARMA models fail to recognize the autoregressive element in the simulated ARMA(1,0) process. With the available tools for identification, the process is recognized as white noise. It is at $\phi = 0.30$ that we have the glimpse of the actual simulated process of ARMA(1,0). For sample size 100 recognizable patterns start to immerge at parameter value 0.25 and similar thing happens for sample size 150. The study concludes that in case of small parameter values and in the case of small sample size the investigations are needed to supplement the existing identification techniques. Only this can ensure the greater accuracy in the identification procedure for ARMA processes with small parameter values.

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بعض طرائق التحديد الدارجة لنموذج ARMA(1,0) بقيم معلمية صغيرة : دراسة محاكاة

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إن طرائق تحديد درجة نموذج ARMA(1,0) مثل طرائق دوال الترابط الذاتي للعينة (Autocorrelation Function) والترابط الذاتي الجزئي (Partial Autocorrelation Function) وكذلك معيار أكايك الإعلامي (AIC) ومعيار شفارتز البيزي (SBC) كل هذه الطرائق صالحة لأحجام عينة أكبر من 50 مشاهدة ولقيم معالم ϕ بعيدة عن القيم المتطرفة في هذه الدراسة بواسطة المحاكاة ، تم توليد نماذج مختلفة من ARMA(1,0) بأحجام عينة تساوي 200, 100, 50 مشاهدة ولقيم المعالم ϕ 0.10, 0.15, 0.20, 0.25, 0.30 ، وتم تطبيق طرائق التحديد السابقة عليها . وقد ركزنا هنا على القيمة المتطرفة صفر ، ولهذا أخذنا قيم ϕ المذكورة ، وبهذا نكون قد استقصينا كامل نموذج ARMA(1,0) عند حدود الضجة البيضاء . وقد لوحظ في هذا البحث أنه لأحجام عينة صغيرة (أقل من 50 مشاهدة) ، فإن الطرائق الدارجة لا تمكن من التعرف على النموذج ARMA(1.0) لقيم $\phi \leq 0.3$ ، خلافا لما هو معروف نظرياً ، بل تعد ضجة بيضاء . أما بالنسبة لأحجام عينة متوسطة (حوالي 100 مشاهدة) فإن الطرائق الدارجة تمكن من تمييز النموذج من قيم $\phi \geq 0.25$.