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A Simple Method of Depth Determination from Numerical Horizontal Magnetic Gradients Due to Thin Dikes

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ABSTRACT. The present paper deals with a simple and rapid approach to depth determination of a buried thin dike using numerical horizontal magnetic gradients. The problem of depth determination has been transformed into finding the zero anomaly distances from observed gradients. Formulas are also given to estimate the amplitude coefficient and the index parameter. The method has been applied to synthetic data and the validity of the method is tested on the Pima copper mine magnetic anomaly, Arizona.

The thin dike model is frequently used in magnetic interpretations to find the depth, and the magnitude and direction of magnetization of intrusive igneous rocks in the forms of dikes and veins. Several excellent methods have been developed for interpreting residual magnetic anomalies due to thin dikes such as those given by Werner (1953), Parker Gay (1963), Stanley (1977), Prakasa Rao *et al.* (1986), Thompson (1982). However, effective quantitative interpretation based on the analytical expression of horizontal magnetic gradient anomalies obtained by numerical differentiation of the observed residual magnetic anomalies are yet to be developed.

The aim of the present study is to introduce a new interpretive technique based on using a thin dike model convolved with the same gradient filter as applied to the observed data. Procedures are also formulated to estimate the index parameter (the effective angle of magnetization) and the amplitude coefficient. The method is tested on synthetic data and on a field example from Arizona.

Theory

Following Parker Gay (1963) and Atchuta Rao *et al.* (1980), the general expression, F, for the magnetic anomaly either in total, vertical, or horizontal field at a point P along the x-axis (Fig. 1) of an arbitrary magnetized thin dike (2-D) is given by

$$F(x,z,\theta) = zM \frac{(x\sin\theta + z\cos\theta)}{(x^2 + z^2)} \qquad \text{......................} (1)$$

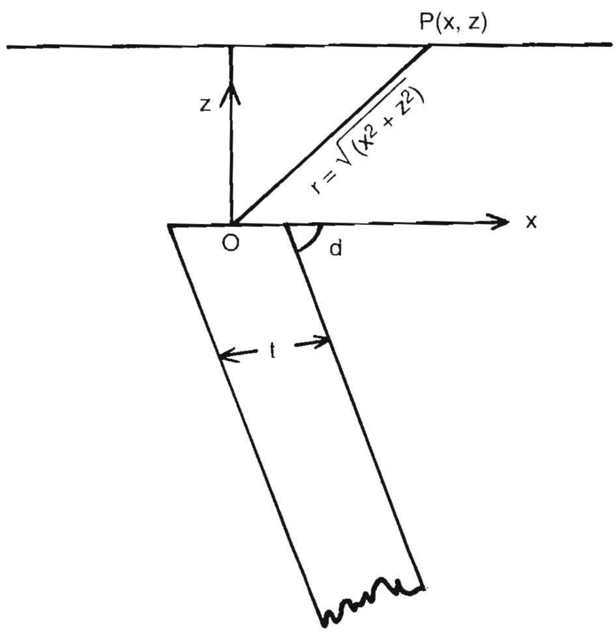

Fig. 1. Cross-sectional view of the thin dike model (2-D).

where z is the depth, x is a position coordinate, M is the amplitude coefficient, and θ is the index parameter. The values of M and θ for the anomalies in the total, vertical and horizontal fields are given in Table 1. The index parameter θ is related to the effective inclination of polarization I'_o.

Table 1. Characteristic amplitude coefficient M and index parameter θ in vertical (ΔV), horizontal (ΔH) and total (ΔT) magnetic anomalies due to thin dikes (after Parker Gay 1963)

Anomaly (F)	**Amplitude Coefficient (M)**	**Index Parameter (θ)**
ΔV	$2kT'_0 t/z$	$I'_0 - d$
ΔH	$2kT'_0 t \sin\alpha/z$	$I'_0 - d\text{-}90°$
ΔT	$\dfrac{2kT'_0 t \sin I_0}{z \sin I_0}$	$2I'_0 - d\text{-}90°$

In Table 1, t is the thickness of the dike, k is the magnetic susceptibility contrast, d is the dip angle of the dike, T'_0 and I'_0 are the values of effective total intensity and effective inclination of magnetic polarization in the vertical plane normal to the strike of the body, and α is the strike of the dike measured clockwise from magnetic north. T'_0 and I'_0 are related to the true total intensity T_0 and true inclination I_0 by

$$\tan I'_0 = \frac{(\tan I_0)}{(\sin\alpha)} \qquad \text{and} \qquad \frac{T'_0}{T_0} = \frac{(\sin I_0)}{(\sin\ I'_0)}$$

The gradient method is an important and simple technique to emphasize magnetic anomalies of small areal extent. The method has a high resolving power particularly when the graticule spacing is very small.

Let us consider three observation points $x_i - s$, x_i, $x_i + s$ along the anomaly profile where s = 1,2,3 ...., L spacing units and is called the window length or graticule spacing. The simplest numerical horizontal gradient field is

$$F_x(x_i, z, \theta) = \frac{zM}{2S}\left[\frac{[(X_i - s)\sin\theta + z\cos\theta]}{((x_i - s)^2 + z^2)} - \frac{[(X_i - s)\sin\theta + z\cos\theta]}{((x_i + s)^2 + z^2)}\right] \quad \text{........ (2)}$$

where i = 1,2,3, ..., N.

Setting equation (2) to zero, we obtain the following equation

$$\tan(\theta) = \frac{2x_0 z}{s^2 + z^2 - x_0^2} \qquad \text{................ (3)}$$

where x_0 is the horizontal distance from the origin ($x_i = 0$) to the point at which the anomaly is zero. However, because there are two horizontal distances along the anomaly profile where the magnetic gradient anomaly is zero, namely, x_{01} and x_{02} (Fig. 2), we conclude from equation (3) that

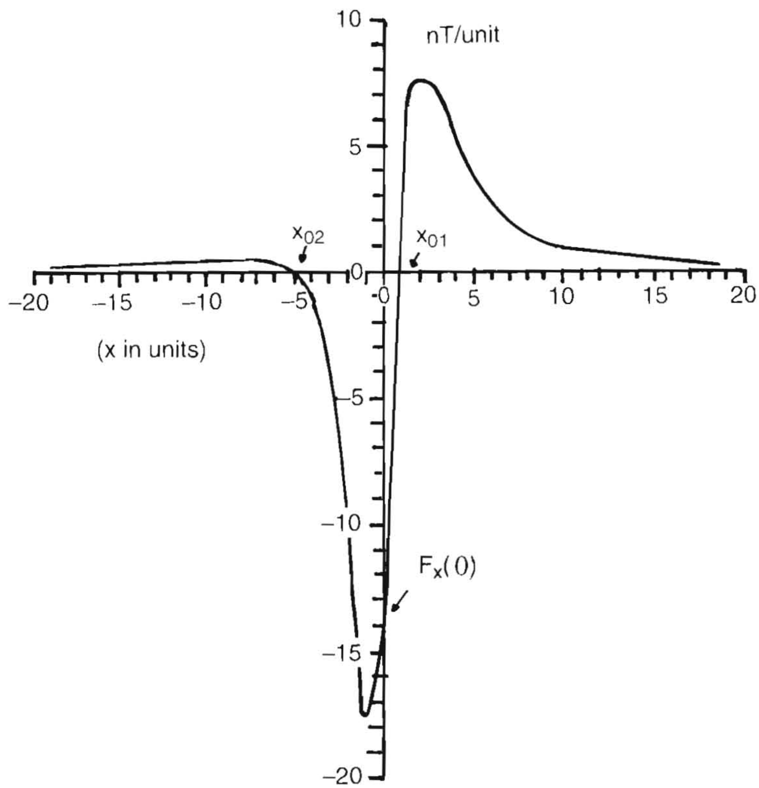

Fig. 2. A typical numerical horizontal gradient profile over a thin dike (profile length = 40 units, z = 2 units, M = 100 nT, $\theta = -135$ degrees, sampling interval = 1 unit, and window length (s) = 1 unit). The zero-gradient anomaly distances x_{01} and x_{02}, and the anomaly value at the origin $F_x(0)$ are illustrated.

$$\frac{x_{01}}{s^2 + z^2 - x_{01}^2} = \frac{x_{02}}{s^2 + z^2 - x_{02}^2} \qquad \text{............. (4)}$$

and finally if the equation is solved for z, we obtain

$$z = (-(S^2 + x_{01}x_{02}))^{1/2} \qquad \text{.............. (5)}$$

Thus knowing the positive horizontal distance x_{01} and the negative horizontal distance x_{02} from a given gradient profile, the depth parameter can be obtained uniquely from equation (5).

Because z is known, the θ value can be determined from equation (3). From θ, the effective angle of magnetization and the dip of the dike can be estimated. The value of θ thus obtained lies between –90 and 90 degrees, although in reality it can be any value between 0 and –360 degrees (Parker Gay 1963). The actual value can be obtained from examination of the field profile. Positions of the dominant positive and the dominant negative determine the actual value by following the rules given in Table 2.

Table 2. Criteria for determining the actual value of θ

Position and sign of dominant anomaly	Actual value of θ
Dominant positive to the south	θ
Dominant negative to the north	θ – 180°
Dominant positive to the north	θ – 360°
Dominant negative to the south	θ – 180°

At the origin ($x_i = 0$), equation (2) yields

$$M = -\left[\frac{F_x(0)\,(s^2 + z^2)}{z\sin\theta}\right] \qquad \text{.............. (6)}$$

where $F_x(0)$ is the gradient anomaly value at $x_i = 0$. Since z, θ, and $F_x(0)$ are known, the amplitude coefficient M can be determined from equation (6). Because α and I_o are assumed known $(2KT'_0t)$ can also be determined using the criteria in Table 1. By assuming susceptibility contrast k, the thickness of the dike t can be estimated.

To this stage, we have assumed knowledge of the origin of the magnetic profile. In practice, a field traverse will have an arbitrary origin, in which case the position of the structure ($x_i = 0$) in equation (1) must first be determined. The position of the turning point, *i.e.* the main maximum value of the profile and the main minimum value of the profile can be used to obtain the correct location of the $x_i = 0$. A straight line joining the maximum to the minimum of the profile will intersect the anomaly curve at the point $x_i = 0$ (Stanley 1977).

Finally, since magnetic methods are often sensitive to the assumptions (*e.g.* is it really a thin dike?) then it is important to give a criterion by which an interpreter can judge whether the method is applicable in a given observed magnetic anomaly. Careful study of equation (1) indicates that the anomaly profile due to a thin dike will always have three zeros, two at infinity and one finite. Thus, when the magnetic profile has only one finite zero, then the assumption of thin dike holds. When the magnetic profile has two finite zeros, then the assumption of thin dike is rejected and the assumption of a horizontal cylinder (2-D) holds (Parker Gay 1965).

Synthetic Examples

Because the present method is a rapid technique developed for ready use in the field, an error response in depth, amplitude coefficient, and index parameter has been evaluated and is presented here.

The procedure of interpretation using the gradient method begins with fixing the origin. The position of x_{01} and x_{02} depends upon this origin. In effect x_{01} and x_{02} take errors from the determined origin and the anomaly values of extrema to produce erroneous results of the lateral position, depth and other parameters.

Thus, in studying the error response of the gradient method, synthetic examples of a thin dike (profile length = 40 units, $\theta = -135°$, $z = 2$ units, $M = 100$ units and sampling interval = 1 unit) is considered. When we apply a simple numerical differentiation filter of window length $s = 1$ unit to the observed magnetic data, it can be shown that the actual x_{01} and x_{02} values are 1 and –5 (Fig. 2), respectively. In each case errors of ±10% are imposed in both x_{01} and x_{02}. Following the interpretation method, values of the three parameters were computed and the percentage of error in each parameter are tabulated (Table 3). The interpreted values were found to differ, in general, from the true values, depending on the error magnitude and signs imposed in both x_{01} and x_{02}, simultaneously.

When x_{01} and x_{02} both have errors of equal magnitude and sign simultaneously, the interpreted z and M will differ widely from the actual values. The maximum error in z and M is ± 12.5 and ± 6.6%, respectively. In the case of the index parameter the results will not vary much from the true values. When x_{01} and x_{02} both possess error of equal magnitude and opposite sign simultaneously, the interpreted z

Table 3. Numerical results

% of error in x_{01}	% of error in x_{02}	% of error in z	% of error in M	% of error in θ
10	10	12.36	6.56	–0.45
10	0	6.07	–0.39	–1.78
10	–10	–0.63	–7.30	–3.30
0	10	6.07	6.90	1.25
0	0	0.00	0.00	0.00
0	–10	–6.46	–6.84	–1.42
–10	10	–0.63	7.74	3.01
–10	0	–6.46	0.92	1.94
–10	–10	–12.68	–5.81	0.64

value will not differ much from the true values. In case of M and θ the interpreted values will vary from the true values. The maximum error is $\pm$ 8 and $\pm$ 4%, respectively. Finally, when x_{01} or x_{02} is kept undisturbed, the percentage error in the model parameters, is always smaller than the imposed error. This demonstrates that the present approach is less sensitive to knowing the exact values of both x_{01} and x_{02}. It is also verified that our method would yield more accurate results than the methods of Stanley (1977) because the maximum and minimum anomaly values do not have to be precisely known and at the same time, our method does not depend on the base line determination as Stanley's does. The horizontal gradient filter with any window length removes the effect of the base line.

Field Example

Fig. 3 shows a vertical magnetic anomaly from the Pima copper mine, Arizona (Parker Gay 1963, Fig. 10, p. 198). It represents the anomaly due to thin dike. This profile of 675 m length was digitized at interval of 22.5 m. Five successive gradient windows were applied to the input data (Fi.g 4). In each case a simple linear interpolation technique is used to determine x_{01} and x_{02} from the observed gradients (Davis 1973). The model parameters (z, M, θ) obtained from each gradient profile are given in Table 4. The average depth obtained by our method is 70 m. It agrees well with the depth of 64 m obtained from drilling.

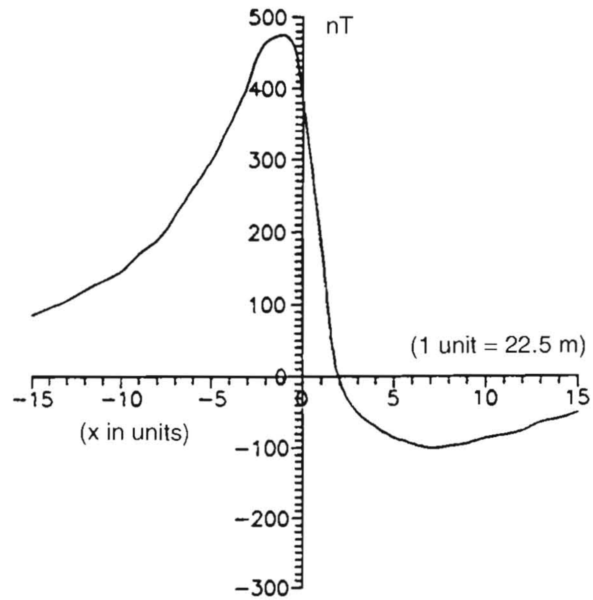

Fig. 3. Vertical magnetic anomaly over Pima copper mine in Arizona.

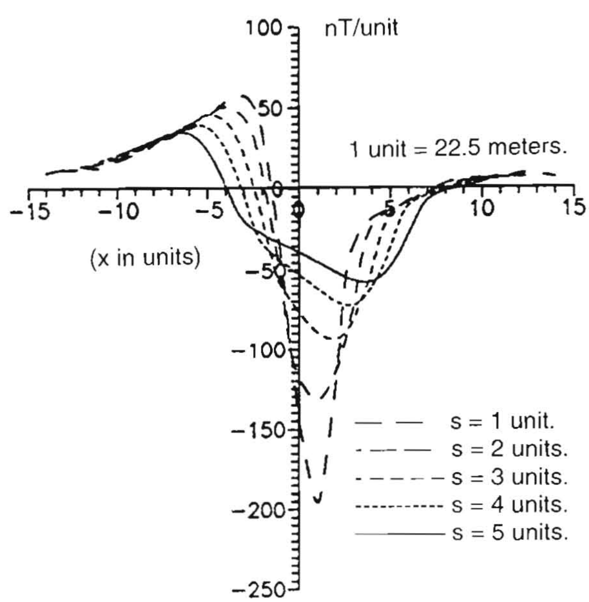

Fig. 4. Horizontal magnetic gradients over Pima copper mine in Arizona.

Table 4. Interpreted depth, amplitude coefficient, and index parameter as computed from numerical horizontal magnetic gradients of Pima copper magnetic anomaly, Arizona, using the present method

Window Length (s) (Spacing Units)	Depth (Meters)	Amplitude Coefficient (nT)	Index Parameter (Degrees)
1	72.4	678.9	–47.7
2	71.2	699.0	–48.0
3	72.3	597.4	–50.1
4	70.4	555.9	–51.0
5	65.3	567.4	–51.2
Average	70.4	616.1	–49.6

Conclusion

The depth determination problem, assuming a buried thin dike-like structure, using numerical horizontal magnetic gradients has been transformed into finding the zero anomaly distances. Our method involves using a simple dike model convolved with the same gradient filter as applied to the observed data. As a result, our method does not depend on a prior knowledge of the base line and hence it gives more reliable results than some of the existing methods. A case study demonstrates the efficiency of the present technique where highly distorted magnetic gradients yield reliable depth estimates.

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طريقة بسيطة لتعيين العمق للجدد الرفيعة من معدل التغير الأفقي العددي للمجال المغناطيسي

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نقدم في هذا البحث طريقة بسيطة وسريعة يمكن تطبيقها في الحقل لتعيين العمق للجدد الرفيعة المدفونة تحت سطح الأرض من معدل التغير الأفقي العددي لشذات المجال المغناطيسي الأرضي وقد حولت مشكلة تعيين العمق إلى إيجاد قيم المسافات الأفقية التي يبلغ قيمة التغيير الأفقي العددي لشذات المجال المغناطيسي الأرضي صفر .

تم اعطاء معادلات لتقدير بقية عناصر النمذجة وتشمل السعة المغناطيسية والميل المغناطيسي كما تم تطبيق الطريقة على بيانات نظرية بها خطأ يقدر ± ١٠ بالمائة في قيم كل من المسافات الأفقية وجد أن نسبة الخطأ في تعيين العمق من شذات معدل التغير الأفقي العددي للمجال المغناطيسي لجده رفيعه لاتزيد عن ١٢ بالمائة عندما تحتوي كل من المسافات الأفقية على نسبة خطأ مقدارها ١٠ بالمائة ولها نفس الأشارة في نفس الوقت أما في حالة تضاد الأشارة في كل من المسافات الأفقية فوجد أن نسبة الخطأ في تقدير الأعماق تقترب من الصفر .

تتميز الطريقة عن معظم الطرق الأخرى المنشورة بأنها لا تعتمد على تعيين خط الأساس حيث أن أي فلتر عددي لإيجاد معدل التغير الأفقي لشذات المجال المغناطيسي يذيل مباشرة تأثير خط الأساس في تعيين العمق وبقية عناصر النموذج .

تم تطبيق الطريقة بنجاح على مثال حقلي من أريزونا وشمل تحليل الشذات المغناطيسية الرأسية لجده بمنجم من النحاس ووجد أن متوسط العمق إلى قمة الجده الناتج من استخدام عدة فلاتر مختلفة على البيانات المغناطيسية هو ٧٠ متراً وهو يتفق مع العمق المعلوم من بيانات الحفر والذي يبلغ ٦٤ متراً .