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Discharge Equation for Simultaneous Flow Over Rectangular Weirs and Below Inverted Triangular Weirs

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ABSTRACT. A combined weir consisting of a sharp-crested rectangular contracted weir and an inverted triangular weir below is a flow regulation device used in irrigation works for diverting the flow from a main channel to a secondary channel. The present practice for computing the discharge for classical rectangular and triangular weirs is from curves or equations relating the discharge coefficient and the upstream depth/gate opening ratio parameter including weir geometry, and tailwater depth/gate opening ratio being the fourth parameter which is needed to be updated. The discharge for simultaneous flow still in addition involves few more parameters: the distance between the weirs, the slope of the apron, the opening depth and weir angle. In this paper one generalized equation including all the important variables is obtained from the experimental investigations. This equation is suitable for both horizontal and sloping channel bed under submerged flow and for the free flow, the tailwater depth / weir opening ratio term being neglected. The predictions of the equation agreed well with the experimental data.

Weirs are the oldest hydraulic devices used for flow diversion and measurement purposes as head regulators for canals, branches, silt-flushing in a raw water intake in alluvial river or a power canal forebay and so on. The weirs operate commonly under the conditions of underflow, overflow and occasionally simultaneous underflow - overflow. Previous studies indicate that Swamee (1988, 1992) presented general equations separately for rectangular weirs and a sluice gate in terms of coefficient of discharge, C_d, for both free and submerged flows. The information regarding the discharge measurement for the simultaneous condition of rectangular and triangular weirs are rarely reported (Ackers *et al.* 1978), Ahmed (1985), Bos (1976), British Standards Institution BSI 3680 (1965), Charles (1956), Chow (1959), Hickox (1944), Majcherek (1985)). A few cases of simultaneous underflow - overflow gates were reported by Naudascher (1991). Also, recently it was reported in East Africa that withdrawal of raw water from alluvial rivers with high rates of sediment transport causes major problems. The raw water intake needs to be rehabilitated due to bed load entering and the raw water pumping system experiences major problems. Two-dimensional or 3-dimensional combined weir systems are desired to prevent such sedimentation (Drewes *et al.* 1994). In this experimental program a combination of rectangular weir and an inverted triangular weir below is suggested and the discharge measurement for the combined weirs considering different weir geometry and flow conditions is investigated in order to obtain generalization.

Theoretical Background

Figure 1 shows the definition sketch for free flow below an inverted triangular and over a sharp-crested rectangular weir system. The conventional flow equation for contracted sharp-crested rectangular weir is given by:

$$Q_w = \frac{2}{3} C_w \sqrt{2g}(b-0.1\ nh)h^{1.5} \qquad (1)$$

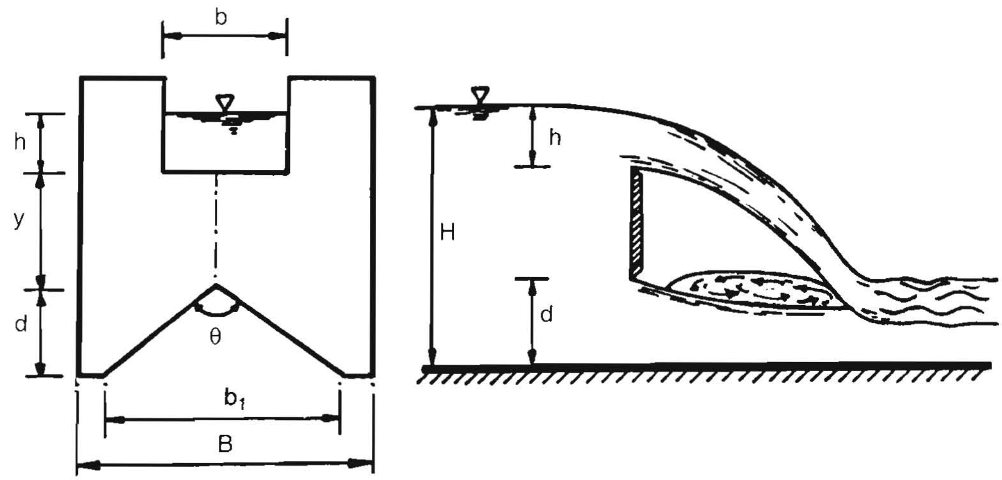

Fig. 1. Definition sketch.

where Q_w is the discharge over the weir; C_w is the discharge coefficient; b is the weir width; n is the number of side contraction; h is the upstream head and g is the acceleration due to gravity.

Similar to the case of a normal sluice gate, the discharge equation for the contracted sharp-edged inverted triangular weir can be written as (Henry 1950, Swamee 1992 and Rajaratnam and Subramanya 1967):

a- For free flow

$$Q_{tf} = \frac{1}{2} C_t b_l d \sqrt{2g} \sqrt{d+y+h} \quad \text{.......} (2)$$

b- for submerged flow

$$Q_{ts} = \frac{1}{2} C_t b_l d \sqrt{2g} \sqrt{d+y+h-h_d} \quad \text{.......} (3)$$

in which Q_t is the flow rate; suffices f and s represent free and submerged flow conditions; C_t is the discharge coefficient; b_l is the bottom weir width; d is the opening depth of triangular weir; h_d is tailwater depth and y is the distance between the bottom edge of upper weir and upper edge of the traingular weir.

Combining the two equations (1) and (3), the total combined discharge, Q_a, for the weir system can written as:

$$Q_a = \frac{2}{3} C_w \sqrt{2g}\,(b-0.1\,nh)h^{1.5} + \frac{1}{2} C_t b_l d \sqrt{2g} \sqrt{d+y+h-h_d} \quad \text{.......} (4)$$

Equation (4) can also be written:

$$\frac{Q_a}{\sqrt{2g}\, b_l d^{1.5}} = \frac{2}{3} C_w \frac{b}{b_l} \left(1-0.2\frac{h}{b}\right)\left(\frac{h}{d}\right)^{1.5} + C_t \sqrt{\frac{y}{d} + \frac{h}{d} + 1 - \frac{h_d}{d}} \quad \text{.......} (5)$$

The values of the discharge coefficient C_w and C_t for the combined weir flow identified by equation (5) are not easy to determine analytically. Thus, from equation (5), the following non-dimensional functional relationship can be written as:

$$\frac{Q_a}{\sqrt{2g}\, b_l d^{1.5}} = f\left(\frac{y}{d}, \frac{h}{d}, \frac{b}{b_l}, \frac{h}{b}, \frac{h_d}{d}\right) \quad (6)$$

Multiplying both sides by b_l/d, equation (6) can be written as:

$$\frac{Q_a}{\sqrt{2g}\, d^{2.5}} = f\left(\frac{y}{d}, \frac{h}{d}, \frac{b}{b_l}, \frac{b_l}{d}, \frac{h}{b}, \frac{h_d}{d}\right) \quad (7)$$

Hence equation (7) is the functional relationship for the non-dimensional discharge of the combined weir system in submerged flow conditions. Similarly for the free flow case (excluding h_d/d) the combined discharge equation is in the form:

$$\frac{Q_a}{\sqrt{2g}\, d^{2.5}} = f\left(\frac{y}{d}, \frac{h}{d}, \frac{b}{b_l}, \frac{b_l}{d}, \frac{h}{b}\right) \quad (8)$$

Experimental Set-Up

Experiments on a combined weir were conducted in a glass sided tilting recirculating flume of 9 m long, 0.30 m wide and 0.30 m deep. The water depths were measured by point gauges having an accuracy of ± 0.1 mm. The discharge through the combined weir was measured by a calibrated V-notch installed in a measuring tank. The measuring tank is located below the outlet of the flume downstream and is connected directly to the underground sump. The tail gate was provided at the downstream end to control the tailwater depth in the flume. A centrifugal pump is used to supply water from the basement sump to the flume inlet for recirculation.

Thirteen combined weir models were tested. Table 1 gives the range of various parameters covered in the present study. The weir models were made of 12 mm thick perspex sheets with all the edges were bevelled at 45° to form sharp edges of about 1 mm thickness. The sides of the weir models were provided with rubber sheets to prevent leakage. The models were fixed to the flume at the middle using two supports from downstream sides. The supports are made of perspex sheets 10 cm long, 2.5 cm wide and 12 mm thick, stuck to the flume side walls using silicon rubber. In addition to the horizontal bed case, two bed slopes were used (0.77% and 1.61%). The selection of model materials and bed slopes were based on the limitations of available facilities. The effect due to the viscosity and surface tension on the discharge over the weir was neglected as their effect is insignificant

(Kindsvater and Carter 1957). The flow conditions were maintained similar to Swamee (1992) as:

a- free flow, $h_d < d$; b- submerged flow, $h_d < h + y + d < 2.5\ h_d(h_d/d)^{0.2}$.

The test procedures were as follows:

(a) The channel was adjusted to the horizontal position.

(b) The weir model was fixed at the middle position from upstream of the flume.

(c) The tailgate was lowered to its minimum position and allowed a certain discharge to pass.

(d) The flow stability was attained, *i.e.* the upstream water level becomes constant, then the following parameters were measured: the discharge, the upstream water level at a distance four times the head over the weir crest and the tailwater depth far away from the model.

(e) The tail gate was raised until the upstream water level just starts to increase, then the tail depth was measured again.

(f) The procedures were repeated for each weir model and bed slope.

Table 1. The tested models dimensions and details

Model	b	b_1	d	y	θ	b_1/d	y/d	b/b_1
1	10	20	10	10	90	2	1	0.5
2	20	11.54	10	12	60	1.15	1.2	1.37
3	15	11.54	10	15	60	1.15	1.5	1.3
4	10	20	10	7	90	2	0.7	0.5
5	15	20	10	10	90	2	1	0.75
6	20	20	10	10	90	2	1	1.01
7	20	20	10	12	90	2	1.2	1
8	15	20	10	15	90	2	1.5	0.75
9	20	20	10	14.5	90	2	1.45	1
10	5	20	10	10	90	2	1	0.25
11	20	8.28	10	12	45	0.83	1.2	2.42
12	20	5.37	10	12	30	0.54	1.2	3.72
13	20	28.53	10	12	110	2.85	1.2	0.7

Analysis of Results and Discussions

During the simultaneous underflow and overflow for the combined contracted sharp-crested weir system, the discharge was found to be affected by a number of variables (Eq. 7). A general non-dimensional equation for predicting the discharge through the combined weir (over a rectangular weir and a triangular weir) below) can therefore be written as:

a- submerged flow:

$$\frac{Q_a}{\sqrt{2g}d^{2.5}} = C_1 + C_2\frac{y}{d} + C_3\frac{h}{d} + C_4\frac{b}{b_1} + C_5\frac{h}{b} + C_6\frac{b_1}{d} + C_7\frac{h_d}{d} \quad \text{..............} \quad (9)$$

where C_1 through C_7 are constants.

b- free flow:

$$\frac{Q_a}{\sqrt{2g}d^{2.5}} = C_1 + C_2\frac{y}{d} + C_3\frac{h}{d} + C_4\frac{b}{b_1} + C_5\frac{h}{b} + C_6\frac{b_1}{d} \quad \text{........................} \quad (10)$$

the values of C_1 through C_7 for the submerged flow and C_1 through C_6 for the free flow cases were determined using multiple linear regression analysis of the experimental data.

A discussion of the effect of the upstream head, channel bed slope, weir angle and overflow head over the weir on the variation of the combined discharge is presented below:

a- Channel Bed Slope

It is a well known fact that the discharge through the rectangular weir and triangular weir on the sloping channel bed may be due to the combined effect of channel bed slope and the upstream head. The variation of the combined discharge was studied for all the sets of data and typically two sets of data (b = 20 cm, y = 12 cm; b = 12 cm and y = 7 cm) using 90° degree triangular weir with two different bed slopes (0.77% and 1.61%) are presented in Fig. 2. The figure shows the variation of discharge ratio through the combined system with the upstream head using the channel bed slope as a third parameter. It is noted from the figure that the discharge ratio increases linearly with upstream head and no significant variation of the discharge is observed with the increase in the bed slope, which may be due to the low range of slopes considered in this study.

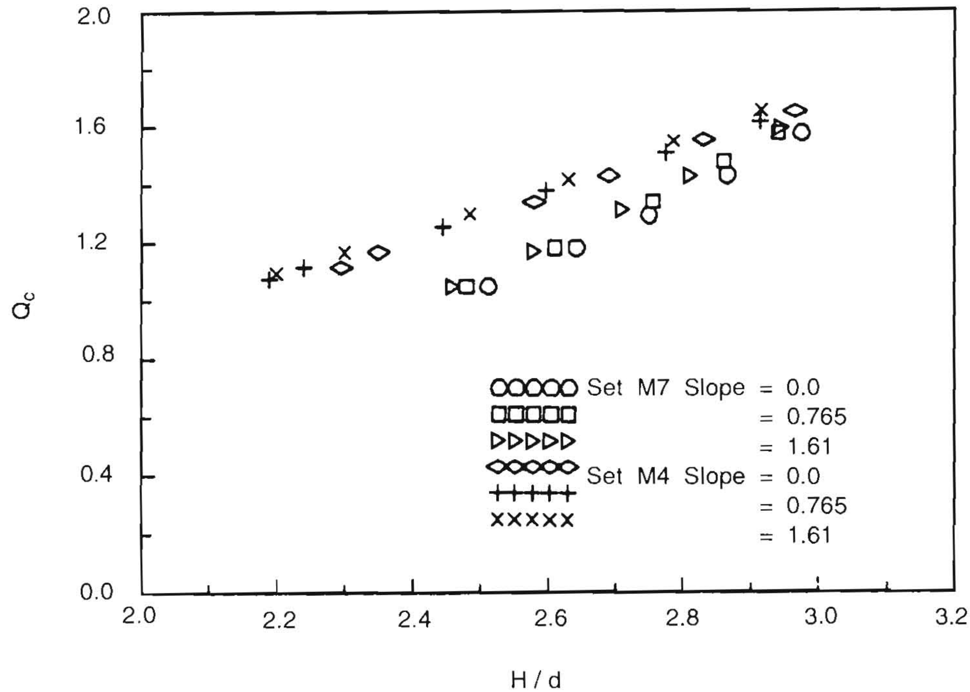

Fig. 2. Effect of upstream head ratio, H/d, and the bed slope, S_0, on the non-dimensional free discharge ratio for typical two sets of data with $\theta = 90°$.

b - Angle of Inverted Triangular Weir

Figure 3 shows the effect of overflow head on the discharge ratio through the combined weir system using different angles of inverted triangular weir below for free flow with constant y/d and width parameters. From the perusal of the figure, it is seen that the discharge increases linearly with the overflow head and there is a significant effect of the angle of the inverted triangular weir on the discharge variation such that the bigger the angle the larger the discharge obtained.

Using a 95% confidence limit, the values of the constants of the equations (9) and (10) obtained from the multiple linear regression analysis of tested data are:

a- For submerged flow ($b_1/d = 2$ and $2.28 < h_d/d < 2.48$):

$C_1 = 1.326$, $C_2 = -0.505$, $C_3 = -0.098$, $C_4 = 0.494$, $C_5 = 0.265$, $C_6 = 0$, $C_7 = -0.026$ ($R^2 = 0.957$, SEE = 0.021).

b- For free flow ($0.54 < b_1/d < 2.85$):

$C_1 = -0.423$, $C_2 = 0.187$, $C_3 = 1.27$, $C_4 = 0$, $C_5 = -0.466$, $C_6 = 0.469$ ($R^2 = 0.971$, SEE = 0.054).

The limiting conditions of equations (9) and 10) are: $0.7 < y/d < 1.5$, $0.5 < b/b_1 < 3.72$ and $30° < \theta < 110°$.

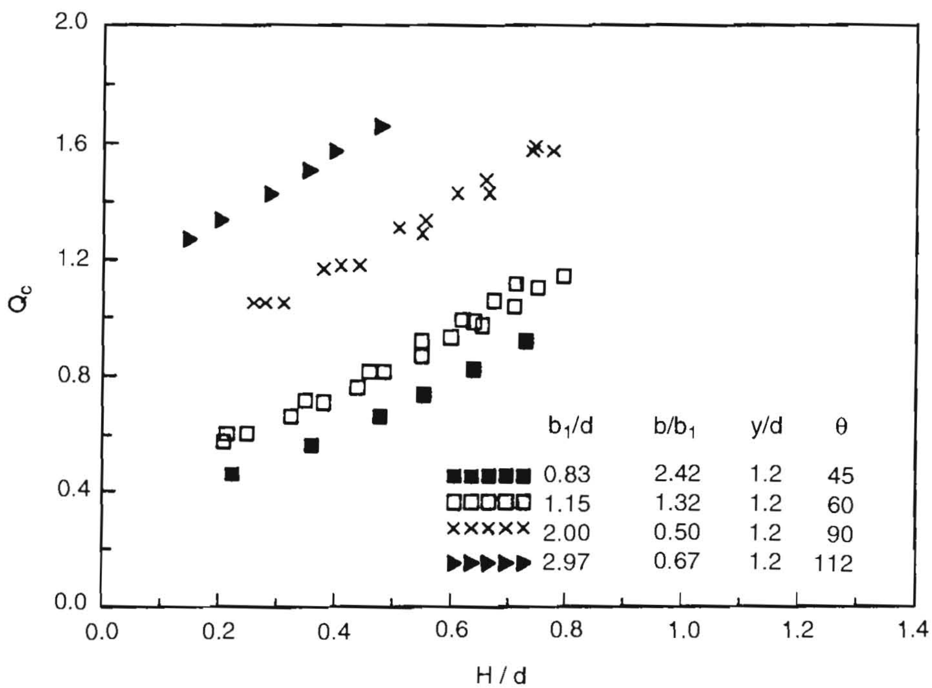

Fig. 3. Effect of overflow head ratio, h/d, and the angle of the inverted triangular weir, θ, on the non-dimensional free discharge ratio for constant y/d and b = 12 cm.

The mean absolute error resulted by the proposed prediction equation is about 5% compared to the present data. Figures 4 and 5 present the verification of equations (9) and (10). A good fit is observed in the figures by comparing the nondimensional discharge ratio predicted by the equations to those observed for the submerged and free flows.

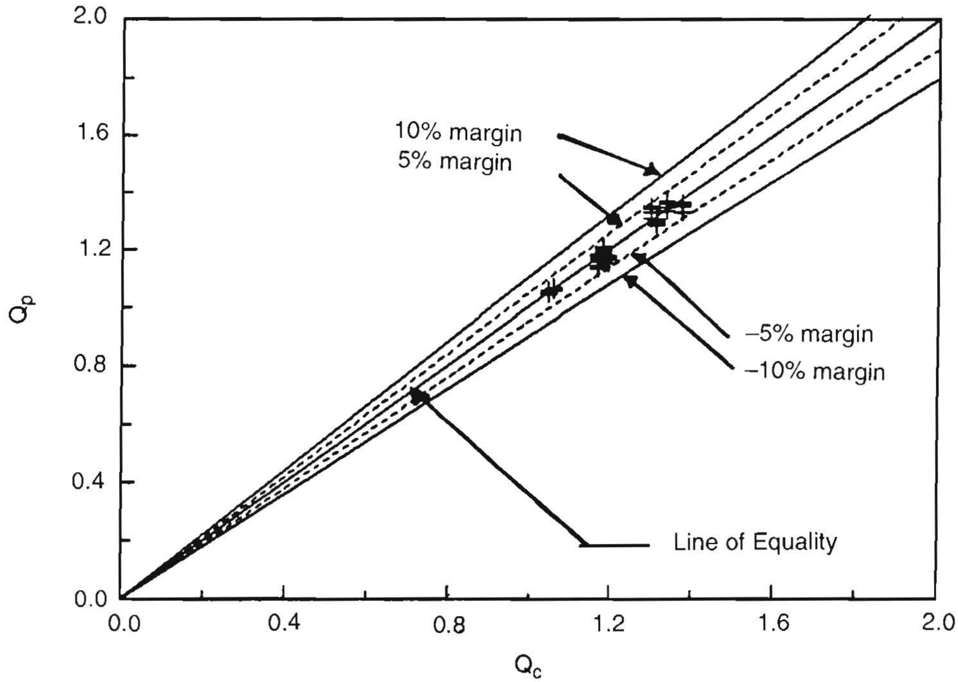

Fig. 4. Verification of prediction eq (9) for submerged flow at $b_1/d = 2$.

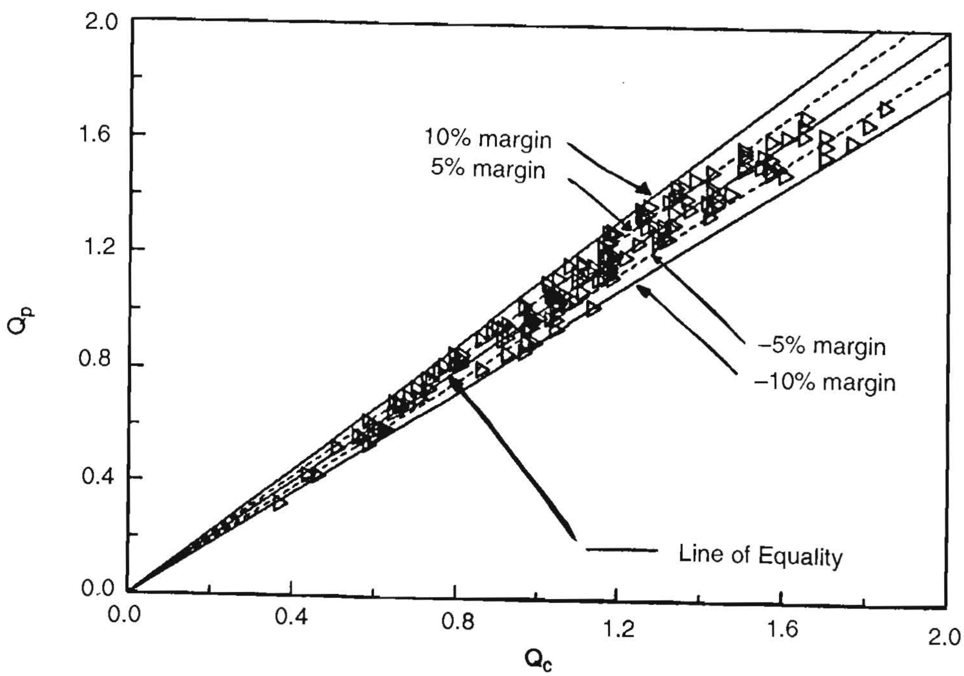

Fig. 5. Verification of prediction eq (10) for free flow.

Conclusions

1. The channel bed slope has a negligible effect on the discharge variation through the combined weir system.
2. The influence of angle of triangular weir below has a significant effect on the combined discharge through the weir system and the bigger the angle, the larger the discharge resulted.
3. Non-dimensional equations for the combined discharge over a contracted sharp-crested and an inverted triangular weir below under free and submerged flow conditions were obtained. Using the equations (9) and (10) the discharge through the combined weir system on both horizontal and sloping floors can be computed accurately.

Notations

B	width of the flume.
C	regression constant (with suffix 1 to 7).
b	width of the contracted rectangular weir.
b_1	bottom width of the inverted V-notch.
C_t	discharge coefficient of the inverted triangular weir (f for free flow and s for submerged flow).
C_w	discharge coefficient of the contracted rectangular weir only.
d	depth or height of the inverted V-notch.
g	acceleration due to gravity.
H	upstream flow depth.
h	the measured head over the weir.
h_d	the measured head downstream the structure.
n	number of sides contractions of the weirs.
Q_a	the measured total combined discharge.
Q_c	the non-dimensional discharge term, ($Q_c = Q_a / \sqrt{2g}\, d^{2.5}$).
Q_p	the predicted non-dimensional discharge term using eq (9) or eq (10).
Q_t	discharge below the inverted triangular weir only (f for free flow and s for submerged flow).
Q_w	discharge over the contracted rectangular weir only.
R^2	coefficient of determination of the regression equations.
y	the solid distance between the upper opening and the lower one.
θ	the angle of the inverted V-notch.

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معادلة التصرف للتدفق الأني فوق الهدارات المستطيلة وتحت الهدارات المقلوبة المثلثة الشكل

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هذه الدراسة عبارة عن قياسات معملية لكمية التصرف للتدفق الأني من خلال هدار مزدوج . هذا الهدار عبارة عن فتحة علوية مستطيلة الشكل وفتحة سفلية مثلثة الشكل . مثل هذا النوع من الهدارات يمكن إستخدامه في أعمال الري لتحويل الماء من القنوات الرئيسية إلى القنوات الفرعية وفي التحكم في كمية التدفق وقياسه .

من المعلوم أن كمية التصرف العابر فوق الهدارات التقليدية يتم حسابها من منحنيات أو معادلات تربط بين معامل التصرف وشكل الهدار وعمق الماء قبل الهدار . أما بالنسبة للهدار المزدوج فإن كمية التصرف للتدفق الأني تتأثر بعوامل أخرى إضافة إلى ما سبق ذكره بالنسبة للهدارات التقليدية . هذه العوامل هي المسافة بين الهدار العلوي والسفلي ، إرتفاع فتحة الهدار السفلي ، زاوية الهدار إضافة إلى ميل القناة .

في هذه الدراسة تم إستخدام عدد ضخم من التجارب المعملية باستخدام ١٣ نموذج من الهدارات المزدوجة بأبعاد مختلفة لدراسة تأثير هذه الأبعاد على كمية

التصرف لهذا النوع من الهدارات . وقد تم التوصل إلى معادلة عامة ، تشمل جميع المتغيرات المؤثرة في كمية التصرف للتدفق الأني . ويمكن إستخدام هذه المعادلة تحت الظروف المختلفة للتدفق تحت الهدار السفلي ، كما يمكن إستخدامها للقنوات المائلة إضافة إلى القنوات الأفقية . هذا وقد أظهرت هذه المعادلة دقة عالية مقارنة بالنتائج المعملية .