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A Simple Method for Depth Determination from Moving Average Residual Gravity Anomalies

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ABSTRACT. A new approach to depth determination from normalized moving average residual gravity anomalies is developed. The depth estimation has been transformed into a solution of a nonlinear equation of the form $z = f(z)$. Formulas have been derived for a sphere, horizontal cylinder, vertical cylinder, and vertical fault. The method can be applied to residuals and Bouguer gravity data of a short profile length. The method is applied to synthetic data with and without random errors and its validity is tested on two field examples: (1) The Abu Roash Dome gravity anomaly, west of Cairo, Egypt and (2) The Louga gravity anomaly, West Senegal, in West Africa.

Simple geometrically shaped models are very useful in depth determination from gravity data acquired in a small area over a buried structure. Several methods of depth estimation are currently in use, such as characteristic points and distances techniques (Nettleton 1976, Abdelrahman and El-Araby 1992), Kelvin transformation (Nedelokov and Burnev 1962), Fourier transform (Odegard and Berg 1965), Mellin transform (Mohan *et al.* 1986), Walsh transforms (Shaw and Agarwal 1990), ratio techniques (Bowin *et al.* 1986, Abdelrahman *et al.* 1989), least-squares minimization approaches (Gupta 1983, Lines and Treitel 1984, Adelrahman and El-Araby 1991, Abdelrahman and Sharafeldin 1995). On the other hand, the moving average (grid) method is a simple technique for the separation of gravity anomalies into residual and regional components. The method is originally developed by Griffin (1949) and application of least-squares to moving average method is described by Agocs (1951).

Abdelrahman and El-Araby (1993) developed a least-squares minimization approach to estimate the depth of a buried structure from a moving average residual gravity anomaly. Abdelrahman and El-Araby (1994) showed that the correlation factors between different moving average residual gravity anomalies can be used in quantitative studies of gravity data. More recently, Abdelrahman and El-Araby (1995) developed a numerical approach to estimate the depth of a buried sphere and a horizontal cylinder from the zero anomaly distance of the moving average residual gravity anomaly profile. In the present paper, we introduced a new interpretive technique to determine the depth of a buried structure from a normalized moving average residual anomaly value. The method is based on simple models convolved with the same moving average filter as applied to the measured gravity. Formulas are derived for a sphere, horizontal cylinder, vertical cylinder and the first horizontal derivative of the gravity effect of a two-dimensional (2-D) thin faulted layer. The validity of the method is tested on synthetic data with and without random errors and on two field examples from Egypt and Senegal.

The Method

The gravity effects of the vertical cylinder (semi-infinite vertical line element approximation), the horizontal cylinder, the sphere, and the first horizontal derivative (FHD) of the gravity effect of a thin faulted layer are defined as (Abdelrahman *et al.* 1989):

$$g(x,z) = \frac{Az^m}{(x^2 + z^2)^q}, \qquad \text{.................................(1)}$$

where z is the depth; x is a position coordinate; and A, m and q are defined in Table 1. The geometries are shown in Fig. 1.

Table 1. Definition of A, m, and q in equation (1). G is the universal gravitational constan σ is the density contrast; R is the radius, and t is the thickness of the fault

Model	A	m	q
Sphere	$4/3\pi G\sigma R^3$	1	1.5
Horizontal cylinder	$2\pi G\sigma R^2$	1	1
Vertical cylinder	$\pi G\sigma R^2$	0	0.5
Vertical fault (FHD)	$2G\sigma t$	1	1

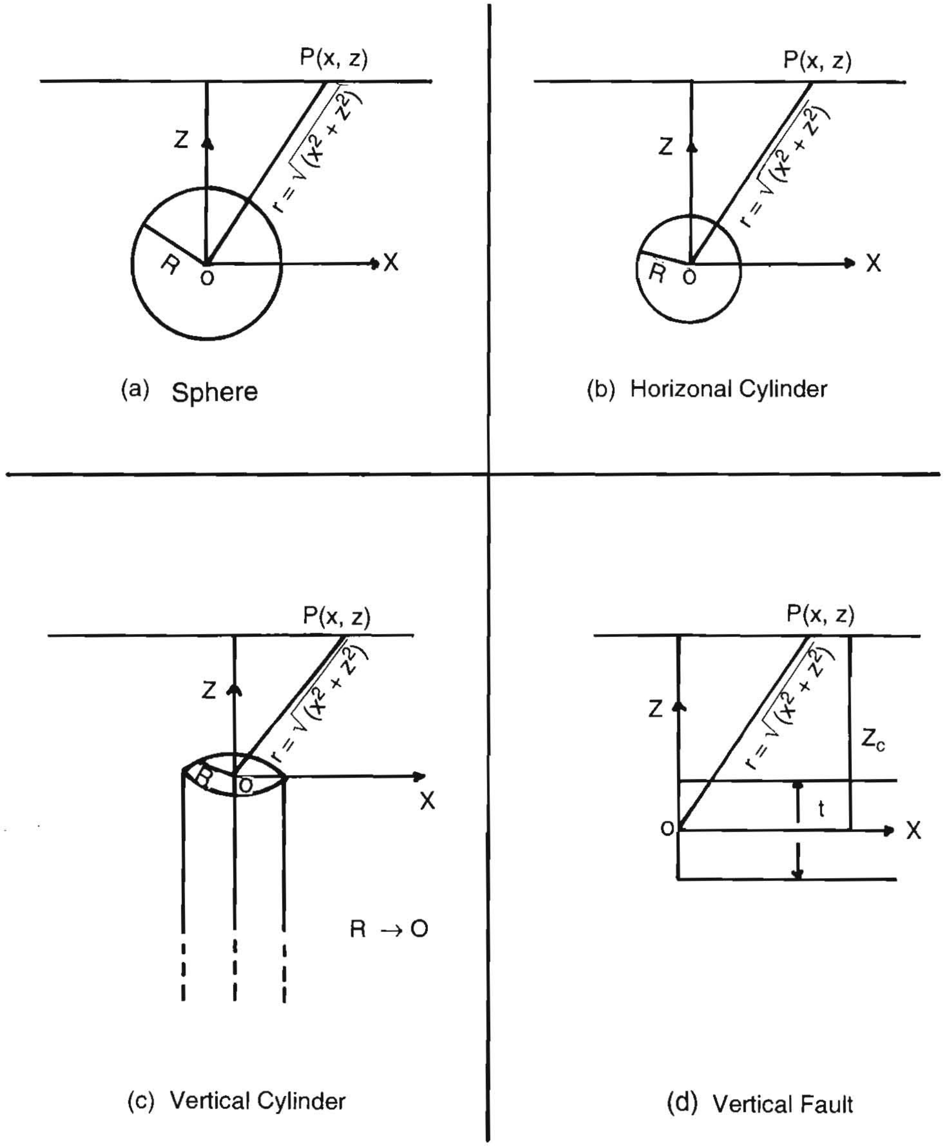

Fig. 1. Simple geologic structures (after Gupta 1983).

Consider three observation points $x_i - s$, x_i, and $x_i + s$ on the anomaly profile where $s = 1, 2, 3, ..., M$ spacing units and is called the window length or graticule spacing. The moving average regional gravity field $Z(x_i, z, s)$ is defined as the average of $g(x_i - s, z)$ and $g(x_i + s, z)$, which is for the above mentioned four simple shapes can be written as

$$Z(x_i, z, s) = \frac{Az^m}{2}\left\{((x_i - s)^2 + z^2)^q + ((x_i + s)^2 + z^2)^q\right\} \quad \text{.................................} (2)$$

The moving average residual gravity anomaly $g_{res}(x_i, z, s)$ at the point x_i is

$$g_{res}(x_i, z, s) = \frac{Az^m}{2}\left\{2(x^2 + s^2)^{-q} - ((x_i - s)^2 + z^2)^{-q} - ((x_i + s)^2 + z^2)^{-q}\right\} \quad \text{.......} (3)$$

Equation (3) represents a simple high pass filter to separate residual anomalies from observed data.

At $x_i = 0$, equation (2) takes the form:

$$g_{res}(0) = \frac{Az^m((s^2 + z^2)^q - z^{2q})}{z^{2q}(s^2 + z^2)^q} \quad \text{..} (4)$$

where $g_{res}(0)$ is the anomaly value at the origin ($x_i = 0$).

Using equation (4), equation (3) can be written in a normalized form as

$$g_{res}(x_i, z, s)_n = \frac{z^{2q}(s^2 + z^2)}{2((s^2 + z^2)^q - z^{2q})}\left\{2(x_i^2 + s^2)^{-q} - ((x_i - s)^2 + z^2)^{-q} - ((x_i + s)^2 + z^2)^{-q}\right\} \text{....} (5)$$

where $g_{res}(x_i, z, s)_n = g_{res}(x_i, z, s) / g_{res}(0)$.

Let the value $g_{res}(x_i, z, s)_n$ at the point $x_i = s$ be denoted by $g_{res}(z, s)_n = F$, then from equation (5), we obtain

$$F = \frac{2z^{2q}(4s^2 + z^2)^q - z^{2q}(s^2 + z^2)^q - (s^2 + z^2)^q(4s^2 + z^2)^q}{2((s^2 + z^2)^q - z^{2q})(s^2 + z^2)^q} \quad \text{.....................} (6)$$

when it is solved for z, is gives the following nonlinear equation

$$z = \left\{ \frac{2F((s^2 + z^2)^q - 2z^{2q})(4s^2 + z^2)^q + (s^2 + z^2)^q (4s^2 + z^2)^q}{2(4s^2 + z^2)^q - (s^2 + z^2)^q} \right\}^{1/2q} \quad \text{.....................(7)}$$

The parameter z can be obtained using simple iteration method (Gerald and Wheatley 1989). The iterative form of equation (7) is given as

$$z_f = f(z_j) , \quad \text{..(8)}$$

where z_j is the initial depth and z_f is the revised depth. z_f will be used as z_j for the next iteration. The iteration stops when $|z_f - z_j| \leq e$, where (e) is a small predetermined real number close to zero.

The source body depth is determined by solving one nonlinear equation in z. Any initial guess for z works well because there is always one global minimum. In all cases, in order to overcome the difficulty in evaluating the moving average residual values at the end points of the anomaly profile, the minimum number of data points along the profile must be equal to 4s + 1.

Since z and $g_{res}(0)$ are known, the amplitude coefficient (A) can be determined from equation (4). Because the density contrast (σ) is assumed to be known from density logs or core samples, the radius (R) of the sphere or of the cylinder and the thickness of the fault can be obtained from (A) and the relations given in Table 1. Real data contain measurement errors which may be compounded by errors in computing the depth. For this reason, it is recommended to use $F = \{g_{res}(z, -s) + g_{res}(z, s)\}/2$ when dealing with real data. The average of the F values would reduce the error in depth estimation.

Synthetic Examples

Numerical results for various test cases are shown in Tables 2 and 3. It was verified numerically that equation (7) gives accurate values of z when synthetic data with and without noise are analyzed. The depth obtained is within 4 percent in case of the sphere model and 7 percent in case of horizontal cylinder model. Moreover, it was numerically verified that the model parameters obtained by the present method are independent of the window length. Any value for the window length works well for various test cases (Table 3).

Table 2. Theoretical results in the case of profile length of 50 units with a station separation of 1 unit and the moving average window length of 4 units

Model depth	Sphere			Horizontal cylinder	
	Using synthetic data	Using synthetic data with 5% random error	Percentage of error in depth	Using synthetic data with 5% random error	Percentage of error in depth
1.00	1.00	1.002	0.184	1.003	0.343
2.00	2.00	1.983	−0.843	1.968	−1.581
3.00	3.00	2.932	−2.262	2.890	−3.668
4.00	4.00	3.992	−0.208	3.999	−0.036
5.00	5.00	4.804	−3.923	4.657	−6.855
6.00	6.00	5.975	−0.410	5.955	−0.746
7.00	7.00	7.068	0.974	7.109	1.551

Table 3. Theoretical results showing the effect of window length on depth determination. Profile length = 50 units, depth = 2 units, sampling interval = 1 unit

Window length	Sphere		Horizontal cylinder	
	Depth computed using 5% random error	Percentage of error in depth	Depth computed using 5% random error	Percentage of error in depth
1.00	1.949	−2.539	1.908	−4.617
2.00	1.971	−1.463	1.956	−2.196
3.00	1.994	−0.301	1.991	−0.449
4.00	1.992	−0.415	1.988	−0.602
5.00	1.959	−2.071	1.932	−3.386
6.00	1.995	−0.263	1.987	−0.669
7.00	1.966	−1.680	1.949	−2.550
8.00	2.018	0.898	2.030	1.504
9.00	1.971	−1.455	1.953	−2.342
10.00	1.999	−0.069	2.002	0.120

The present method can be applied not only to "true residual" but also to Bouguer anomaly profile of a very short length, because the moving average filter automatically removes the linear regional trends.

Field Examples

Abu Roash Dome anomaly

The Bouguer gravity map of the Abu Roash Dome, west Cairo, Egypt (after Abdelrahman *et al.* 1985) is given in Fig. 2. A gravity profile of 8.5 km length along

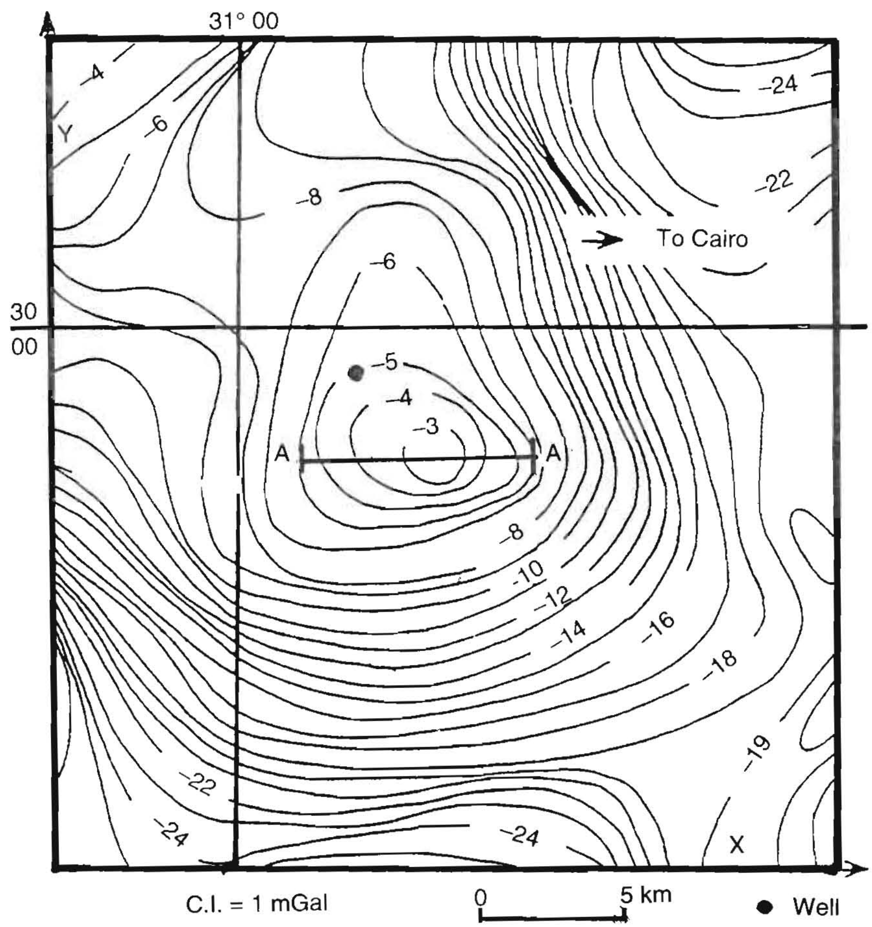

Fig. 2. Observed Bouguer gravity map of Abu Roash, west Cairo, Egypt. (Abdelrahman *et al.* 1985).

the line AA, Fig. 3, was digitized at interval of 0.425 km, Table 4. The 21 Bouguer gravity values have been subjected to a separation technique using the moving average method. Moving average windows, s, of 2, 3, and 4 spacing units were used and applied to the data (Fig. 3). The present method, equation (7), was applied to each of the three residual profiles, yielding three depth solutions (Table 6) from assumed vertical cylinder target. The average depth is 1.76 km. This value agrees very well with the drill hole data given by Said (1962) where the depth to the basement was found to be 1.81 km in Abu Roash well # 1.

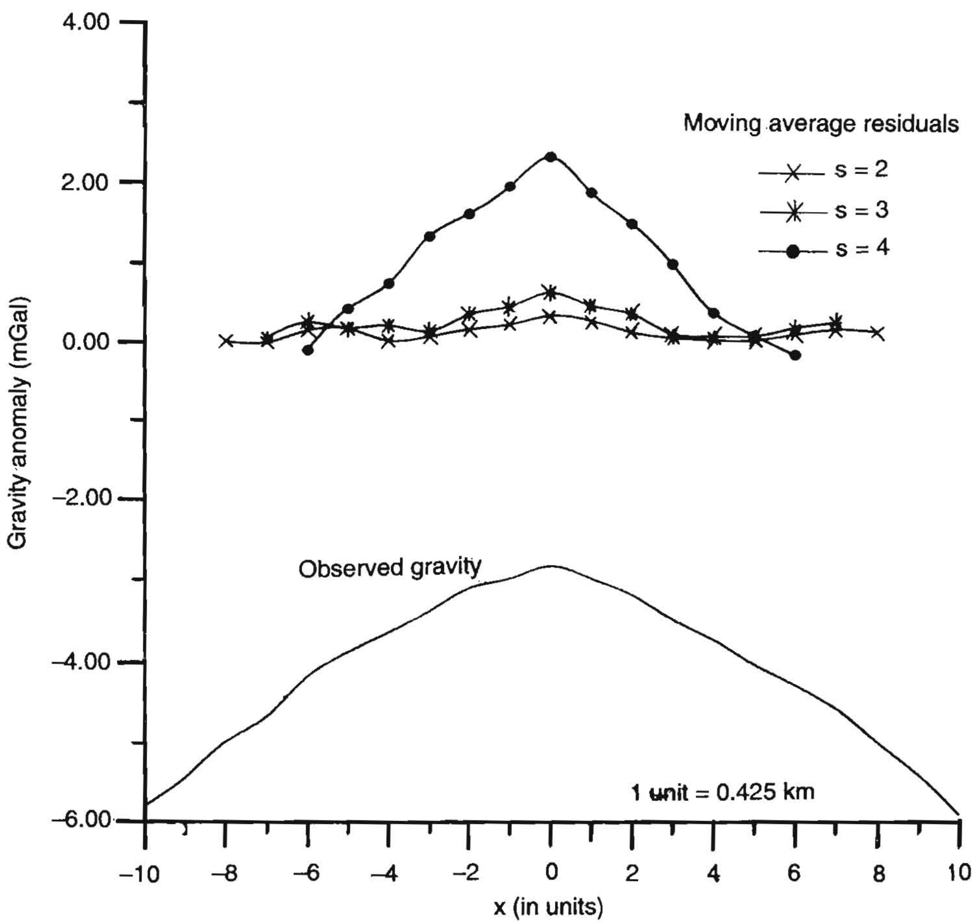

Fig. 3. A gravity profile of 8.5 km length along line AA' over Abu Roash Dome, West Cairo, Egypt, and its moving average residual anomalies.

Table 4. Observed gravity anomaly values and the moving average residuals of Abu Roash Dome, west Cairo, Egypt. Station separation is 0.425 km

x-coordinate	Bouguer values (mGal)	Moving average residuals (mGal)		
		$g_{res}(x_i, z, 2)$	$g_{res}(x_i, z, 3)$	$g_{res}(x_i, z, 4)$
–10.00	–5.80			
–9.00	–5.45			
–8.00	–5.00	0.00		
–7.00	–4.70	–0.02	0.03	
–6.00	–4.20	0.13	0.23	–0.12
–5.00	–3.90	0.15	0.16	0.40
–4.00	–3.66	0.00	0.19	0.72
–3.00	–3.40	0.05	0.12	1.31
–2.00	–3.12	0.14	0.33	1.59
–1.00	–3.00	0.20	0.43	1.93
0.00	–2.85	0.31	0.60	2.30
1.00	–3.00	0.25	0.44	1.85
2.00	–3.20	0.10	0.33	1.46
3.00	–3.50	0.03	0.08	0.96
4.00	–3.75	0.00	0.05	0.35
5.00	–4.05	0.00	0.05	0.06
6.00	–4.30	0.07	0.15	–0.18
7.00	–4.60	0.13	0.22	
8.00	–5.00	0.10		
9.00	–5.40			
10.00	–5.90			

The Louga anomaly

Figure 4 shows the gravity anomaly of an area on the west coast of Senegal in West Africa (Nettleton 1962, Fig. 4). (The picture is incomplete on the west side because the area is along the coast and the western part would be over the Atlantic Ocean). A north - south gravity profile of 40 km length along the line BB' is shown in Fig. 5. The anomaly was digitized at an interval of 2.5 km (Table 5). Three successive moving average windows were applied (Fig. 5). The depth obtained from each residual anomaly, assuming a spherical target, is given in Table 6. The average depth is 10.2 km. The depth according to Nettleton (1962, 1976) is 9.3 km.

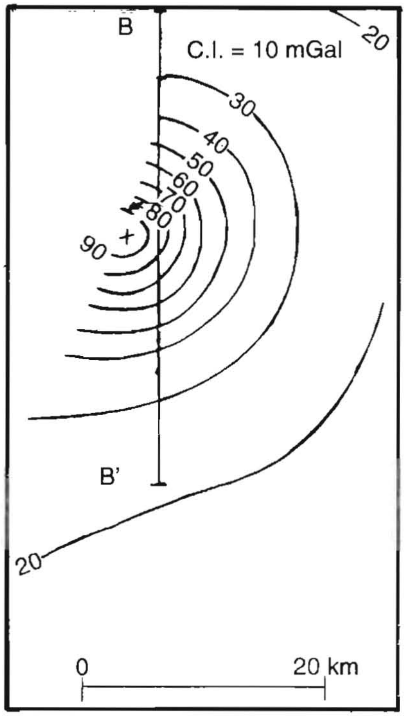

Fig. 4. Observed Bouguer gravity map of Louga area, West Senegal, West Africa (Nettleton 1962).

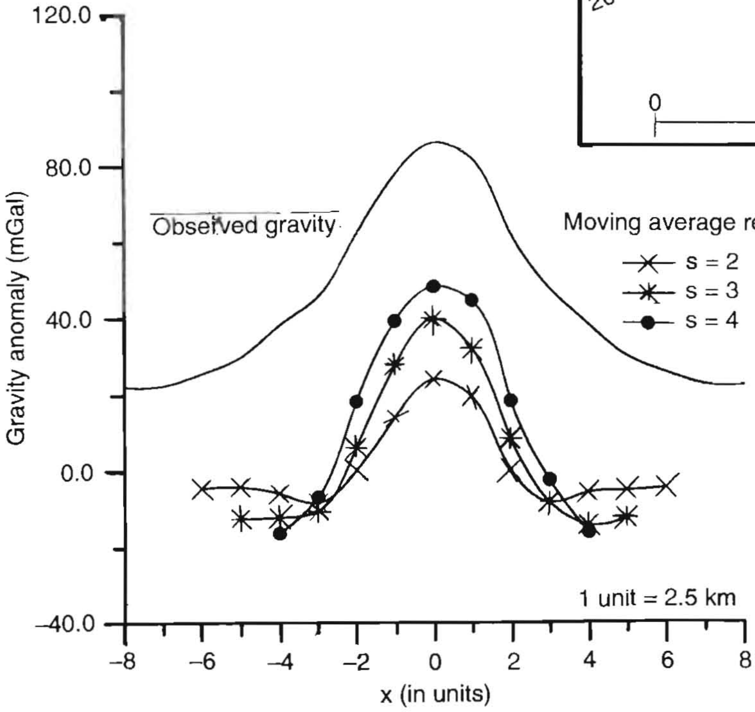

Fig. 5. A north-south gravity profile of 30 km over the Louga area, West Senegal, West Africa, and its moving average residual anomalies.

Table 5. Observed gravity anomaly values and the moving average residuals of the Louga area, West Senegal, West Africa. Station separation is 2.5 km

x-coordinate	Bouguer values (mGal)	Moving average residuals (mGal)		
		$g_{res}(x_i, z, 2)$	$g_{res}(x_i, z, 3)$	$g_{res}(x_i, z, 4)$
−8.00	22.20			
−7.00	22.47			
−6.00	25.58	−4.48		
−5.00	29.74	−4.16	−12.33	
−4.00	37.94	−5.83	−12.07	−16.09
−3.00	45.33	−8.31	−10.39	−6.75
−2.00	61.95	0.06	6.23	18.18
−1.00	77.53	14.03	27.59	38.96
0.00	85.84	23.90	39.48	47.89
1.00	81.69	19.22	31.73	44.16
2.00	61.95	0.04	8.31	18.18
3.00	47.40	−8.31	−8.31	−2.60
4.00	37.97	−5.79	−14.10	−16.05
5.00	29.74	−5.20	−12.33	
6.00	25.58	−4.50		
7.00	22.47			
8.00	22.20			

Table 6. Interpreted depth as computed from moving average residuals of Abu Roash gravity anomaly, West Cairo, Egypt, Louga gravity anomaly, West Senegal, West Africa, using the present method

Window length (Spacing units)	Computed depth (km)	
	(Abu Roash Anomaly)	Louga Anomaly
2	1.135	7.00
3	1.904	10.75
4	2.227	12.95
Average values	**1.755**	**10.23**

For both field examples of non-cylindrical and/ or non-spherical buried structures, this method gives (1) the position quite accurately, (2) a fair approximation of depth, and (3) a rough measure of the diameter and therefore the deeper flanks of the structure when the density contrast is known.

Conclusions

The depth determination problem, assuming a simple buried structure and using normalized moving average residual gravity anomalies, has been transformed into the problem of finding a solution of a nonlinear equation of the form $z = f(z)$. The simple models were convolved with the same moving average filter as applied to the observed data. As a result, the method can be applied not only to "true residuals" but also to Bouguer anomaly profiles of short length.

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طريقة بسيطه لتعيين العمق من شذات متبقيات الوسط المتحرك التثاقلية

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نقدم في هذا البحث طريقة جديده لتعيين الأعماق للتراكيب الجيولوجيه من شذات متبقيات الوسط المتحرك التثاقلي المعاير وقد حولت مشكلة تعيين العمق إلى مشكلة حل لمعادلة واحده غير خطية في شكل F(z)=0 .

وقد تم استنتاج معادلات للكره والاسطوانه الرأسيه والأفقية والصدع الرأسي ويمكن أن يتم تطبيق هذه الطريقة ليس فقط على شذات المتبقيات الحقيقية التثاقلية للتراكيب الجيولوجية لكن أيضا يمكن تطبيقها على شذات بوجير التثاقليه المأخوذه على بروفيل قصير الطول تقع تماما فوق التركيب الجيولوجي .

تم تطبيق الطريقة على بيانات نظرية بها خطأ عشوائي مقداره ٥٪ وبدون خطأ عشوائي أما في حالة وجود خطأ عشوائي ووجد أن الطريقة تعطي القيمه الفعليه للعمق في حالة عدم وجود خطأ عشوائي فأن نسبة الخطأ في تقدير العمق لا تزيد عن ٧٪ . وثبت أيضاً أن الطريقة لا تعتمد على أي افتراض مبدئي للعمق كما أنها لا تعتمد على طول نافذة الفلتر المستخدم في تعيين شذات الوسط المتحرك التثاقلي .

تم تطبيق الطريقة على بيانات حقليه للمجال التثاقلي تمثل في شذات بوجير على قبه أبو رواش الموجوده غرب القاهره بمصر وكذلك على منطقة اللوجه بغرب السنغال الواقعة بغرب أفريقيا ووجد في الحالتين أن متوسط العمق الناتج من تطبيق الطريقة يتفق مع بيانات الحفر أو البيانات التي تم الحصول عليها بواسطة علماء آخرين بأستخدام طرق أخرى .