O-Ring Static Sealing Reliability Model and Influence Factors Analysis

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Sp. Issue 2013/ # Paper 4

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KEYWORDS

O-ring; Static sealing reliability; Stress relaxation factor; Sealing influence factors; Stress relaxation

ABSTRACT

Sealing reliability plays a very important role in the aviation field. This paper put forward a new sealing reliability analysis method which can be applicable to most of the static seals. For the long-term using rubber seals, it will emerge stress relaxation which may lead to sealing failure. So, it needs to define a stress relaxation factor, which will analyze the sealing reliability taking the stress relaxation into account. Then, set the O-ring as a case, it calculated the static sealing reliability of the O-ring in operating conditions. And it also analyzed the influence of design parameters on the sealing reliability. Finally we found: (1) When the design parameters is certain, the stress relaxation factor, variance and correlation coefficient of the random variables will make a big influence on the sealing reliability; (2) When the design parameters is not certain, the compression ratio and media pressure also have a big influence on the sealing reliability.

Introduction

Air sealing failures may result in a waste of energy, the aircraft fault, and cannot complete the task, while sometimes may lead to a catastrophe, also causing huge economic losses for the airlines. Sealing failure is a complex and difficult problem, the sealing reliability makes a serious impact on the safety of people. Therefore, it has a very important significance for researching the sealing reliability.

This paper put forward a new viable sealing reliability analysis method, which is suitable for most static seals. Then set the o-ring as a case, it calculated the theoretical value of the maximum contact stress and maximum shear stress in FEA software (Abaqus). Finally, it analyzed the sealing reliability of O-ring based on sealing reliability model. The study has a great significance not only for O-ring but for sealing reliability.

Material and Methods

At present, in the field of sealing design and analysis, we often use following conditions as the criterion of reliable sealing:

(1) the contact stress of sealing surface is greater than the medium pressure;

(2) The maximum shear stress is less than the shear strength of the sealing material. Sealing ring must meet two requirements at the same time.

(1) The Random Variables Selection

Since there are manufacture errors and assembly errors in the actual processing, which make errors to exist in the equipments (Huang and Zhang, 2009; Wang *et al.*, 2010; Xiao, *et al.*, 2011). So, the maximum contact stress and the maximum shear stress of seals does not completely equal to the theoretical value, but subjects to a certain distribution. Therefore, in the static sealing reliability analysis, we select the maximum contact stress X_1 and the maximum shear stress X_2 as the random variables.

Assuming maximum contact stress and maximum shear stress following the normal distribution, that is $X_1 : N(\mu_1,\sigma_1^2)$ and $X_2 : N(\mu_2,\sigma_2^2)$. μ_1 and μ_2 are the expectation of their respective normal distribution; σ_1 and σ_2 are the variance of their respective normal distribution. The following is the reliability of maximum contact stress and maximum shear stress to meet the sealing conditions, and finally calculated the overall reliability of the static seal.

(2) The Reliability of Maximum Contact Stress to Meet the Sealing Conditions

For the long-term using rubber seals, due to the permanently compression deformation of the sealing ring, which will make the maximum contact stress decreased. So, it may lead to the sealing failure.

Wang Wei and Zaho Shu, gao stated that "after the stress relaxation, the maximum contact stress of the O-ring reduced obviously, and it decreased rapidly within 200 seconds." & "when the compression ratio is large, it should consider the reduction of the maximum contact stress (Wang and Zhao, 2008). Otherwise, it may lead to the sealing failure (Wang and Zhao, 2008)." The stress relaxation curves can be seen in fig. 1.

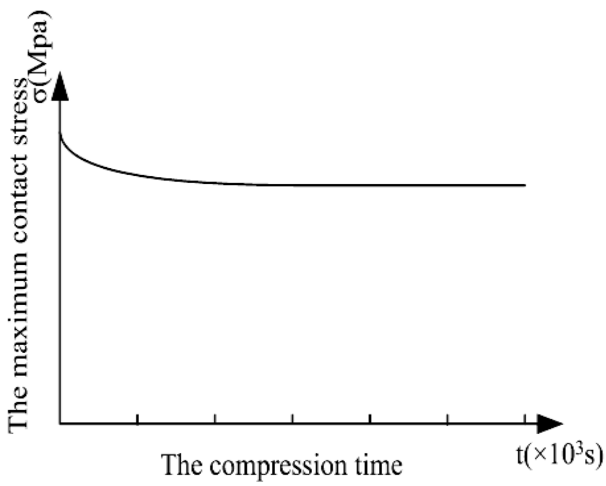


Figure 1: The Stress Relaxation of Rubber Seals

Therefore, we need to define a stress relaxation factor ξ: for the certain sealing device, under the certain temperature and certain compression ratio, the difference of the maximum contact stress between the initial time and t' time divided by the maximum contact stress of initial time, it as seen below.

$$\xi = \frac{\sigma_0 - \sigma_{t'}}{\sigma_0}$$

Wherein, σ_0 is the maximum cont act stress of the initial time; and $\sigma_{t'}$ is the maximum contact stress of time *t*'.

Stress relaxation factor ξ is related to the sealing forms, sealing materials, the compression ratio, the temperature and time, etc. The larger of stress relaxation factor is, the more serious of stress relaxation is.

Assuming that the maximum contact stress X_1, according to the first requirement of reliable sealing: maximum contact stress should be greater than the pressure medium. So the reliability of maximum contact stress to meet the sealing condition is $p_{r1} = p(x_1 \geq p_0)$, wherein, p_0 is the medium pressure; the expectation of X_1 is the maximum contact stress in the time t $\mu_1 = \sigma_t = (1-\xi)\tilde{x}_1$, $\tilde{x}_1$ is the maximum contact stress in the initial time. According to the 3σ criteria, the probability of maximum contact stress in the interval $[\mu_1 - 3\sigma_1, \mu_1 + 3\sigma_1]$ is 99.74%, we assume $3\sigma_1 = k \cdot \mu_1$, the variance reflects the seals process performance.

The geometrical meaning of the maximum contact stress reliability is the shaded area in Fig. 2; and its physical meaning is the probability of the maximum contact stress which is greater than medium pressure.

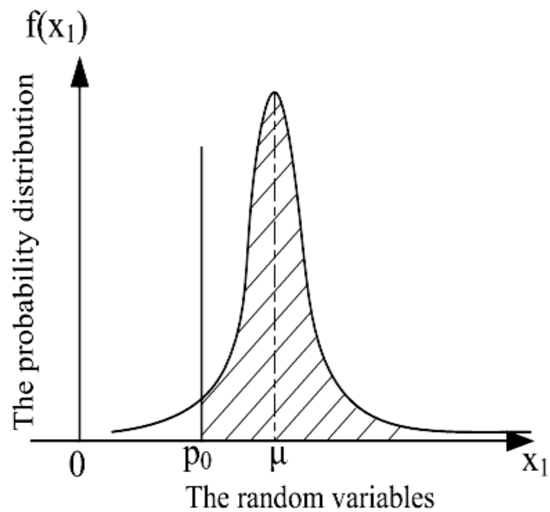


Figure 2: The Probability Distribution of the Maximum Contact Stress

For the maximum contact stress, the probability density function of maximum contact stress reliability is as follows:

$$f(x_1) = \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left[-\frac{(x_1 - \mu_1)^2}{2\sigma_1^2} \right]$$

Where: x_1 is the maximum contact stress; μ_1 is the expectation of maximum contact stress; σ_1 is standard variance of the maximum contact stress.

According to the reliable sealing criterion, we can get the reliability of maximum contact stress to meet the sealing conditions

$$p_{r1} = p(x_1 \geq p_0) = \int_{p_0}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left[-\frac{(x_1 - \mu_1)^2}{2\sigma_1^2} \right] dx_1$$

Simplify the above formula, we can get

$$p_{r1} = \Phi\left(\frac{\mu_1 - p_0}{\sigma_1} \right) \quad (4) \quad \text{Let} \quad \beta_p = \frac{\mu_1 - p_0}{\sigma_1}$$

β_p is the reliability index of maximum contact stress to meet the sealing conditions. The larger of the reliability index is, the higher of the sealing reliability is. We can seen that, the greater of the maximum contact stress expectation is, the more reliable of the seal is; the smaller of the maximum contact stress variance is, the more reliable of the seal is.

(3) The Reliability of Maximum Shear Stress to Meet the Sealing Conditions

Assumed the maximum shear stress according to the second reliable sealing requirement, the reliability of maximum shear stress to meet the sealing condition is $p_{r2} = p(x_2 \leq \tau_0)$, wherein, τ_0 is shear strength of the sealing material; the expectation of X_2 is $\mu_2 = (1-\xi)\tilde{x}_2$, $\tilde{x}_2$ is the maximum shear stress in the initial time. Similarly, assuming $3\sigma_2 = k \cdot \mu_2$, the variance reflects the seals process performance.

The geometrical meaning of the maximum shear stress reliability is the shaded area in Fig. 3; and its physical meaning is the probability of the maximum shear stress which is less than the shear strength of sealing material.

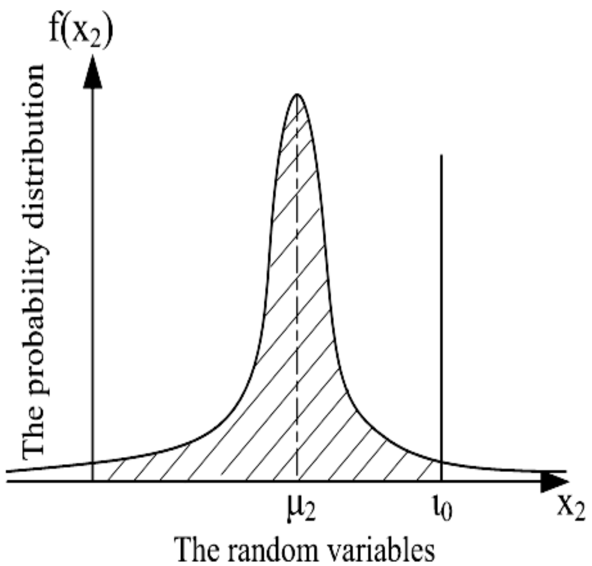


Figure 3: The Probability Distribution of the Maximum Shear Stress

Maximum shear stress X_2 following normal distribution with mean value μ_2 and standard deviation σ_2 then the probability density function of the maximum shear stress reliability is as follows:

$$f(x_2) = \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left[-\frac{(x_2 - \mu_2)^2}{2\sigma_2^2} \right]$$

Where, x_2 is the maximum shear stress; μ_2 is the expectation of maximum shear stress; σ_2 is the standard variance of maximum shear stress.

According to the second requirement of the reliable sealing criterion, the reliability of maximum shear stress to meet the sealing conditions is:

$$p_{r2} = p(x_2 \leq \tau_0) = \int_{-\infty}^{\tau_0} \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left[-\frac{(x_2 - \mu_2)^2}{2\sigma_2^2} \right] dx_2$$

Transform the above formula, we get:

$$p_{r2} = \int_{-\infty}^{\frac{\tau_0 - \mu_2}{\sigma_2}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2} \right) dt$$

It is the standard normal distribution, so

$$p_{r2} = \Phi\left(\frac{\tau_0 - \mu_2}{\sigma_2} \right) \quad \text{Let} \quad \beta_\tau = \frac{\tau_0 - \mu_2}{\sigma_2}$$

β_τ is the reliability index of maximum shear stress to meet the sealing conditions. The greater of β_τ is, the higher of the reliability is. We can see that, the smaller of the maximum shear stress expectations is, the more reliable of seals is; the smaller of the variance is, the more reliable of seals is.

(4) The Overall Sealing Reliability

Theoretically, the reliable static sealing must meet the two requirements. But two random variables are not independent, and we cannot determine its overall probability distribution. So, we assume that the random vector follows binary normal distribution, try to calculate its reliability.

Let the random vector $X = (X_1, X_2)^T$, and X_2 following normal distribution with mean value μ and covariance matrix Σ Assumed the correlation coefficient is r_{12}, so the probability density function of sealing reliability is:

$$f(x) = \frac{1}{(2\pi)|\Sigma|^{\frac{1}{2}}} \exp\left[-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu) \right]$$

where $x = (x_1, x_2)^T$- is the random vector; $\mu = (\mu_1, \mu_2)$ is the expectation of random vector; Σ- is the covariance of random vector. Here, we must ensure that the covariance matrix is positive definite.

$$\Sigma = \begin{pmatrix} \sigma_1^2 & r_{12}\sigma_1\sigma_2 \\ r_{12}\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$$

We get the overall sealing reliability is:

$$p_r = \int_{p_0}^{+\infty}\int_{-\infty}^{\tau_0} \frac{1}{(2\pi)|\Sigma|^{\frac{1}{2}}} \exp\left[-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)\right] dx_1 dx_2$$

That is the double integral on infinite interval, and its integrand is also very complex. So, we need to use the gauss-laguerre quadrature formula to solve it. Then, transform the formula into standard format of the gauss-laguerre quadrature formula.

Let $u = x_1 - p_0$, $v = -x_2 + \tau_0$, $x = \begin{pmatrix} u + p_0 \\ -v + \tau_0 \end{pmatrix}$, So

$$p_r = \int_0^{+\infty}\int_0^{+\infty} f(x) du dv$$

Wherein, $f(x) = \frac{1}{(2\pi)|\Sigma|^{\frac{1}{2}}} \exp\left[-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)\right]$

Now, we can use the gauss-laguerre quadrature formula to solve it.

Results

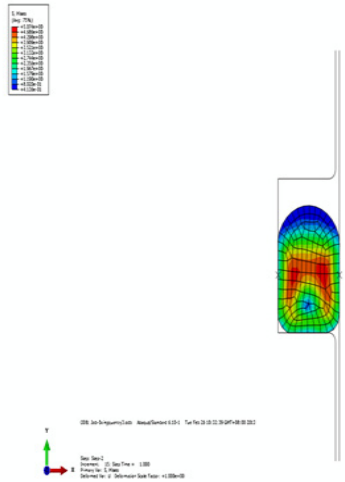
(1) The O-ring Finite Element Calculation

Following, it sets the o-ring as a case which is most widely used. Firstly, it sets up the finite element model of O-ring, and get the theoretical value of maximum contact stress and maximum shear stress in the working condition.

The initial compression ratio of O-ring is 20%, and the medium pressure is 4MPa. It calculated the theoretical value of the random variables of the O-ring in working conditions.

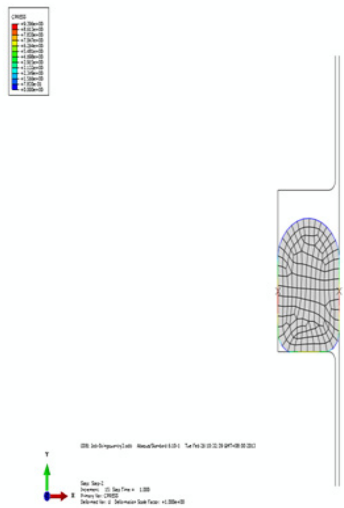
The O-ring stress distribution is shown in Figs. 4,5,6.

From these figures 4,5,6 we can see, the theoretical value of maximum contact stress is 9.396MPa, the theoretical value of maximum shear stress is 2.195MPa. Then, it calculated O-ring sealing reliability.



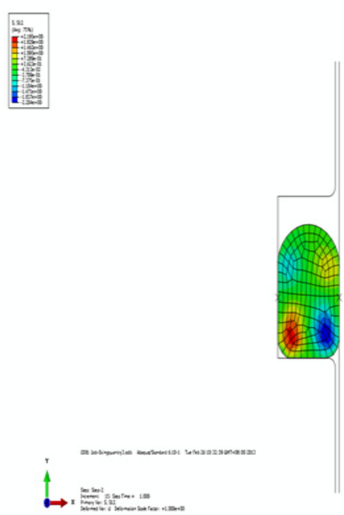
Maximum Mises stress: 5.074MPa

Figure 4: O-ring the Mises Stress Distribution



Maximum contact stress: 9.396MPa

Figure 5: O-ring Contact Stress Distribution



Maximum shear stress: 2.195MPa

Figure 6: O-ring Shear Stress Distribution

(2) The O-ring Sealing Reliability

(2.1) The Reliability of Maximum Contact Stress Meeting the Sealing Condition

According to the theoretical value of maximum contact stress, assumed the stress relaxation factor is $\xi = 0.1$, we could get the expectation of maximum contact stress is $\mu_1 = (1-\xi)x_1 = 8.456$. The variance of the maximum contact stress takes $3\sigma = 30\%\mu_1$, so, $\sigma = 0.8456$.

So, the reliability of maximum contact stress to meet the sealing condition is:

$$p_{r1} = \Phi\left(\frac{\mu_1 - p_0}{\sigma_1}\right) = \Phi(5.27), \beta = 5.27$$

When the reliability index $\beta > 4.99$, the reliability $p_r > 0.9999996981$.

Following, it calculates the reliability respectively for the different stress relaxation factors and variances, and also analyzes their influence on the reliability.

When the stress relaxation factor is $\xi = 0.15$, the expectation of the maximum contact stress is $\mu_1 = (1-\xi)x_1 = 7.987$, and $\sigma = 0.7987$, the reliability of maximum contact stress to meet the sealing conditions is

$$p_{r1} = \Phi\left(\frac{\mu_1 - p_0}{\sigma_1}\right) = \Phi(4.99), \beta = 4.99$$

When the stress relaxation factor $\xi = 0.2$, the expectation of the maximum contact stress is $\mu_1 = (1-\xi)x_1 = 7.517$, and $\sigma = 0.7517$, the reliability of maximum contact stress to meet the sealing conditions is

$$p_{r1} = \Phi\left(\frac{\mu_1 - p_0}{\sigma_1}\right) = \Phi(4.68), \beta = 4.68$$

From the above calculation, the greater of the stress relaxation factor is, the lower of the reliability of maximum contact stress to meet the sealing conditions is. It indicates that the stress relaxation is disadvantageous for the maximum contact stress reliability.

In the case of the stress relaxation factor $\xi = 0.1$, when the variance is $3\sigma = 20\%\mu_1$, and $\sigma = 0.564$, the reliability of maximum contact stress to meet the sealing conditions is

$$p_{r1} = \Phi\left(\frac{\mu_1 - p_0}{\sigma_1}\right) = \Phi(7.90), \beta = 7.90$$

When the variance is $3\sigma = 10\%\mu_1$, and $\sigma = 0.282$, the reliability of maximum contact stress to meet the sealing conditions is

$$p_{r1} = \Phi\left(\frac{\mu_1 - p_0}{\sigma_1}\right) = \Phi(15.8), \beta = 15.8$$

It can be seen from the above calculation, when the variance of the maximum contact stress is reduced, the reliability of maximum contact stress to meet the sealing conditions improved significantly.

(2.2) The Reliability of Maximum Shear Stress to Meet the Sealing Conditions

Similarly, assuming the stress relaxation factor is $\xi = 0.1$, according to the theoretical value of maximum shear stress, we get the expectation of maximum shear stress $\mu_2 = (1-\xi)x_2 = 1.976$; the variance of the maximum shear stress takes $3\sigma = 30\%\mu_2$, so $\sigma = 0.1976$.

The shear strength of the O-ring is $\tau_0 = 4MPa$, the reliability of maximum shear stress to meet the sealing conditions is:

$$p_{r2} = \Phi\left(\frac{\tau_0 - \mu_2}{\sigma_2}\right) = \Phi(10.24), \beta = 10.24$$

It also calculated the reliability respectively for different values of the stress relaxation factor and variance, and also analyzed their influence on the reliability.

When stress relaxation factor is $\xi = 0.15$, the expectation of the maximum shear stress is $\mu_2 = (1-\xi)x_2 = 1.866$, and $\sigma = 0.1866$, the reliability of maximum shear stress to meet the sealing conditions is

$$p_{r2} = \Phi\left(\frac{\tau_0 - \mu_2}{\sigma_2}\right) = \Phi(11.44), \beta = 11.44$$

When the stress relaxation factor $\xi = 0.2$, the expectation value of maximum shear stress is $\mu_2 = (1-\xi)x_2 = 1.756$, and $\sigma = 0.1756$. The reliability of maximum shear stress to meet the sealing conditions is

$$p_{r2} = \Phi\left(\frac{\tau_0 - \mu_2}{\sigma_2}\right) = \Phi(12.78), \beta = 12.78$$

According to the reliability index of the maximum shear stress, the greater of the stress relaxation factor is, the higher of the reliability is. It indicates the stress relaxation is advantageous for the maximum shear stress reliability.

In the case of the stress relaxation factor $\xi = 0.1$, when the variance is $3\sigma = 20\%\mu_2$, and $\sigma = 0.1318$, the reliability of maximum shear stress to meet the sealing conditions is

$$p_{r2} = \Phi\left(\frac{\tau_0 - \mu_2}{\sigma_2}\right) = \Phi(15.36), \beta = 15.36$$

When the variance is $3\sigma = 10\%\mu_2$, and $\sigma = 0.0659$, the reliability of maximum shear stress to meet the sealing conditions is

$$p_{r2} = \Phi\left(\frac{\tau_0 - \mu_2}{\sigma_2}\right) = \Phi(30.72), \beta = 30.72$$

As can be seen from the above calculation, when the variance of the maximum shear stress is reduced, the reliability of maximum shear stress to meet the sealing conditions can improve significantly.

(3) The O-ring Overall Sealing Reliability

Random vector is $X = (X_1, X_2)^T$ and X following normal distribution with mean value μ_2 and standard deviation σ_2, Assumed the correlation coefficient is $r_{12} = 0.4$, the expectation of random vector is $\mu = (\mu_1, \mu_2) = (8.456, 1.976)$; the

$$\Sigma = \begin{pmatrix} \sigma_1^2 & r_{12}\sigma_1\sigma_2 \\ r_{12}\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix} = \begin{pmatrix} 0.715 & 0.067 \\ 0.067 & 0.039 \end{pmatrix}$$

According to $p_0 = 4MPa$ and $\tau_0 = 4MPa$, we get the sealing reliability which meet both two requirements of the reliable sealing.

$$p_r = \int_0^{+\infty}\int_0^{+\infty} f(x)dudv$$

where, $f(x) = \frac{1}{(2\pi)|\Sigma|^{\frac{1}{2}}} \exp\left[-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)\right]$

And, $x = \begin{pmatrix} u+4 \\ -v+4 \end{pmatrix}$

Then use the gauss-laguerre quadrature formula to solve it, and got its reliability:

$p_r \approx 0.5641011$

When the correlation coefficient is $r_{12} = 0.3$, the covariance of random vector is

$$\Sigma = \begin{pmatrix} \sigma_1^2 & r_{12}\sigma_1\sigma_2 \\ r_{12}\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix} = \begin{pmatrix} 0.715 & 0.050 \\ 0.050 & 0.039 \end{pmatrix}$$

The sealing reliability is: $p_r \approx 0.8464141$

When the correlation coefficient is $r_{12} = 0.2$, the covariance of random vector is

$$\Sigma = \begin{pmatrix} \sigma_1^2 & r_{12}\sigma_1\sigma_2 \\ r_{12}\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix} = \begin{pmatrix} 0.715 & 0.0335 \\ 0.0335 & 0.039 \end{pmatrix}$$

The sealing reliability is: $p_r \approx 0.89426842$

From the above calculation, when we assume the random vector following the binary normal distribution, the smaller of the correlation coefficient is, the higher of the sealing reliability is.

Since we assumed that the random vector following binary normal distribution (actually it may be not correct), so the sealing reliability only has a relative sense, no absolute sense.

Discussion

This section mainly discussed the influence of design parameters on reliability, including compression ratio and media pressure.

(1) The Compression Ratio

In the medium pressure of 4MPa, assuming the stress relaxation factor $\xi = 0.1$ and the variance $3\sigma = 30\%\mu$, it calculated the sealing reliability index of the different compression ratios, then used Excel to map out the curve of the sealing reliability index, including the sealing reliability index of the maximum contact stress and maximum shear stress (Figs. 7 and 8).

It can be seen from Fig. 7, the greater of the compression ratio is, the greater of the maximum contact stress reliability index is. It means the reliability of the maximum contact stress to meet the sealing conditions is higher with the compression ratio increased.

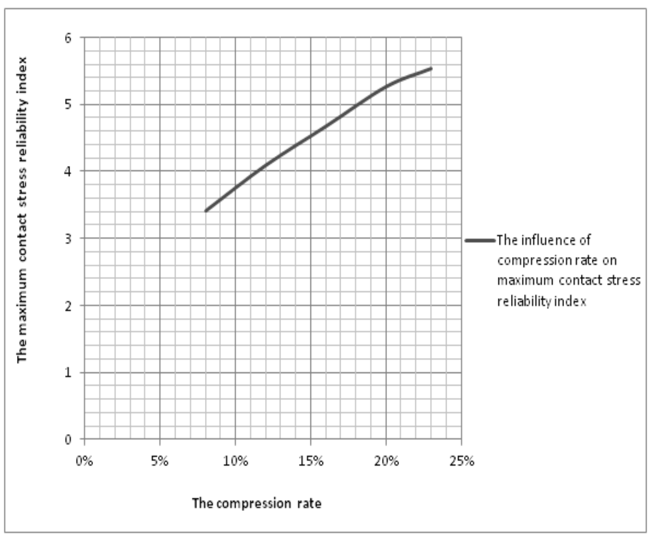


Figure 7: The Compression Ratio Influence on the Maximum Contact Stress Reliability Index

From Fig. 8, the maximum shear stress reliability index is highest when compression ratio is in the range of 15% to 20%, it means the reliability of the maximum shear stress to meet the sealing conditions is highest.

When the compression ratio is decreased, the maximum shear stress reliability is also lower. It isn't same with what we thought, and we analyzed its reason: when the compression ratio is decreased, the O-ring was squeezed into the gap by the medium pressure, which made the maximum shear stress increased.

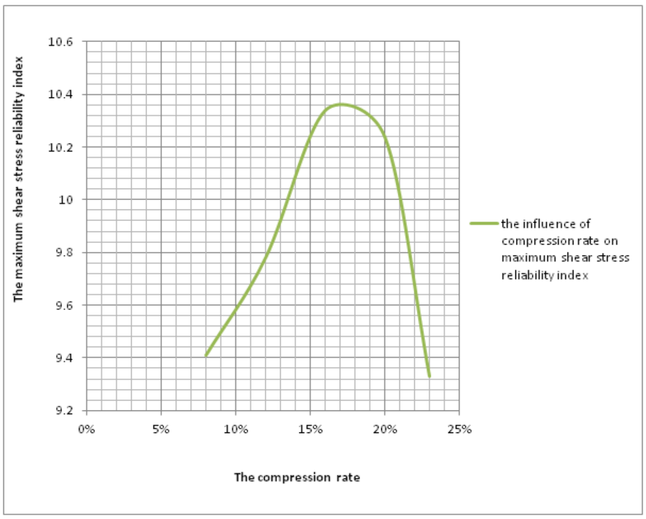


Figure 8: The Compression Ratio Influence on the Maximum Shear Stress Neliability Index

(2) The Medium Pressure

In the compression ratio of 20%, assuming the stress relaxation factor $\xi = 0.1$ and the variance $3\sigma = 30\%\mu$, it calculated the sealing reliability index of the different medium pressure. Similarly, it used Excel to map out the curve of the sealing reliability index (Fig. 9 and Fig. 10).

As can be seen from Fig. 9, the greater of the medium pressure is, the greater of the maximum contact stress reliability index is. It means the reliability of maximum contact stress to meet the sealing conditions is higher with medium pressure increased.

However, it can be seen from Fig. 10, the greater of the medium pressure is, the smaller of the maximum shear stress reliability index is. It means the reliability of the maximum shear stress to meet sealing conditions is smaller with medium pressure increased.

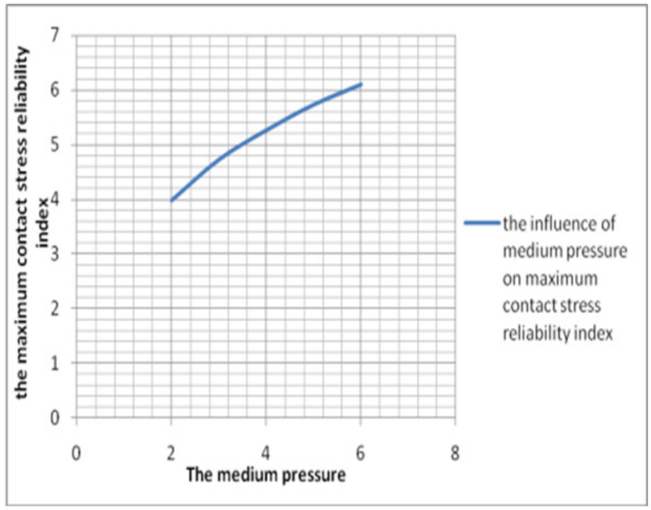


Figure 9: The Medium Pressure Influence on the Maximum Contact Stress Reliability Index

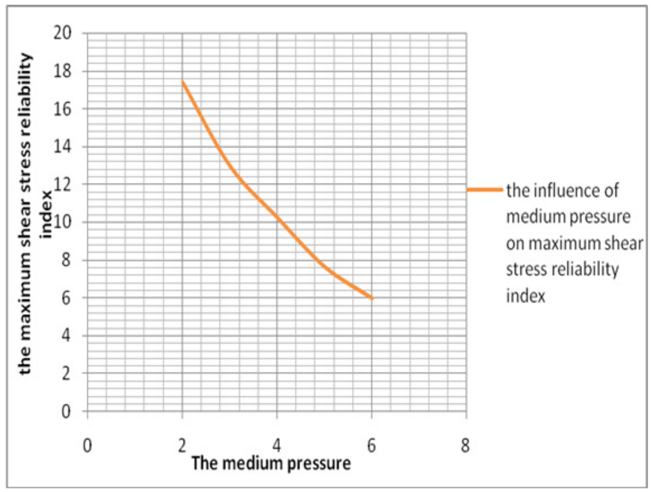


Figure 10: The Medium Pressure Influence on the Maximum Shear Stress Reliability Index

Conclusions

This paper put forward a new viable sealing reliability analysis method, which took the stress relaxation into account. Then it set the o-ring as a case, calculated the theoretical value of the maximum contact stress and maximum shear stress in FEA software. Finally, it calculated the sealing reliability, and analyzed the design parameters influence on the sealing reliability. And it got the conclusions:

(1) When the design parameters are certain, (a) the greater of the stress relaxation factor is, the lower of the maximum contact stress reliability is, but the higher of the maximum shear stress reliability is; (b) when the variances of

maximum contact stress and the maximum shear stress are smaller, both the maximum contact stress reliability and the maximum shear stress reliability are higher significantly; (c) when we assume the random vector following the binary normal distribution, the smaller of the correlation coefficient is, the higher of the overall sealing reliability is.

(2) When the design parameters are not certain, (a) the greater of the compression ratio is, the higher of the maximum contact stress reliability is, but when the compression ratio is in the range of 15% to 20%, the maximum shear stress reliability is highest;
(b) when the medium pressure is increased, the maximum contact stress reliability is higher, but the maximum shear stress reliability is lower.

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