Adaptive Maintenance Optimization for Mechanical Systems with Non-stationary Wiener Degradation

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Non-stationary degradation; Change point; Wiener process; Online detection; Offline

ABSTRACT

Online and offline adaptive maintenance policies are presented for the systems with non-stationary Wiener degradation processes. In the policies, preventive maintenance threshold will change with the degradation indicator adaptively. The cumulative sum algorithm is used in the online model to detect the change point of the degradation process. On this basis, an analytical model of the offline model is developed. A gearbox case based on the vibration-based degradation signals is studied to show the performance of the maintenance policy. It shows that it is important to consider the change point and to be of high effectiveness using the adaptive maintenance model.

Introduction

In reality, the degradation processes for many systems are non-stationary, such as the two-stage degradation process (Wang, 2010; Jiang, 2011; Wang *et al.*, 2011; Liu and Huang, 2010; Liu and Huang, 2010), the degradation rate is small in the first stage and large in the second stage. The modeling of degradation process is important to the condition-based maintenance (CBM). However, much less efforts are devoted to consider the existence of change point, and this may cause the maintenance optimization results not credible. Therefore, it is beneficial to consider the degradation change point for the maintenance program to maintain the system with high availability.

Many researchers have mainly investigated the maintenance optimization problem based on stationary degradation processes (Noortwijk and Frangopol, 2004; Mohamed, 1987; Grall *et al.*, 2002; Noortwijk and Kallen, 2006; Wang, 2002). However, the degradation processes for many systems are non-stationary due to environment influences or ageing factors etc. (Mitra *et al.*, 2008). To identify the degradation change point is very significant for the maintenance planning, including the development of the degradation model, the inspection scheme, and the preventive maintenance threshold. Some researchers have considered the change point detection in the CBM settings. (Fouladirad and Grall, 2011) proposed an adaptive PM threshold model for a system with a wear rate transition. The cumulative sum (CUSUM) algorithm is used to detect the abrupt change time. Saassouh (Saassouh *et al.*, 2007) proposed an activation zone to plan the maintenance action for a deteriorating system with random change of mode and the deteriorating level and the change mode can be continuously and perfectly monitored. Ponchet (Ponchet *et al.*, 2010) developed two condition-based maintenance optimization models with and without considering the sudden changes in their degradation processes, respectively. The numerical results show that the change of degradation mode strongly influences the choice of the best decision rule structure. Zhao (Zhao *et al.*, 2010) presented a predictive maintenance method considering the system with two degradation modes, and the maintenance actions were scheduled based on two reliability thresholds.

In this paper, online and offline adaptive maintenance optimization models considering the non-stationary Wiener degradation process will be investigated. The online model is adapted to the situation that the time of change point is non-informative, and the offline model can be used when the change time can be known instantly. The main contributions of this study are: (a) considering the non-stationary Wiener degradation process in the maintenance optimization; (b) developing two adaptive maintenance models for the Wiener degradation process with change point.

This paper is organized as follows. Section 2 is devoted to model the non-stationary Wiener degradation process. Section 3 describes an online change detection algorithm. The detailed maintenance policy is presented in Section 4. In Section 5, the evaluations of the two adaptive maintenance models are presented. A numerical example is used to illustrate and analyze the proposed maintenance policies in Section 6. Conclusions are made in Section 7.

Reliability Analysis

(1) Degradation Model

The system degradation process is assumed to be non-stationary Wiener processes. At each time *t,* the deterioration level of the system can be summarized by a random variable $X(t)$. The degradation process $\{X(t)\}_{t\geq 0}$ is an non-monotone process with initial state $X(0)=0$. Define the failure time as $T_L := \inf\{t: X(t)\geq L, t\geq 0\}$, which means that the system will be declared as failed when $X(t)$ exceeds a critical level L. The system in degradation failure state does not mean that the system cannot be in operation, but means that it is unacceptable for economic and safety reasons. Due to the internal mechanism or the external environment influences, the deterioration rate can undergo a sudden change at an unknown time T_c during the system life cycle. T_c is a random variable with probability density function f_c. Before T_c, the system is evolving with a nominal mode M_1. The deterioration rate after T_c suddenly increases from a nominal mode to an accelerated mode M_2, see figure 1. Wiener process is used to describe the deterioration process for its flexible characteristics to describe the non-monotone degradation processes and its explicit mathematical properties (Noortwijk, 2009).

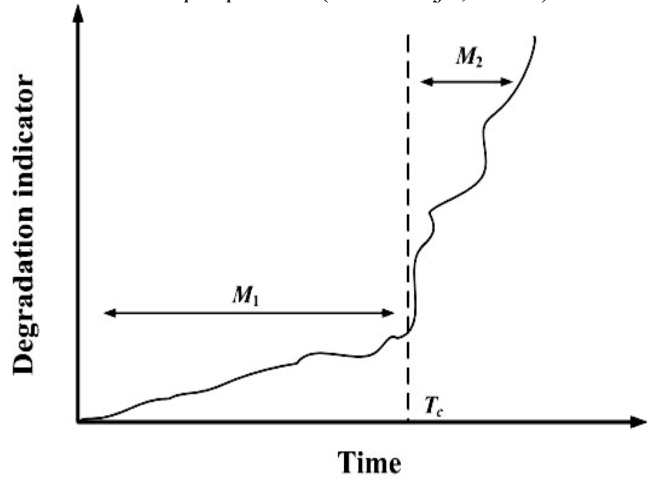

Figure 1: Degradation Path with Change Point

Assume the deteriorating process $\{X_t^i\}_{t\geq 0}$ in mode M_i (i=1,2), can be expressed by a non-stationary Wiener process:

$$X(t)=\begin{cases}\mu_1 t+\sigma_1 W(t),\ t\leq T_c\\ \mu_2(t-T_c)+\sigma_2 W(t-T_c)+X^1_{T_c},\ t>T_c\end{cases}\quad (1)$$

where $W(t)$ is a standard Brownian motion; $\mu_i\geq 0$ is the slope of the linear drift in mode M_i and $\sigma_i>0$ is the diffusion coefficient in mode M_i.

The increment $Y^i_{t-s}=X^i_t-X^i_s\ (0\leq s\leq t)$ follows a normal probability density function (pdf),

$$f_{\mu_i(t-s),\sigma_i^2(t-s)}(y)=\frac{1}{\sigma_i\sqrt{2\pi(t-s)}}Exp\left(-\frac{(y-\mu_i(t-s))^2}{2\sigma_i^2(t-s)}\right)\quad (2)$$

The average of deterioration rate in M_i is μ_i and the variance is $(\sigma_i)2/t$. For the sudden change in the deterioration process, the mean deterioration rate in the accelerated mode M_2 is larger than the mean deterioration rate in nominal mode M_1, that is $\mu_2>\mu_1$. According to the definition of the deterioration rate transition, the system state at time t can be represented as

$$X(t)=Y^1_t\mathbf{I}_{\{t\leq T_c\}}+(Y^1_{T_c}+Y^2_{t-T_c})\mathbf{I}_{\{t>T_c\}}\quad (3)$$

where I{E}=1 if E is true and 0 otherwise.

The increment *X(*t*)-X(*s*)* depends not only on the time interval t-s but also on the deterioration mode.

(2) Time to Failure Distribution

According to the properties of Wiener process, TL for single mode Mi has an inverse Gaussian distribution with probability density function (Whitmore and Schenkelberg, 1997)

$$f_{T_L}^i(t) = \frac{L}{\sqrt{2\pi\sigma_i^2 t^3}} \exp\left(-\frac{(L-\mu_i t)^2}{2\sigma_i^2 t}\right), t > 0 \quad (4)$$

The corresponding cumulative distribution function (cdf) is

$$F_{T_L}^i(t) = P(T_L^i < t) = \Phi\left(\frac{-L+\mu_i t}{\sigma_i\sqrt{t}}\right) + \exp\left(\frac{2\mu_i L}{\sigma_i^2}\right)\Phi\left(\frac{-L-\mu_i t}{\sigma_i\sqrt{t}}\right) \quad (5)$$

Denote the distribution of TL by $F_L(t)$, which has the following form

$$\begin{aligned} F_L(t) &= P(T_L \le t) \\ &= \underbrace{P(X(t) \ge L \mid T_c \ge t)P(T_c \ge t)}_{Without\ change\ point} + \underbrace{P(X(t) \ge L \mid T_c < t)P(T_c < t)}_{With\ change\ point} \\ &= \int_t^\infty \int_0^t f_{T_L}^1(z) f_c(s) dz ds + \int_0^t \int_0^{t-s} \int_0^L f_{\mu_1 s, \sigma_1^2 s}(\omega) f_{T_{L-\omega}}^2(z) f_c(s) d\omega dz ds \\ &= F_{T_L}^1(t)(1-F_c(t)) + \int_0^t \int_0^{t-s} \int_0^L f_{\mu_1 s, \sigma_1^2 s}(\omega) f_{T_{L-\omega}}^2(z) f_c(s) d\omega dz ds \end{aligned} \quad (6)$$

where Fc (resp. fc) denotes the distribution (resp. density) function of Tc,

The probability density function of TL is given by

$$f_L(t) = \frac{\partial F_L(t)}{\partial t} \quad (7)$$

Online CUSUM Algorithm

Change point problems deal with anomaly detection, or more generally detection of changes in the statistical behavior of process. There are many studies on the change point detection problems. Lai (Lai, 1995) reviewed varieties of sequential detection procedures and introduced a unified theory of sequential change point detection, which is intended to optimize the detection delay with respect to false alarm rate. Lorden (Lorden, 1971) proved that the mean detection delay derived by using the CUSUM algorithm subject to the average run length is asymptotically minimized with a given false alarm rate. Wang (Wang, 2007) considered a system with two stage degradation mode and developed a probability model to predict the initiation point of the second degradation stage. Jiang (Jiang, 2011) derived the change point by fitting a general piecewise model to the observed degradation data.

The online change detection uses the online information to detect the change time. CUSUM algorithm is one of the widely used change-point detection algorithms (Basseville and Nikiforov, 1993). In this paper, we choose CUSUM algorithm as the detection procedure when there is no priori information about the change time.

Let Y_1, Y_2, … , Y_v, denote the subsequent deterioration increments in a fixed time interval Δt before T_c with the density function $f_{\mu_1\Delta t,\sigma_1^2\Delta t}(y_j)$, and Y_{v+1}, Y_{v+2},… denotes the subsequent deterioration increments in a fixed time interval Δt after T_c with density function $f_{\mu_2\Delta t,\sigma_2^2\Delta t}(y_j)$, where $v \in \mathrm{N}$ and y_j is the realization of Y_j. Define

$$Q_n = \max_{1\le k\le n} \sum_{j=1}^n \log\left(\frac{f_{\mu_2\Delta t,\sigma_2^2\Delta t}(y_j)}{f_{\mu_1\Delta t,\sigma_1^2\Delta t}(y_j)}\right), Q_0 = 0.$$

According to the one-sided CUSUM scheme, the stopping rule for the sequential observations $\{Y_j\}_{1\le j\le n}$.

$$N = \min\{n : \max_{1\le k\le n} \sum_{j=k}^n \log\frac{f_{\mu_2\Delta t,\sigma_2^2\Delta t}(y_j)}{f_{\mu_1\Delta t,\sigma_1^2\Delta t}(y_j)} \ge c_\gamma\} \quad (8)$$

When $Q_n \ge c_\gamma$, the system will be judged in M_2. Then the detection time of the system mode change is $t_{detect} = N\Delta t$. The threshold c_γ is chosen such that $E_{M_1}[N] = \gamma$ when $v=\infty$ or $P_{M_1}(N \le \infty) \le a$, where γ and a are fix constants.

As stated by (Lorden, 1971), the online CUSUM algorithm minimizes the worst mean detection delay when $\gamma \to \infty$,

$$\tau = \sup_{v\ge1}\ ess \sup\ E_v(N - v + 1 \mid N \ge T_c, Y_1, ..., Y_{v-1}) \quad (9)$$

Where *ess* sup denotes an essential supremum (Basseville and Nikiforov, 1993).

The minimal number of observations required for the detection of T_c in the worst case, τ^*, satisfies the following relation

$$\tau^* = \min \tau \sim \frac{|\log a|}{d_{1,2}}, \text{when } a \to 0 \quad (10)$$

where $d_{1,2}$ is the Kullback-Liebler information between the two normal probability density function, $f_{\mu_1\Delta t,\sigma_1^2\Delta t}(y)$ and $f_{\mu_2\Delta t,\sigma_2^2\Delta t}(y)$, respectively

$$d_{1,2} = E_{M_2}\left[\log\left(\frac{f_{\mu_2\Delta t,\sigma_2^2\Delta t}(y)}{f_{\mu_1\Delta t,\sigma_1^2\Delta t}(y)}\right)\right] = \ln\frac{\sigma_1}{\sigma_2} + \frac{(\mu_1-\mu_2)^2\Delta t + \sigma_2^2 - \sigma_1^2}{2\sigma_1^2} \quad (11)$$

As the observations interval is assumed to be fixed Δt, the value $\tau^*\Delta t$ represents the mean time delay for the detection.

Maintenance Policies

There are three possible maintenance actions, respectively, including inspection, preventive replacement and corrective replacement. The preventive/corrective replacement restores the system to be as good as new with negligible time. For the cost reason, the system is periodically inspected without error. The system degradation status can only be revealed by inspection. The inspection times $\{t_k\}_{k\in N}$ defined by $t_k=k\Delta t$.

According to whether considering the change point in the degradation process, two maintenance policies are considered in this paper adaptive maintenance policy and change-blind maintenance policy.

(1) Adaptive Maintenance Policy

The preventive maintenance threshold varies for different degradation modes in the adaptive maintenance policy (Fig. 2). The detailed adaptive maintenance policy is as follows.

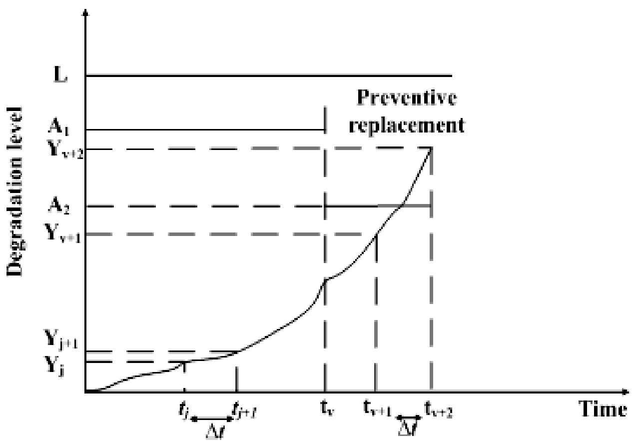

Figure 2: Adaptive PM threshold

(a) If one of the following exclusive events $\{X(t_{k-1})<A_1\cap X(t_k)\geq L\cap t_{k-1}<t_k\leq t_c\}$, $\{X(t_{k-1})<A_2\cap X(t_k)\geq L\cap t_c<t_{k-1}\leq t_k\}$ or $\{X(t_{k-1})<A_1\cap X(t_k)\geq L\cap t_{k-1}<t_c\leq t_k\}$ occurs, the system fails and a corrective replacement is carried out at time t_k with cost C_c. t_c is the realization of T_c.

(b) If one of the following exclusive events $\{X(t_{k-1})<A_1\cap A_1\leq X(t_k)<L\cap t_{k-1}<t_k\leq t_c\}$, $\{X(t_{k-1})<A_1\cap A_2\leq X(t_k)<L\cap t_{k-1}<t_c\leq t_k\}$ or $\{X(t_{k-1})<A_2\cap A_2\leq X(t_k)<L\cap t_c<t_{k-1}\leq t_k\}$ occurs, a preventive replacement is triggered with cost C_p.

(c) If one of the following exclusive events $\{X(t_k)<A_1\cap t_k\leq t_c\}$ or $\{X(t_k)<A_2\cap t_k\geq t_c\}$ occurs, the system is left unchanged, and the maintenance decision is postponed to the next inspection time t_{k+1}. Each inspection incurs a cost C_I.

(d) If the degradation failure happens between two inspections, there will be a period of unavailability for the system, and the cost rate of the unavailability is C_D.

Considering the change-point effect, the maintenance policy has two preventive replacement thresholds, A_1 and A_2 for mode M_1 and M_2, respectively. As mode M_2 is an accelerated degradation mode, we have $A_2<A_1$.

(2) Change-blind Maintenance Policy

With the aim to show the importance of considering the degradation rate change, we also investigate the change-blind maintenance policy. The decision rule of the change-blind maintenance policy neglects the change of degradation rate and only considers the system stay in mode M_1. The change-blind maintenance policy is as follows:

(a) If $X(t_k)<A$, the system is left as it is until the next inspection time.

(b) If $A\leq X(t_k)<L$, the system is replaced preventively.

(c) If $X(t_k)$, the system is considered as failed and a corrective replacement will take place. Before the replacement, there will be a period that the system stays in unavailability.

A and Δt are two maintenance decision variables in the change-blind maintenance policy. The corresponding costs for different maintenance actions are the same as the adaptive maintenance policy.

Maintenance Policy Evaluation

Considering the maintenance cost incurred for each maintenance action, the maintenance policy is evaluated using the expected long run cost rate over an infinite time span. Because the system is restored to as good as new after each preventive/corrective replacement, we can use the renewal reward theory (Sheldon, 1996) to compute the expected long-run cost rate as

$$E[C_\infty]=\lim_{t\to\infty}\frac{E[C(t)]}{t}=\frac{E[C(T)]}{E[T]} \quad (12)$$

where $C(t)$ is the global maintenance cost at time t, T is the length of a renewal cycle.

The accumulated cost on a renewal cycle T can be written as

$$E[C(T)]=C_I E[N_I(T)]+C_P P_P(\Delta t,\mathbf{A})+C_C P_C(\Delta t,\mathbf{A})+C_D E[W(T)] \quad (13)$$

where

$E[N_I(T)]$ is the expected number of inspections during a renewal cycle T;

$P_p(\Delta t, A)$ is the probability that the renewal cycle ends with a preventive replacement and $\boldsymbol{A}$ is the PM threshold set $\{A_1, A_2\}$;

$P_c(\Delta t, A)$ is probability that the renewal cycle ends with a corrective replacement;

$E[W(T)]$is the expected unavailability time in a renewal cycle.

The maintenance optimization problem is reduced to find the value of Δt, A_1 and A_2 that minimize the expected long-run cost rate $E[C_\infty]$.

According to whether the priori distribution of the change point is known, two configurations for the adaptive maintenance policy evaluation are considered: online evaluation and offline evaluation.

(1) Online Evaluation

When the change time is completely unknown, CUSUM algorithm is used to detect the change point in the degradation process and there will be a detection delay for the change time. In this configuration, the expected long-run cost rate is evaluated based on the online CUSUM algorithm.

The threshold A_1 can be obtained by minimizing the maintenance cost in mode M_1 and A_2 can be obtained in mode M_2 by the same way.

(2) Offline Evaluation

If the distribution of the change time is known before the application of the adaptive maintenance, the analytical expression of the expected long-run cost rate can be obtained. There will be no detection delay of the change time in this case. The detailed analytical expressions are as follows.

(2.1) Expression of $P_c(\Delta t, \mathrm{A})$

The corrective replacement can be caused by degradation failure. According to the change point occasions (see Figure 3), the probability for a corrective replacement in a renewal cycle is denoted as

$$P_C(\Delta t, \mathbf{A}) = \sum_{k=1}^{\infty} \begin{matrix} P(X(t_{k-1}) < A_1 \cap X(t_k) \geq L \cap t_{k-1} < t_k \leq t_c) \\ +P(X(t_{k-1}) < A_2 \cap X(t_k) \geq L \cap t_c \leq t_{k-1} < t_k) \\ +P(X(t_{k-1}) < A_1 \cap X(t_k) \geq L \cap t_{k-1} \leq t_c < t_k) \end{matrix} \tag{14}$$

The detailed analytic formulas of $P_c(\Delta t, \mathrm{A})$ are expressed as follows.

If $t_{k-1}<t_k<t_c$, the degradation failure founded at t_k is

$$\begin{aligned} & P(X(t_{k-1}) < A_1 \cap X(t_k) \geq L \cap t_{k-1} < t_k \leq t_c) \\ & = P(X(t_{k-1}) < A_1 \cap X(t_k) \geq L) P(t_{k-1} < t_k \leq t_c) \\ & = P(X(t_{k-1}) < A_1 \cap X(t_k) - X(t_{k-1}) \geq L - X(t_{k-1})) P(t_{k-1} < t_k \leq t_c) \\ & = P(X(t_{k-1}) < A_1 \cap X(\Delta t) \geq L - X(t_{k-1})) P(t_{k-1} < t_k \leq t_c) \\ & = \int_{t_k}^{\infty} \int_0^{A_1} \int_{L-z}^{\infty} f_{\mu_1 \Delta t, \sigma_1^2 \Delta t}(u) f_{\mu_1 t_{k-1}, \sigma_1^2 t_{k-1}}(z) f_c(t_c) \, du\, dz\, dt_c \end{aligned} \tag{15}$$

If $t_c< t_{k-1}< t_k$, the probability for the degradation failure founded at t_k is

$$\begin{aligned} & P(X(t_{k-1}) < A_2 \cap X(t_k) \geq L \cap t_c \leq t_{k-1} < t_k) \\ & = P(X(t_{k-1}) < A_2 \cap X(t_k) - X(t_{k-1}) \geq L - X(t_{k-1}) \cap t_c \leq t_{k-1} < t_k) \\ & = P(X(t_{k-1}) < A_2 \cap X(\Delta t) \geq L - X(t_{k-1})) P(t_c \leq t_{k-1} < t_k) \\ & = \int_0^{t_{k-1}} \int_0^{A_2} \int_0^{z} \int_{L-z}^{\infty} f_{\mu_1 t_c, \sigma_1^2 t_c}(u) f_{\mu_2 (t_{k-1}-t_c), \sigma_2^2 (t_{k-1}-t_c)}(z-u) f_{\mu_2 \Delta t, \sigma_2^2 \Delta t}(\omega) f_c(t_c) \, d\omega\, du\, dz\, dt_c \end{aligned} \tag{16}$$

If $t_{k-1}< t_c< t_k$, the probability for the degradation failure founded at t_k is

$$\begin{aligned} & P(X(t_{k-1}) < A_1 \cap X(t_k) \geq L \cap t_{k-1} \leq t_c < t_k) \\ & = P(X(t_{k-1}) < A_1 \cap X(t_k) - X(t_{k-1}) \geq L - X(t_{k-1}) \cap t_{k-1} \leq t_c < t_k) \\ & = \int_{t_{k-1}}^{t_k} \int_0^{A_1} \int_{L-u}^{\infty} \int_0^{z} f_{\mu_1 (t_c - t_{k-1}), \sigma_1^2 (t_c - t_{k-1})}(\omega) f_{\mu_2 (t_k - t_c), \sigma_2^2 (t_k - t_c)}(z-\omega) f_{\mu_1 t_{k-1}, \sigma_1^2 t_{k-1}}(u) f_c(t_c) \, d\omega\, dz\, du\, dt_c \end{aligned} \tag{17}$$

(2.2) Expression of $P_p(\Delta t, \mathrm{A})$

Under the considered maintenance policy, the probability for a renewal cycle ended by a preventive replacement is

$$P_P(\Delta t, \mathbf{A}) = \sum_{k=1}^{\infty} \begin{matrix} P(X(t_{k-1}) < A_1 \cap A_1 \leq X(t_k) < L \cap t_{k-1} < t_k \leq t_c) \\ +P(X(t_{k-1}) < A_2 \cap A_2 \leq X(t_k) < L \cap t_c \leq t_{k-1} < t_k) \\ +P(X(t_{k-1}) < A_1 \cap A_2 \leq X(t_k) < L \cap t_{k-1} \leq t_c < t_k) \end{matrix} \tag{18}$$

where

$$\begin{aligned} & P(X(t_{k-1}) < A_1 \cap A_1 \leq X(t_k) < L \cap t_{k-1} < t_k \leq t_c) \\ & = P(X(t_{k-1}) < A_1 \cap A_1 - X(t_{k-1}) \leq X(t_k) - X(t_{k-1}) < L - X(t_{k-1}) \cap t_{k-1} < t_k \leq t_c) \\ & = P(X(t_{k-1}) < A_1 \cap A_1 - X(t_{k-1}) \leq X(\Delta t) < L - X(t_{k-1}) \cap t_{k-1} < t_k \leq t_c) \\ & = \int_{t_k}^{\infty} f_c(t_c) dt_c \int_0^{A_1} \int_{A_1-u}^{L-u} f_{\mu_1 \Delta t, \sigma_1^2 \Delta t}(\omega) f_{\mu_1 t_{k-1}, \sigma_1^2 t_{k-1}}(u) \, d\omega\, du \end{aligned} \tag{19}$$

$$\begin{aligned} & P(X(t_{k-1}) < A_2 \cap A_2 \leq X(t_k) < L \cap t_c \leq t_{k-1} < t_k) \\ & = P(X(t_{k-1}) < A_2 \cap A_2 - X(t_{k-1}) \leq X(t_k) - X(t_{k-1}) < L - X(t_{k-1}) \cap t_c \leq t_{k-1} < t_k) \\ & = P(X(t_{k-1}) < A_2 \cap A_2 - X(t_{k-1}) \leq X(\Delta t) < L - X(t_{k-1}) \cap t_c \leq t_{k-1} < t_k) \\ & = \int_0^{t_{k-1}} \int_0^{A_2} \int_0^{z} \int_{A_2-u}^{L-u} f_{\mu_1 t_c, \sigma_1^2 t_c}(u) f_{\mu_2 (t_{k-1}-t_c), \sigma_2^2 (t_{k-1}-t_c)}(z-u) f_{\mu_2 \Delta t, \sigma_2^2 \Delta t}(\omega) f_c(t_c) \, d\omega\, du\, dz\, dt_c \end{aligned} \tag{20}$$

$$\begin{aligned} & P(X(t_{k-1}) < A_1 \cap A_2 \leq X(t_k) < L \cap t_{k-1} \leq t_c < t_k) \\ & = P(X(t_{k-1}) < A_1 \cap A_2 - X(t_{k-1}) \leq X(t_k) - X(t_{k-1}) < L - X(t_{k-1}) \cap t_{k-1} \leq t_c < t_k) \\ & = \int_{t_{k-1}}^{t_k} f_c(t_c) dt_c \int_0^{A_1} \int_{\max(0, A_2-u)}^{L-u} \int_0^{z} f_{\mu_1 (t_c - t_{k-1}), \sigma_1^2 (t_c - t_{k-1})}(\omega) f_{\mu_2 (t_k - t_c), \sigma_2^2 (t_k - t_c)}(z-\omega) f_{\mu_1 t_{k-1}, \sigma_1^2 t_{k-1}}(u) \, d\omega\, dz\, du \end{aligned} \tag{21}$$

(2.3) Expression of $E[N_I(T)]$

Based on the formulas of the probability for the corrective/preventive replacement, the expected number of inspection during a renewal cycle $E[N_I(T)]$ is given as

$$\begin{aligned} E[N_I(T)] = & \sum_{k=1}^{\infty} \begin{pmatrix} P(X(t_{k-1}) < A_1 \cap X(t_k) \geq L \cap t_{k-1} < t_k \leq t_c) \\ +P(X(t_{k-1}) < A_2 \cap X(t_k) \geq L \cap t_c \leq t_{k-1} < t_k) \\ +P(X(t_{k-1}) < A_1 \cap X(t_k) \geq L \cap t_{k-1} \leq t_c < t_k) \end{pmatrix} k \\ & + \sum_{k=1}^{\infty} \begin{pmatrix} P(X(t_{k-1}) < A_1 \cap A_1 \leq X(t_k) < L \cap t_{k-1} < t_k \leq t_c) \\ +P(X(t_{k-1}) < A_2 \cap A_2 \leq X(t_k) < L \cap t_c \leq t_{k-1} < t_k) \\ +P(X(t_{k-1}) < A_1 \cap A_2 \leq X(t_k) < L \cap t_{k-1} \leq t_c < t_k) \end{pmatrix} k \end{aligned} \tag{22}$$

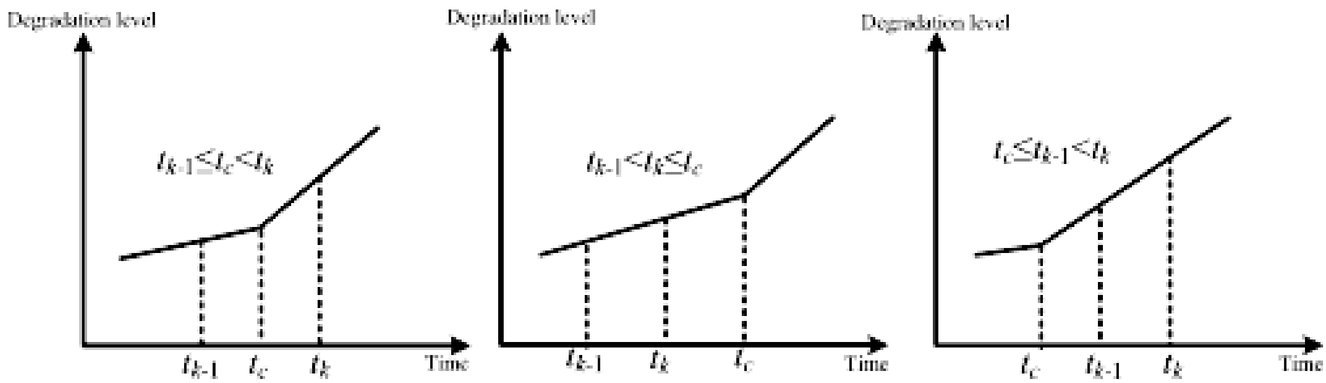

Figure 3: Different Locations of tc

(2.4) Expression of $E[W(T)]$

Under the considered maintenance policy, the degradation failure is non-self-announcing, and there will be a period of unavailability after degradation failure for the system. Denote the expected unavailability time in a renewal cycle by $E[W(T)]$. The detailed expression of $E[W(T)]$ can be found in Appendix A.

(2.5) Expression of $E[T]$

Under the considered maintenance policy, the system lifecycle can be ended by a preventive replacement and a corrective replacement. The expected length of the system lifecycle is given as

$$E[T]=\sum_{k=1}^{\infty}\begin{pmatrix} P(X(t_{k-1})<A_1\cap X(t_k)\ge L\cap t_{k-1}<t_k\le t_c) \\ +P(X(t_{k-1})<A_2\cap X(t_k)\ge L\cap t_c\le t_{k-1}<t_k) \\ +P(X(t_{k-1})<A_1\cap X(t_k)\ge L\cap t_{k-1}\le t_c<t_k) \end{pmatrix} t_k$$
$$+\sum_{k=1}^{\infty}\begin{pmatrix} P(X(t_{k-1})<A_1\cap A_1\le X(t_k)<L\cap t_{k-1}<t_k\le t_c) \\ +P(X(t_{k-1})<A_2\cap A_2\le X(t_k)<L\cap t_c\le t_{k-1}<t_k) \\ +P(X(t_{k-1})<A_1\cap A_2\le X(t_k)<L\cap t_{k-1}\le t_c<t_k) \end{pmatrix} t_k \quad (23)$$

Numerical Examples

In this Section, the optimal maintenance decision values of Δt, A_1 and A_2 are investigated through a gearbox run to failure case. Gearboxes are the most important components of machines for transmitting mechanical power, and always degrade gradually during their operating processes. The health condition of gears can be monitored by vibration monitoring. Through wavelet transformation, we can obtain the evolution of vibration energy signals in a gearbox experiment (Teng *et al.*, 2012; Zhao and Feng, 2011). Figure 4 shows the degradation process of the vibration signal of a gearbox. This is similar to the bearing degradation signals developed in (Wang, 2007; Gebraeel and Pan, 2008).

As Wiener processes can describe the non-monotone processes, we use the Wiener process to model the evolution of the vibration signal. From Figure 4, we can see that the degradation process is composed by two distinct stages. The degradation rate is small in the first stage and becomes larger in the second stage.

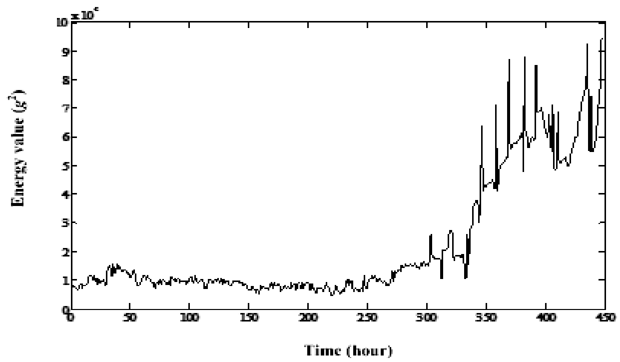

Figure 4: The Evolution of Vibration Degradation Signal

According to the experiment data in Fig.4, the estimated degradation model parameters are (μ_1,σ_1)=(51.89,1336.6), and (μ_1,σ_1)=(640.88,9859.6). The system degradation failure threshold L=95000 g^2. The maintenance costs are, respectively, C_I=5, C_p=50, C_c=100, C_D=250. The change time T_c is assumed to follow uniform distribution U(0,450).

Considering the system with a single degradation mode, we use Monte Carlo method or the method in reference (Wang *et al.*, 2011) to calculate the optimized maintenance values with fixed inspection interval. For mode M_1, the minimum expected cost rate is $E\left[C_\infty^1\right]=0.10$, which is achieved at A_1=43000 g^2, Δt_1 =180h; For mode M_2, the minimum expected cost rate is $E\left[C_\infty^2\right]=2.60$, which is achieved at A_2=30000 g^2, Δt_2=10h.

As the mode M_2 is an accelerated degradation mode compared with the mode M_1, it can be noticed that $\Delta t_2 < \Delta t_1$, and $A_2 < A_1$. For the clarity of the detection algorithm, we choose Δt_2 as the inspection interval to evaluate the online maintenance policy. Because Δt_2 not only can achieve the optimal result in mode M_2, also can assure the system operating with high reliability in mode M_1.

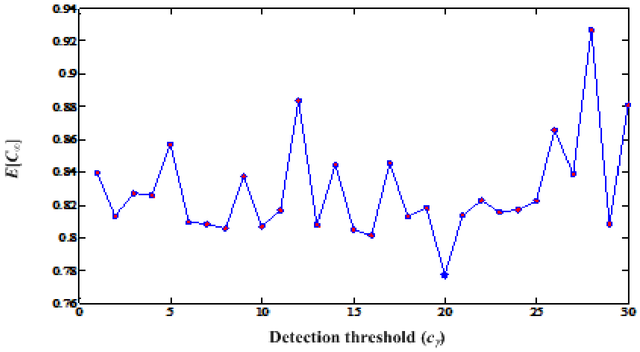

Figure 5: False Alarm Rate vs. Detection Threshold

(1) Online Optimization

For the CUSUM algorithm, the detection threshold c_γ affects the expected maintenance cost rate and the detection accuracy. The evolution of false alarm rate with varied c_γ is shown in Figure 5. It is noticed that the false alarm rate approaches zero with c_γ increasing.

In order to obtain the minimum $E[C_\infty]$ for the online adaptive maintenance policy, the expected cost rate for different values of c_γ is calculated and the optimal c_γ is corresponding to the minimum $E[C_\infty]$. Considering the online adaptive maintenance policy with A_1=43000 g^2, A_2=30000 g^2 and Δt =10 h, the PM threshold changes adaptively from A_1 to A_2 when the change point is detected and the lowest $E[C_\infty]$=0.78 is achieved at the optimal c_γ=20 see Figure 6.

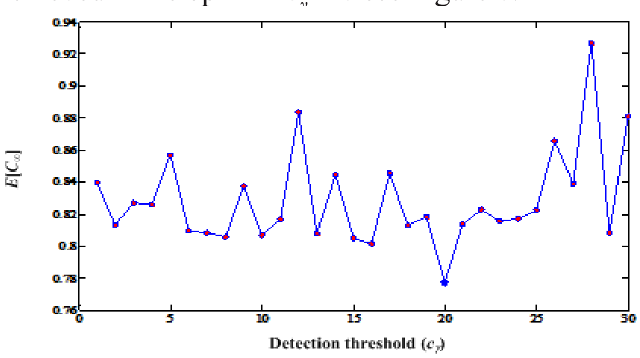

Figure 6: E[C∞] with cγ Under the Online Maintenance Policy

(2) Offline Optimization

Under the offline adaptive maintenance policy, the change point can be known instantly during the evolution of the degradation process, and the PM threshold will change with the degradation mode. Through Monte Carlo simulations, the optimal expected cost rate $E[C_\infty]$=0.77 is achieved with A_1=43000, A_2=30000 andΔt =13h. Figure 7 presents the contour curves of $E[C_\infty]$ with Δt =13h.

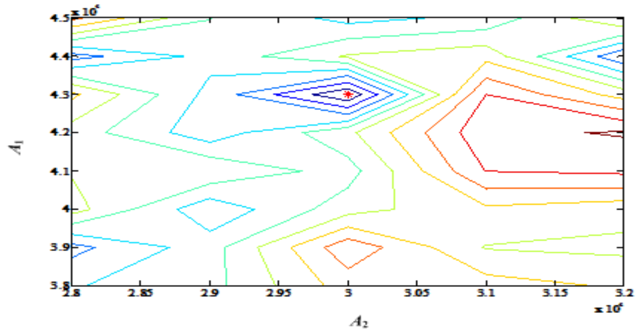

Figure 7: Contour Curves of E[C∞] with Δt =13 h

Compared with the online optimization result, it is noticed that the inspection interval of the offline optimization is a little larger than the online inspection interval and the achieved offline minimum expected cost rate is lower than the result of online model. Because the larger inspection interval for the offline model will decrease the total maintenance cost compared with the online model. Meantime, the optimal expected long run cost rates are similar. This proves that the online model results are consistent with the offline model and the online adaptive maintenance policy is applicable and effective for the non-informative change point occurrence situation.

(3) Change-blind Maintenance Policy

Under the change-blind maintenance policy, the change point influences will not be considered in the maintenance optimization model. The optimal maintenance decision variables for mode M_1, A_1=43000 and Δt =180h, will continue to be used in mode M_2. The minimum expected cost rate obtained by Monte Carlo simulations is $E[C_\infty]$=41.57.

The result of change-blind maintenance model reveals that change mode monitoring has great influences to the maintenance decisions and it is necessary to consider the degradation mode change in the maintenance model.

Appendix A

The expected unavailability time in a renewal cycle, $E[W(T)]$ is given as

$$E[W(T)] = \sum_{k=1}^{\infty}(E[W(T,k)]) \qquad (A.1)$$

where

$$\begin{aligned} E[W(T,k)] = & \underbrace{\int_0^{A_1} f_{\mu_1 t_{k-1},\sigma_1^2 t_{k-1}}(u)du \int_0^{\Delta t}(\Delta t-\omega) f_{L-u}(\omega \mid \mu_1,\sigma_1)d\omega \int_{t_k}^{\infty} f_c(t_c)dt_c}_{t_{k-1}<t_k \le t_c} \\ & + \underbrace{\int_0^{t_{k-1}}\int_0^{A_2}\int_0^{z}\int_0^{\Delta t}(\Delta t-\omega) f_{L-z}(\omega \mid \mu_2,\sigma_2) f_{\mu_1 t_c,\sigma_1^2 t_c}(u) f_{\mu_2 (t_{k-1}-t_c),\sigma_2^2 (t_{k-1}-t_c)}(z-u) f_c(t_c)d\omega du dz dt_c}_{t_c \le t_{k-1}<t_k} \\ & + \underbrace{\int_{t_{k-1}}^{t_k}\int_0^{A_1}\int_0^{(t_c-t_{k-1})}(\Delta t-\omega) f_{L-u}(\omega \mid \mu_1,\sigma_1) f_{\mu_1 t_{k-1},\sigma_1^2 t_{k-1}}(u) f_c(t_c)d\omega du dt_c}_{t_{k-1}<T_L \le t_c<t_k} \qquad (A.2) \\ & + \underbrace{\int_{t_{k-1}}^{t_k}\int_0^{A_1}\int_{(t_c-t_{k-1})}^{\Delta t}(\Delta t-\omega) f_{L-u}(\omega \mid \mu_1,\sigma_1,\mu_2,\sigma_2) f_{\mu_1 t_{k-1},\sigma_1^2 t_{k-1}}(u) f_c(t_c)d\omega du dt_c}_{t_{k-1}<t_c<T_L \le t_k}, \end{aligned}$$

where

$$f_{L-u}(t \mid \mu_1,\sigma_1) = \frac{\partial F_{L-u}(t \mid \mu_1,\sigma_1)}{\partial t}, \quad F_{L-u}(t \mid \mu_1,\sigma_1) = \int_{L-u}^{\infty} f_{\mu_1 t,\sigma_1^2 t}(\omega)d\omega,$$

$$f_{L-z}(t \mid \mu_2,\sigma_2) = \frac{\partial F_{L-z}(t \mid \mu_2,\sigma_2)}{\partial t}, \quad F_{L-z}(t \mid \mu_2,\sigma_2) = \int_{L-z}^{\infty} f_{\mu_2 t,\sigma_2^2 t}(\omega)d\omega,$$

$$F_{L-u}(t \mid \mu_1,\sigma_1,\mu_2,\sigma_2) = \int_{L-u}^{\infty}\int_0^{z} f_{\mu_1 (t_c-t_{k-1}),\sigma_1^2 (t_c-t_{k-1})}(\omega) f_{\mu_2 (t-t_c),\sigma_2^2 (t-t_c)}(z-\omega)d\omega dz,$$

$$f_{L-u}(\omega \mid \mu_1,\sigma_1,\mu_2,\sigma_2) = \frac{\partial F_{L-u}(t \mid \mu_1,\sigma_1,\mu_2,\sigma_2)}{\partial t}.$$

Conclusions

For the non-stationary Wiener degradation system, this paper presents two adaptive maintenance optimization models, online model and offline model to take into account the degradation mode change influences. CUSUM algorithm is used to detect the change point assuming the time of change of mode is unknown completely. The online adaptive maintenance model is adapted to the situation that the change time is non-informative, and the offline model can be applied when the change point time distribution is known. Through the comparison among the maintenance models, we validate the applicability and effectiveness of the adaptive maintenance model.

The numerical analysis from a gearbox case proves that 1) it is necessary to consider the degradation mode change in the maintenance model and 2) the adaptive maintenance model can deal with the change of degradation mode efficiently and provide an optimal maintenance schedule.

References

Basseville M; and **Nikiforov IV** (1993) *Detection of Abrupt Changes: Theory and Application: Prentice-Hall Information and System Sciences Series*, Prentice-Hall, London, UK, pp1-528.

Gebraeel N; and **Pan J** (2008) Prognostic Degradation Models for Computing and Updating Residual Life Distribution in a Time-Varying Environment. *IEEE Transactions on Reliability*, **57** (4): 539-550.

Grall A; Dieulle L; Brenguer C; and **Roussignol M** (2002) Continuous-time Preventive Maintenance Scheduling for A Deteriorating System. *IEEE Transactions on Reliability*, **51** (2): 141-150.

Jiang R (2011) Degradation Change Point and its Application in CBM Decision. *Proceedings of 2011 Prognostics & System Health Management Conference*, Shenzhen 24-25 May 2011, City University of Hong Kong, Hong Kong, China. pp1-6.

Lai TL (1995) Sequential Change Point Detection in Quality Control and Dynamical Systems. *Journal of the Royal Statistical Society. Series B (Methodological)*, **57** (4): 613-658.

Liu Y; and **Huang HZ** (2010) Optimal Replacement Policy for Multi-state System under Imperfect Maintenance. *IEEE Transactions on Reliability*, **59** (3): 483-495.

Liu Y; and **Huang HZ** (2010) Optimal Selective Maintenance Strategy for Multi-state Systems under Imperfect Maintenance. *IEEE Transactions on Reliability*, **59** (2): 356-367.

Lorden, G (1971) Procedures for Reacting To a Change in Distribution. *The Annual of Mathematical Statistics*, **42** (6): 1897-1908.

Mohamed AH (1978) Inspection and Maintenance Policies of Devices Subject to Deterioration.

Advances in Applied Probability, **19** (4): 917-931.

Mitra F; Antoine G; and **Laurence D** (2008) On the Use of On-Line Detection for Maintenance of Gradually Deteriorating Systems. *Reliability Engineering & System Safety*, **93** (12): 1814-1820.

Mitra F; and **Antoine G** (2011) Condition-based Maintenance for A System Subject to A Non-Homogeneous Wear Process with A Wear Rate Transition. *Reliability Engineering & System Safety*, **96** (6): 611-618.

Noortwijk JMV (2009) A Survey of the Application of Gamma Processes in Maintenance. *Reliability Engineering & System Safety*, **94** (1): 2-21.

Noortwijk JMV; and **Frangopol DM** (2004) Two Probabilistic Life-cycle Maintenance Models for Deteriorating Civil Infrastructures. *Probabilistic Engineering Mechanics*, **19** (4): 345-359.

Noortwijk JMV; and **Kallen MJ** (2006) Optimal Periodic Inspection of A Deterioration Process with Sequential Condition States. *International Journal of Pressure Vessels and Piping*, **83** (4): 249-255.

Ponchet A; Fouladirad M; and Grall A (2010) Assessment of A Maintenance Model of A Multi-Deteriorating Mode System. *Reliability Engineering & System Safety*, **95** (11): 1244-1254.

Saassouh B; Dieulle L; and **Grall A** (2007) Online Maintenance Policy for A Deteriorating System with Random Change of Mode. *Reliability Engineering & System Safety*, **92** (12): 1677-1685.

Sheldon MR (1996) *Stochastic Processes for Insurance and Finance, Wiley Series in Probability and Statistics*. Johon Wiley & Sons, New York, USA, pp1-639.

Teng HZ; Zhao JM; Jia XS; and **Zhang XH** (2012) Gearbox State Recognition Based on Continuous Hidden Markov Model. *Journal of Vibration and Shock*, **31** (5): 92-96.

Wang W (2007) A Two-Stage Prognosis Model in Condition Based Maintenance. *European Journal of Operational Research*, **182**(3): 1177-1187.

Wang HZ (2002) A Survey of Maintenance Policies of Deteriorating Systems. *Europian Journal of Operational Research*, **139**(3): 469-489.

Wang Z; Huang HZ; Li Y; and **Xiao NC** (2011) An Approach to Reliability Assessment under Degradation and Shock Process. *IEEE Transactions on Reliability*, **60** (4): 852-863.

Whitnore GA; and **Schenkelberg F** (1997) Modelling Accelerated Degradation Data using Wiener Diffusion with a Time Scale Transformation. *Lifetime Data Analysis*, **3** (1): 27-45.

Zhao JM; and **Feng TL** (2011) Remaining Useful Life Prediction Based on Nonlinear State Space Model. *Proceedings of 2011 Prognostics & System Health Management Conference 24-25 May 2011,* Shenzhen, China, pp1-6.

Zhao Z; Wang F; Jia M; and **Wang S** (2010) Preventive Maintenance Policy Based on Process Data. *Chemometrics and Intelligent Laboratory Systems*, **103** (2): 137-143.