Bayesian Estimation of Products with Wiener Process Degradation Based on Linex Loss Function

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ABSTRACT

Bayesian estimation for parameters and the reliability of products for which the performance degradation process modeled by wiener process are obtained based on linex loss function. Using both non-informative and conjugate prior distribution, several Bayesian estimates under squared error and linex loss functions are computed. Finally, these Bayesian estimates are compared through the mean squared error (MSE) based on Monte Carlo simulation study. According to these comparisons, it is shown that Bayesian estimators with linex loss function are more flexible.

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KEYWORDS

Wiener process; Linex loss function; Bayesian estimation; Performance degradation data; Life data

Introduction

At present, weapon systems such as underwater vehicles and underwater weapons have the characteristics of long life and high reliability. Therefore, we may only obtain a part of the life data, which results in a great challenge for the evaluation of the product's reliability using traditional methods. Although several techniques have been applied for this issue on the basis of, e.g., censoring, and accelerating the product's lifetime tested at a higher level of stress, these techniques are sometimes invalid for highly reliable products during a given period of time(Tseng, *et al.*, 2009; Tsai, *et al.*, 2011) .

The degradation analysis is an alternative approach for this problem. It assumes that a product has a quality characteristic that degradation over time can be related to the reliability by testing a number of products and by measuring the evolution of their performances. The observed data which will be used to evaluate the product's reliability is called the degradation data containing a rich source of reliability information and consequently offering many advantages over time-to-failure data(Nelson, 1900; Meeker and Escobar,1998; Wang, *et al.*, 2011). General discussions of degradation models and their applications are given by Meeker & Escobar (Meeker and Escobar, 1998) , Singpurwalla (Singpurwalla, 1995), Huang (Huang and An, 2009) and Nikulin (Nikulin, *et al.*, 2009).

Basically, there are two principal methods being widely used: the degradation path approach and the stochastic process approach (Nikulin, *et al.*, 2009; Guo, *et al.*, 2013). The degradation path approach was developed by Lu and Meeker (Lu and Meeker, 1993). It focuses on the inter-item variability and thus can be used to estimate the lifetime distribution for the population, see Park and Bae (Park and Bae, 2010), Fan et al (Fan, et al., 2012). However, the degradation uncertainty for an individual is not taken into account in this kind of models. A stochastic process model focuses on the individual item behavior and can remedy the shortage of general path models (Lu and Meeker, 1993; Pandy, *et al.*, 2009). This model assumes that the degradation is a random process, for instance, the gamma process (Pan and Balakrishnan, 2011; Cheng, *et al.*, 2012) and the wiener process (Brownian motion)(Lim and Yum, 2011; Wang, 2009; Wang, *et al.*, 2011). Because the Wiener process model can describe a nonmonotonic degradation process with Gaussian distribution, and provide a good description for some system behaviors (Barker and Newby, 2010), it has been widely used in previous studies. Guo et al (Guo, *et al.*, 2013) deals with mission-oriented systems subjected to gradual degradation modeled by a Wiener stochastic process within the context of CBM (condition-based maintenance). Wang (Wang, 2009) studies the maximum likelihood inference on a class of Wiener processes with random effects for degradation data. In this paper, therefore, we choose a Wiener process to describe a product's degradation.

The main objective of this study is to obtain the Bayesian estimation of the reliability with wiener process. The loss function is one of the key factors for the Bayesian assessment (Berger, 1980). Typical loss functions include the quadratic loss function (Yu, 2007) and the linear-exponential (Linex) loss function (Doostparast, *et al.*, 2013) . The quadratic loss function is mainly applied in the situation that the over estimation and the under estimation are of equal importance (Jaheen, 2004). In practice, however, the real loss is often not symmetric. For example (Singh, *et al.*, 2008), supposed a producer of integrated circuits aims to estimate the failure rate of his product. If his estimation is larger than the real value, he has to incur additional expenses to improve the technology and to increase the reliability of this integrated circuit. On the other hand, if he underestimates, he may lose customers and reputations in the market. In the extreme case, the under estimation of failure rate may even lead to complete ruin. Linex loss is one of the widely used asymmetric loss function and can cope up with such situations effectively. For this reason, we obtain the Bayesian estimation based on the Linex loss function.

The linex loss function is defined by (Varian, 1975)

$$L(\hat{\theta}-\theta) \propto (e^{\nu(\hat{\theta}-\theta)} - \nu(\hat{\theta}-\theta) - 1), a \neq 0 \quad (1)$$

where $\hat{\theta}$ is the estimation of θ and ν is the shape parameter. The sign and magnitude of ν represent the direction and degree of asymmetry, respectively. When $\nu < 0$, $L(\Delta)$ rises exponentially when $\Delta < 0$ ($\hat{\theta} < \theta$, underestimation) and almost linearly when $\Delta > 0$ ($\hat{\theta} > \theta$, overestimation). Conversely, when $\nu > 0$, the linex loss function is exponential to the right of the origin and linear to the left. It is easy to verify that the Bayesian estimator of θ under the Linex loss function is obtained as

$$\hat{\theta} = -\frac{1}{\nu} \ln E(e^{-\nu\theta} \mid x) \quad (2)$$

where $\boldsymbol{E}$ stands for the posterior expectation.

The remainder of this paper is as follows. Section 2 introduces the Wiener process model. In section 3 we fuse the failure data and the degradation data, and obtain the Bayesian estimation based on the Linex loss function using both non-informative and conjugate prior distributions. Section 4 presents Monte Carlo studies to validate this method. Section 5 makes some concluding remarks.

Degradation Model

Definition 1 (Lindqvist and Skogsrud, 2009) A stochastic process $\{X(t), t \geq 0\}$ is a Wiener process with drift coefficient μ and variance parameter (diffusion coefficient) σ^2 if the following holds

(1) $X(0) = 0$;

(2) $\{X(t), t \geq 0\}$ has stationary and independent increments;

(3) For every $t > 0$, $X(t)$ is normally distributed with a mean of μt and variance of $\sigma^2 t$.

According to the failure physics analysis, when the amount of the performance degradation reaches a pre-specified critical level at the first time, failure occurs. Let T denotes the failure time, then $T(D) = \inf\{t : X(t) = D; t \geq 0\}$. References (Yu, 2003; Wang and Xu, 2010) show that the failure time $T(D)$ follows a Inverse Gaussian(IG) distribution. The density function of IG is written by

$$f(t; D, \mu, \sigma) = \frac{D}{\sqrt{2\pi\sigma^2}} t^{-3/2} e^{-\frac{(D-\mu t)^2}{2\sigma^2 t}} \quad (3)$$

and the cumulative distribution function is

$$F(t; D, \mu, \sigma) = \Phi\left(\frac{\mu t - D}{\sigma\sqrt{t}}\right) + \exp\left(\frac{2\mu D}{\sigma^2}\right)\Phi\left(\frac{-\mu t - D}{\sigma\sqrt{t}}\right) \quad (4)$$

where Φ is the standard normal cumulative distribution function.

Denote the observed performance value by $X(t_{ij})$ for product $i (i = 1, ..., n)$ at time t_{ij} $(0 = t_{i0} < t_{i1} < ... < t_{im_i})$, then degradation increments $\Delta x_{ij} = X(t_{ij}) - X(t_{(i-1)j})$ at the time interval $\Delta t_{ij} = t_{ij} - t_{(i-1)j}$, $(i = 1, ..., n, j = 1, 2, ..., m_i)$ follow the normal distribution in line with the definition of wiener process. The density function of Δx_{ij} is

$$f(\Delta x_{ij}) = \frac{1}{\sqrt{2\pi\sigma^2 \Delta t_{ij}}} \exp\left(-\frac{(\Delta x_{ij} - \mu \Delta t_{ij})^2}{2\sigma^2 \Delta t_{ij}}\right) \qquad (5)$$

The likelihood function based on degradation data is

$$f_1(\Delta x; \mu, \sigma) = \prod_{i=1}^{n}\prod_{j=1}^{m_i} \frac{1}{\sqrt{2\pi\sigma^2 \Delta t_{ij}}} \exp\left(-\frac{(\Delta x_{ij} - \mu \Delta t_{ij})^2}{2\sigma^2 \Delta t_{ij}}\right)$$

$$\propto \left(\frac{1}{\sigma^2}\right)^{N/2} \exp\left\{-\frac{1}{2\sigma^2}\left[\mu^2 \sum_{i=1}^{n}\sum_{j=1}^{m_i} \Delta t_{ij} - 2\mu \sum_{i=1}^{n}\sum_{j=1}^{m_i} \Delta x_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{m_i} \frac{\Delta x_{ij}^2}{\Delta t_{ij}}\right]\right\}$$

where $\sum_{i=1}^{n} m_i = N$.

The likelihood function based on life data is

$$f_2(t; \mu, \sigma) = \prod_{k=1}^{m_0} \frac{D}{\sqrt{2\pi\sigma^2}} t_k^{-3/2} \exp\left(-\frac{(D - \mu t_k)^2}{2\sigma^2 t_k}\right)$$

$$\propto \left(\frac{1}{\sigma^2}\right)^{m_0/2} \exp\left[-\frac{1}{2\sigma^2}\left(\mu^2 \sum_{k=1}^{m_0} t_k - 2\mu D m_0 + \sum_{k=1}^{m_0} \frac{D^2}{t_k}\right)\right].$$

Therefore, the likelihood based on the degradation data and life data is

$$f(\Delta x, t; \mu, \sigma) = f_1(\Delta x; \mu, \sigma) f_2(t; \mu, \sigma)$$

$$\propto \left(\frac{1}{\sigma^2}\right)^{(N+m_0)/2} \exp\left\{-\frac{1}{2\sigma^2}\left[\left(A_1\left(\mu - \frac{A_2}{A_1}\right)^2 + \left(A_3 - \frac{A_2^2}{A_1}\right)\right)\right]\right\} \qquad (6)$$

where $A_1 = \sum_{i=1}^{n}\sum_{j=1}^{m_i} \Delta t_{ij} + \sum_{k=1}^{m_0} t_k$, $A_2 = \sum_{i=1}^{n}\sum_{j=1}^{m_i} \Delta x_{ij} + D m_0$,

$$A_3 = \sum_{i=1}^{n}\sum_{j=1}^{m_i} \frac{\Delta x_{ij}^2}{\Delta t_{ij}} + \sum_{k=1}^{m_0} \frac{D^2}{t_k}.$$

Bayesian Estimation Based On Linex Loss Function

(1) Bayesian Estimation with Non-informative Prior Distribution

When there is no information of parameters μ and σ^2, we select the Jeffreys prior distribution, since this probability density functions has maximum entropy in a given range. Probability density functions of μ and σ^2 are

$$\pi_1(\mu) \propto 1, \ \pi_1(\sigma^2) \propto \frac{1}{\sigma^2}$$

Then their joint prior distribution is

$$\pi_1(\mu, \sigma^2) \propto \frac{1}{\sigma^2}$$

According to the Bayesian theory, the posterior density function of μ and σ^2 is

$$\pi_1(\mu, \sigma^2 \mid x) = \frac{f(\Delta x, t; \mu, \sigma)\pi_1(\mu, \sigma^2)}{\iint_{\Theta} f(\Delta x, t; \mu, \sigma)\pi_1(\mu, \sigma^2) d\mu d\sigma^2}$$

$$\propto \frac{1}{\sigma^2}\left(\frac{1}{\sigma^2}\right)^{(N+m_0)/2}$$

$$\exp\left\{-\frac{1}{2\sigma^2}\left[\left(A_1\left(\mu - \frac{A_2}{A_1}\right)^2 + \left(A_3 - \frac{A_2^2}{A_1}\right)\right)\right]\right\} \qquad (7)$$

$$\propto \left(\frac{1}{\sigma^2}\right)^{B_1+1} \exp\left(-\frac{B_2}{2\sigma^2}\right) \frac{1}{\sigma} \exp\left[-\frac{A_1(\mu - B_3)^2}{2\sigma^2}\right]$$

where $B_1 = \frac{(N + m_0 - 1)}{2}$, $B_2 = \frac{A_3 - A_2^2 / A_1}{2}$, $B_3 = \frac{A_2}{A_1}$

and Θ denotes the parameter space.

Therefore, the posterior density function is given by $\pi_1(\mu, \sigma^2 \mid x) \square IG(B_1, B_2) N(B_3, \sigma^2 / A_1)$ (8)

The posterior density function of σ^2 is written as

$$\pi_1(\sigma^2 \mid x) = \int_0^{\infty} \pi_1(\mu, \sigma^2 \mid x) d\mu \quad IG(B_1, B_2) \qquad (9)$$

The posterior density function of μ is

$$\pi_1(\mu \mid x) = \int_0^{\infty} \pi_1(\mu, \sigma^2 \mid x) d\sigma^2$$

$$= \frac{\Gamma((2B_1+1)/2)}{\Gamma(B_1)\sqrt{2\pi}\sqrt{B_2/(A_1)}}\left[1 + \frac{1}{2B_1}\left(\frac{\mu - B_3}{\sqrt{B_2/(B_1 A_1)}}\right)^2\right]^{-(2B_1+1)/2} \qquad t(2B_1) \qquad (10)$$

The posterior density function of μ subjects to the Student's $t-$ distribution with $2B_1$ degrees of freedom.

Consider the squared error loss function and the Linex loss function (1) and (2). Bayesian estimates of μ, σ^2 and $R(t)$, when the prior distribution is taken to be noninformative prior distribution $\pi_1(\mu, \sigma^2)$ are obtained

Now, we infer Bayesian estimates of μ, σ^2 and $R(t)$ against the squared error loss function and the Linex loss function when the prior distribution is $\pi_1(\mu, \sigma^2)$.

Bayesian estimates of μ, σ^2 and $R(t)$ against the squared error loss function are, separately, obtained

$$\hat{\mu}_{S1} = E_1(\mu \mid x) = B_3 \qquad (11)$$

$$\hat{\sigma}_{S1}^2 = E_1(\sigma^2 \mid x) = \int_0^{\infty} \frac{B_2^{B_1}}{\Gamma(B_1)}\left(\frac{1}{\sigma^2}\right)^{B_1+1} \exp\left(-\frac{B_2}{\sigma^2}\right)\sigma^2 d\sigma^2 = \frac{B_2}{B_1} \qquad (12)$$

where E_2 stands for the posterior expectation based on the noninformative prior distribution.

Thus, the Bayesian estimate of reliability function $R(t)$ with squared error loss function under noninformative prior distribution is written

as
$$\hat{R}_{S1}(t) = 1 - F(t; D, \hat{\mu}_{S1}, \hat{\sigma}_{S1}) \tag{13}$$

Similarly, for the Linex loss function we have
$$\hat{\mu}_{L1} = -\frac{1}{\nu}\ln E_1(e^{-\nu\mu} \mid x)$$
$$= -\frac{1}{\nu}\ln \int_0^{\infty} e^{-\nu\mu} \frac{\Gamma((2B_1+1)/2)}{\Gamma(B_1)\sqrt{2\pi}\sqrt{B_2/(A_1)}}$$
$$[1+\frac{1}{2B_1}(\frac{\mu - B_3}{\sqrt{B_2/(B_1A_1)}})^2]^{-(2B_1+1)/2} d\mu \tag{14}$$

$$\hat{\sigma}^2_{L1} = -\frac{1}{\nu}\ln E_1(e^{-\nu\sigma^2} \mid x)$$
$$= -\frac{1}{\nu}\ln\{\int_0^{\infty} \frac{B_2^{B_1}}{\Gamma(B_1)}(\frac{1}{\sigma^2})^{B_1+1} \exp(-\frac{B_2}{\sigma^2})\exp(-\nu\sigma^2)d\sigma^2\} \tag{15}$$

Therefore, the Bayesian estimate of reliability function $R(t)$ with squared error loss function under non-informative prior distribution is
$$\hat{R}_{L1}(t) = 1 - F(t; D, \hat{\mu}_{L1}, \hat{\sigma}_{L1}) \tag{16}$$

(2) Bayesian Estimation with Conjugate Prior Distribution

We choose the conjugate prior probability density function defined by a normal-inverse gamma distribution. The conjugate prior distribution can be taken as $\pi_2(\mu,\sigma^2) \propto IG(a,b)N(c,d\sigma^2)$ (17)

Therefore, posterior distributions of μ and σ^2 for the degradation data and life data can be obtained in the following form
$$\pi_2(\mu,\sigma^2 \mid x) \propto (\frac{1}{\sigma^2})^{B_1+a+5/2}\exp(-\frac{B_2+b}{\sigma^2})\bullet$$
$$\frac{1}{\sigma}\exp[-\frac{(\mu-c)^2}{2d\sigma^2} - \frac{A_1(\mu-B_3)^2}{2\sigma^2}]$$
$$\propto (\frac{1}{\sigma^2})^{B_1+a+5/2}\exp[-\frac{1}{\sigma^2}(B_2+b+\frac{A_3'-A_2'^2/A_1'}{2})]\bullet$$
$$\frac{1}{\sigma}\exp[-\frac{A_1'(\mu - A_2'/A_1')^2}{2\sigma^2}] \tag{18}$$
$$\propto (\frac{1}{\sigma^2})^{B_1'+1}\exp(-\frac{B_2'}{\sigma^2})\ \frac{1}{\sigma}\exp[-\frac{A_1'(\mu-B_3')^2}{2\sigma^2}]$$
$$IG(B_1',B_2')N(B_3',\sigma^2/A_1')$$

Where

$A_1' = \frac{1}{d} + A_1$, $A_2' = \frac{c}{d} + A_1B_3$, $A_3' = \frac{c^2}{d} + A_1B_3^2$,

$B_1' = B_1 + a + 3/2$, $\quad B_2' = B_2 + b + \frac{A_3' - A_2'^2/A_1'}{2}$

and $B_3' = A_2'/A_1'$.

Then the posterior distribution function of σ^2 will be
$$\pi_2(\sigma^2 \mid x) \quad IG(B_1',B_2') \tag{19}$$
and the posterior distribution function of μ is obtained as
$$\pi_2(\mu \mid x) = \int_0^{\infty}\pi_2(\mu,\sigma^2 \mid x)d\sigma^2$$
$$= \frac{(B_2')^{B_1'}}{\Gamma(B_1')}\sqrt{\frac{A_1'}{2\pi}}\int_0^{\infty}(\frac{1}{\sigma^2})^{B_1'+0.5+1}\bullet\exp(-\frac{2B_2'+A_1'(\mu-B_3')^2}{2\sigma^2})d\sigma^2$$
$$= \frac{\Gamma((2B_1'+1)/2)}{\sqrt{2B_1'\pi}\Gamma(B_1')\sqrt{B_2'/(B_1'A_1')}}\bullet$$
$$[1+\frac{1}{2B_1'}(\frac{\mu-B_3'}{\sqrt{B_2'/(B_1'A_1')}})^2]^{-(2B_1'+1)/2} \quad t(2B_1') \tag{20}$$

Then Bayesian estimates of μ, σ^2 and $R(t)$ against the squared error loss function are, separately, obtained as
$$\hat{\mu}_{S2} = E_2(\mu \mid x) = B_3' \tag{21}$$
$$\hat{\sigma}^2_{S2} = E_2(\sigma^2 \mid x) = B_2'/B_1' \tag{22}$$
and
$$\hat{R}_{S2}(t) = 1 - F(t; D, \hat{\mu}_{S2}, \hat{\sigma}_{S2}) \tag{23}$$
where E_2 stands for the posterior expectation based on the conjugate prior distribution.

Bayesian estimates under conjugate prior distribution $\pi_2(\mu,\sigma^2)$ based on the linex loss function will be
$$\hat{\mu}_{L2} = -\frac{1}{\nu}\ln E_2(e^{-\nu\mu} \mid x)$$
$$= -\frac{1}{\nu}\ln\int_0^{\infty} e^{-\nu\mu}\frac{\Gamma((2B_1'+1)/2)}{\sqrt{2B_1'\pi}\Gamma(B_1')\sqrt{B_2'/(B_1'A_1')}}\bullet \tag{24}$$
$$[1+\frac{1}{2B_1'}(\frac{\mu-B_3'}{\sqrt{B_2'/(B_1'A_1')}})^2]^{-(2B_1'+1)/2}d\mu$$
$$\hat{\sigma}^2_{L2} = -\frac{1}{\nu}\ln E_2(e^{-\nu\sigma^2} \mid x)$$
$$= -\frac{1}{\nu}\ln\{\int_0^{\infty}\frac{B_2'^{B_1'}}{\Gamma(B_1')}(\frac{1}{\sigma^2})^{B'+1}\exp(-\frac{B_2'}{\sigma^2})\exp(-\nu\sigma^2)d\sigma^2\} \tag{25}$$

Finally, the Bayesian estimate of reliability function $R(t)$ under conjugate prior distribution is written as
$$\hat{R}_{L2}(t) = 1 - F(t; D, \hat{\mu}_{L2}, \hat{\sigma}_{L2}) \tag{26}$$

(3) Estimation of Hyper Parameters

For the product i, the observed performance value is $X(t_{ij})$ at time t_{ij} $(0 = t_{i0} < t_{i1} < ... < t_{im_i})$, then degradation increments $\Delta x_{ij} = X(t_{ij}) - X(t_{(i-1)j})$ at the time interval $\Delta t_{ij} = t_{ij} - t_{(i-1)j} (i = 1,\cdots,n; j = 1,2,\cdots,m_i)$ follow the normal distribution in line with the definition of wiener process.

The likelihood function based on the degradation data is

$$f_{3i}(\Delta x;\mu_i,\sigma_i)=\prod_{j=1}^{m_i}\frac{1}{\sqrt{2\pi\sigma_i^2\Delta t_{ij}}}\exp\left(-\frac{(\Delta x_{ij}-\mu_i\Delta t_{ij})^2}{2\sigma_i^2\Delta t_{ij}}\right)$$

Then the log-likelihood function is obtained as

$$l_{3i}(\Delta x;\mu_i,\sigma_i)\propto-\frac{m_i}{2}\ln(\sigma_i^2)-\sum_{j=1}^{m_i}\frac{(\Delta x_{ij}-\mu_i\Delta t_{ij})^2}{2\sigma_i^2\Delta t_{ij}}$$

The Maximum likelihood estimate is obtained as

$$\hat{\mu}_i=\frac{1}{m_i}\sum_{j=1}^{m_i}\frac{\Delta x_{ij}}{\Delta t_{ij}} \tag{27}$$

$$\hat{\sigma}_i^2=\frac{1}{m_i}\sum_{j=1}^{m_i}\frac{(\Delta x_{ij}-\hat{\mu}_i\Delta t_{ij})^2}{\Delta t_{ij}} \tag{28}$$

According to Eq. (17), we can obtain

$\pi_2(\sigma^2)\quad IG(a,b)$,

Using the moments method, estimations of a,b can be written as

$$\begin{cases}\dfrac{b}{a-1}=\dfrac{1}{n}\sum_{i=1}^{n}\hat{\sigma}_i^2=\bar{\sigma}^2\\ \dfrac{b^2}{(a-1)^2(a-2)}=\dfrac{1}{n-1}\sum_{i=1}^{n}(\hat{\sigma}_i^2-\bar{\sigma}^2)^2=S_\sigma^{2*}\end{cases}$$

So

$$\begin{cases}\hat{a}=\dfrac{S_\mu^{2*}}{(\bar{\sigma}^2)^2}+2\\ \hat{b}=(\hat{a}-1)\bar{\sigma}^2\end{cases} \tag{29}$$

Similarly, we can get $\pi_2(\mu\,|\,\bar{\sigma}^2)\quad N(c,d\bar{\sigma}^2)$ and

$$\begin{cases}\hat{c}=\dfrac{1}{n}\sum_{i=1}^{n}\hat{\mu}_i=\bar{\mu}\\ \hat{d}=\dfrac{1}{\bar{\sigma}^2(n-1)}\sum_{i=1}^{n}(\hat{\mu}_i-\bar{\mu})^2\end{cases} \tag{30}$$

Numerical Analysis

In this Section, a numerical study will be performed to compare the differences between Bayesian estimators under squared error loss function and that under Linex loss function using both noninformative and conjugate prior distribution. The accuracy of each estimate is measured by its mean square error (MSE). It should be noted that if the close form of the estimate can not be obtained, a numerical integration technique will be adopted. For the comparison of various Bayesian estimators, we specify different values of n,m_0 and ν. The estimators and MSEs of estimators $\hat{\mu}_{S1}$, $\hat{\sigma}_{S1}^2$, $\hat{\mu}_{L1}$, $\hat{\sigma}_{L1}^2$, $\hat{\mu}_{S2}$, $\hat{\sigma}_{S2}^2$, $\hat{\mu}_{L2}$ and $\hat{\sigma}_{L2}^2$ are displayed in Tables 1, 2 and 3. Furthermore, Figs. 1 and 2 present the Bayesian estimations of reliability functions $\hat{R}_{L1}(t)$ and $\hat{R}_{L2}(t)$ with different prior distributions. All estimates are obtained when $D=5$, $n=8$, $m_0=6$ and $m_1=m_2=\cdots=m_n=21$.

Based on estimates and MSE values, following conclusions can be drawn from these tables and Figs.

(1) The estimate values against Linex loss function approximate to the estimates against the squared error loss function as $\nu\to0$, which indicate the estimate against the squared error loss function can be approximated by that against the Linex loss function. Therefore, the Linex loss function is more widely used than the squared error loss function.

(2) The estimate values against Linex loss function are greater than the values against the squared error loss function when $\nu<0$. Conversely, when $\nu>0$, the estimate values against Linex loss function are smaller than the values against the squared error loss function. Considering the different damages caused by overestimation and underestimation, suitable value of ν can be selected to reduce the risk. Therefore, the linex loss function is more flexible than squared error loss function.

(3) As shown in Fig.1 and Fig.2, the estimations $\hat{R}_{L1}(t)$ and $\hat{R}_{L2}(t)$ are in good agreement with the real reliability function, which shows that the estimates against linex loss function is very promising.

(4) As shown in Tables 2 and 3, the MSE values for the parameter μ against linex loss function are smaller than the MSE values against squared error loss function. And the MSE values for the parameter σ^2 almost equal the MSE against squared error loss function. Therefore, the Bayesian estimates against linex loss function are more suitable to evaluate the location parameter μ.

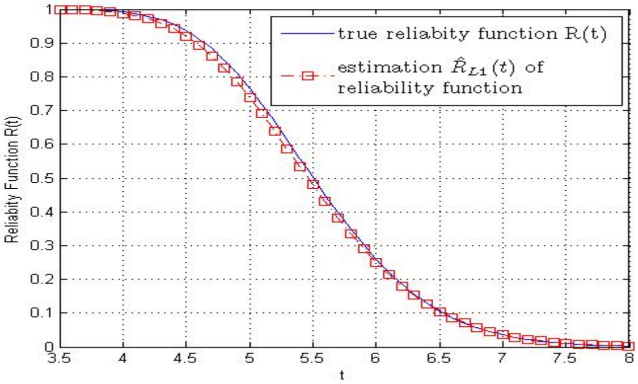

Figure 1: The Bayesian Estimation of the Reliability Function for Wiener Process with Noninformative Prior Distribution Against Linex Loss Function ($\mu = 0.9$, $\sigma^2 = 0.08$, $\nu = 5$).

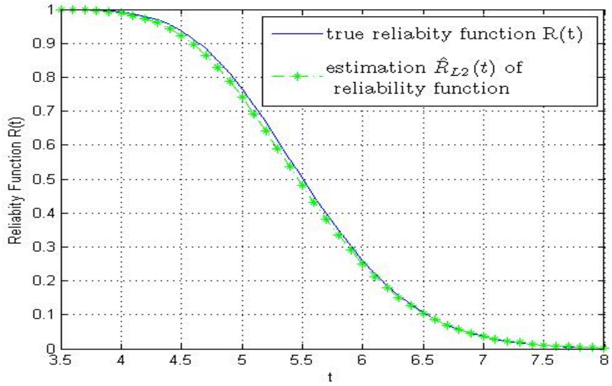

Figure 2: The Bayesian Estimation of the Reliability Function for Wiener Process with Conjugate Prior Distribution Against Linex Loss Function ($\mu = 0.9$, $\sigma^2 = 0.08$, $\nu = 5$).

Table 1: Estimates and MSEs of the Parameters with Different Values of ν ($\mu = 0.9, \sigma^2 = 0.08$)

ν	-5	-3	-1	-0.5	0.5	1	3	5
$\hat{\mu}_{S1}$	0.9065 (1.4169e-004)	0.9065 (1.4169e-004)	0.9065 (1.4169e-004)	0.9065 (1.4169e-004)	0.9065 (1.4169e-004)	0.9065 (1.4169e-004)	0.9065 (1.4169e-004)	0.9065 (1.4169e-004)
$\hat{\mu}_{L1}$	0.9068 (1.4491e-004)	0.9067 (1.4361e-004)	0.9066 (1.4232e-004)	0.9066 (1.4200e-004)	0.9065 (1.4137e-004)	0.9065 (1.4106e-004)	0.9064 (1.3981e-004)	0.9063 (1.3858e-004)
$\hat{\mu}_{S2}$	0.9060 (1.3308e-004)	0.9060 (1.3308e-004)	0.9060 (1.3308e-004)	0.9060 (1.3308e-004)	0.9060 (1.3308e-004)	0.9060 (1.3308e-004)	0.9060 (1.3308e-004)	0.9060 (1.3308e-004)
$\hat{\mu}_{L2}$	0.9090 (1.7954e-004)	0.9078 (1.5969e-004)	0.9071 (1.4915e-004)	0.9070 (1.4824e-004)	0.9061 (1.3560e-004)	0.9060 (1.3482e-004)	0.9053 (1.2684e-004)	0.9041 (1.2684e-004)
$\hat{\sigma}^2_{S1}$	0.0871 (1.4228e-004)	0.0871 (1.4228e-004)	0.0871 (1.4228e-004)	0.0871 (1.4228e-004)	0.0871 (1.4228e-004)	0.0871 (1.4228e-004)	0.0871 (1.4228e-004)	0.0871 (1.4228e-004)
$\hat{\sigma}^2_{L1}$	0.0884 (1.6605e-004)	0.0883 (1.6397e-004)	0.0882 (1.6192e-004)	0.0880 (1.6047e-004)	0.0872 (1.4760e-004)	0.0869 (1.4097e-004)	0.0868 (1.3894e-004)	0.0866 (1.3798e-004)
$\hat{\sigma}^2_{S2}$	0.0866 (1.2862e-004)	0.0866 (1.2862e-004)	0.0866 (1.2862e-004)	0.0866 (1.2862e-004)	0.0866 (1.2862e-004)	0.0866 (1.2862e-004)	0.0866 (1.2862e-004)	0.0866 (1.2862e-004)
$\hat{\sigma}^2_{L2}$	0.0868 (1.3387e-004)	0.0867 (1.3227e-004)	0.0866 (1.3073e-004)	0.0866 (1.3050e-004)	0.0866 (1.2975e-004)	0.0866 (1.2919e-004)	0.0865 (1.2761e-004)	0.0864 (1.2611e-004)

Table 2: Estimates and MSEs of the Parameters with Different Values of μ, σ^2

Estimations		$\hat{\mu}_{S1}$	$\hat{\mu}_{L1}$	$\hat{\mu}_{S2}$	$\hat{\mu}_{L2}$
$\mu = 0.5$	$\sigma^2 = 0.02$	0.5080 (9.1025e-005)	0.5078 (8.9049e-005)	0.5082 (9.4402e-005)	0.5080 (9.2420e-005)
	$\sigma^2 = 0.06$	0.5139 (2.7753e-004)	0.5134 (2.6280e-004)	0.5141 (2.8290e-004)	0.5136 (2.6799e-004)
	$\sigma^2 = 0.1$	0.4738 (0.0050)	0.4729 (0.0050)	0.4740 (0.0050)	0.4732 (0.0050)
$\mu = 1$	$\sigma^2 = 0.02$	1.0027 (3.2410e-005)	1.0025 (3.1705e-005)	1.0017 (2.7976e-005)	1.0016 (2.7736e-005)
	$\sigma^2 = 0.08$	1.0057 (1.2978e-004)	1.0050 (1.2199e-004)	1.0047 (1.1905e-004)	1.0040 (1.1270e-004)
	$\sigma^2 = 0.14$	1.0072 (2.2163e-004)	1.0060 (2.0481e-004)	1.0062 (2.0751e-004)	1.0050 (1.9323e-004)
$\mu = 1.6$	$\sigma^2 = 0.04$	1.6018 (5.2449e-005)	1.6015 (5.1231e-005)	1.5994 (4.9186e-005)	1.5990 (4.9759e-005)
	$\sigma^2 = 0.08$	1.6032 (1.0715e-004)	1.6025 (1.0298e-004)	1.6008 (9.6968e-005)	1.6000 (9.6393e-005)
	$\sigma^2 = 0.16$	1.6041 (2.0773e-004)	1.6026 (1.9796e-004)	1.6016 (1.9291e-004)	1.6001 (1.9035e-004)

Table 3: Estimates and MSEs of the Parameters with Different Values of μ, σ^2

Estimations		$\hat{\sigma}^2_{S1}$	$\hat{\sigma}^2_{L1}$	$\hat{\sigma}^2_{S2}$	$\hat{\sigma}^2_{L2}$
$\mu = 0.5$	$\sigma^2 = 0.02$	0.0281 (8.2267e-005)	0.0291 (5.6885e-004)	0.0279 (7.9056e-005)	0.0288 (6.0436e-004)
	$\sigma^2 = 0.06$	0.1036 (0.0025)	0.1038 (0.0025)	0.1004 (0.0021)	0.1006 (0.0022)
	$\sigma^2 = 0.1$	0.3066 (0.0688)	0.3001 (0.0635)	0.2953 (0.0623)	0.2896 (0.0578)
$\mu = 1$	$\sigma^2 = 0.02$	0.0209 (6.0202e-006)	0.0595 (0.0032)	0.0229 (1.3138e-005)	0.0374 (0.0017)
	$\sigma^2 = 0.08$	0.0848 (1.0393e-004)	0.0851 (1.0691e-004)	0.0842 (9.1957e-005)	0.0845 (9.4486e-005)
	$\sigma^2 = 0.14$	0.1494 (3.4849e-004)	0.1492 (3.3620e-004)	0.1462 (2.7717e-004)	0.1460 (2.6829e-004)
$\mu = 1.6$	$\sigma^2 = 0.04$	0.0399 (1.8728e-005)	0.0399 (4.9115e-005)	0.0508 (1.3393e-004)	0.0510 (1.4151e-004)
	$\sigma^2 = 0.08$	0.0797 (7.4706e-005)	0.0800 (7.4243e-005)	0.0890 (1.5026e-004)	0.0894 (1.5581e-004)
	$\sigma^2 = 0.16$	0.1592 (3.2234e-004)	0.1588 (3.1245e-004)	0.1654 (3.2500e-004)	0.1649 (3.1004e-004)

Conclusions

Based on non-informative prior distribution and conjugate prior distribution respectively, the Bayesian estimates of the parameters μ, σ^2 and the reliability functions $R(t)$ are obtained for the wiener degradation process. Bayesian estimators under squared error loss function and Linex loss function are obtained. Comparisons are made for these estimators based on simulation study. On the basis of this discussion, we may conclude that the proposed estimator performs close to the value of reliability and is more flexible than the estimators against squared error loss function. Thus, the adoption of proposed estimators based on the linex loss function is recommended.

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