Numerical Study of Stagnation Point Flow and Heat Transfer of Micropolar Fluid towards a Surface with Viscous Dissipation Effects

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ABSTRACT

In this paper, we present numerical investigation of the problem of steady laminar two dimensional boundary layer stagnation point flow and heat transfer of a Micropolar fluid towards a heated surface in the presence of viscous dissipation. The governing boundary layer partial differential equations are reduced to a set of ordinary ones using similarity transformations. The solutions for different values of Eckert number, Micropolar parameters and Prandtl number are computed, analyzed and discussed. The study reveals that the reverse flow of heat near the surface may occur due to viscous dissipation which may further be enhanced by the increasing values of the Prandtl number. It may be recommended that the viscous dissipation should not be simply ignored while studying the boundary layer stagnation point flows.

KEYWORDS

Micropolar fluid; viscous dissipation; thermal boundary layer; thermal reversal; Dewey Classification: 510 Mathematics

دراسة عددية لإنسياب درجة التجمد وإنتقال الحرارة في سائل مائيكروبولر Micropolar نحو السطح مع تأثير تبديد اللزوجة

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المُستلخص

تقدم هذه الدراسة تحقيقاً عددياً لمشكلة إنسياب درجة التجمدعلى مُسطح ثابت ثنائي الأبعاد وانتقال الحرارة في سائل مايكروبولر Micropolar نحو السطح مع تأثير تبديد اللزوجة يتم تخفيض الطبقات الحدودية التي تنظم المعادلات التفاضلية الجزئية لمجموعات من الخلايا العادية باستخدام تحولات التشابه. و. يتم احتساب الحلول لقيم مختلفة من عدد إيكرت Eckertو مائيكروبولر براميترز Micropolar parameters وعدد براندتل Prandtl وتحليلها ومناقشتها. تكشف الدراسة أن الإنسياب العكسي لدرجة الحرارة على المستوى الأقرب من السطح قد تحدث نتيجة لتبديد اللزوجة التي قد تزيد من تعزيز القيم المتزايدة لعدد براندتل Prandtl. وعليه قد يكون من التوصيات عدم تجاهل تبديد اللزوجة في دراسات إنسياب درجة التجمد.

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الكلمات الدالة

سائل مائيكروبولر، تبديد لزج، الطبقة الحرارية المتاخمة، الإنعكاس الحراري، تصنيف ديوي: ٥١٠ الرياضيات

Introduction

No one can deny the importance of micropolar fluid model as the Newtonian model is not appropriate to completely describe some modern engineering and industrial processes which involve materials, possessing an internal structure. The non-Newtonian fluid flow problems offer a challenge to researchers. Fluids having polymeric additives, display a significant reduction of shear stress and polymeric concentration, as predicted experimentally by (Hoyt and Fabula, 1964). The deformation of such materials can be well explained by the theory of micropolar fluids given by (Eringen, 1966). The flow of colloidal solutions, liquid crystals, polymeric fluids & blood are some of the examples where the concept of micropolar fluids can be seen in action.

Micropolar fluid is a hot area of research in which different aspects of the problems are being studied, in every possible detail. For example, (Peddieson and McNitt, 1970), (Gorla, 1983), and (Rees and Bassom, 1996), all having studied the flow of micropolar fluids over a flat plate. Simultaneous fluid flow with heat & mass transfer is the most commonly observed phenomenon in many natural, manufacturing & metallurgical processes. Hot rolling, wire drawing, crystal growing & polymer processing are some examples of such processes. The convective flows in fluid saturated porous media have attracted the attention of a wide community of researchers, due to their numerous applications in heat exchanger devices, petroleum reservoirs, insulation systems, filtration, nuclear waste repositories and chemical catalytic reactors. (Pop and Ingham, 2001), (Ingham and Pop, 2005), (Vafai, 2005) and (Nield and Bejan, 2006) are worth mentioning.

A numerical study of steady incompressible micropolar fluid in a two dimensional stagnation point flow towards a stretching sheet was presented by (Roslinda, *et al.* 2004). (Chang, 2006) gave the numerical solution of flow and heat transfer characteristics of mixed convection in micropolar fluid along a vertical flat plate with conduction effects. The problem of two dimensional unsteady boundary layer flow and heat transfer of a viscous incompressible electrically conducting non-Newtonian fluid in the stagnation region in the presence of magnetic field was considered by (Kumari, *et al.*, 2007). Implicit finite difference scheme was used to solve the governing equation of motion. (Lok *et al.* 2007) studied the two dimensional non-orthogonal stagnation flow of a micropolar fluid over a flat plate. (Ayub, *et al.*, 2008) presented the analytic solution of stagnation point flow of a viscoelstic fluid towards a stretching surface by using homotopy method.

The problem of asymmetric flow of a micropolar fluid between two porous disks was numerically studied by (Ashraf, *et al.*, 2009a) by using a finite difference scheme. The study was also extended to the case when one disk is porous and the other is non porous, by (Ashraf, *et al.*, 2009b). The study of two dimensional stagnation point flow of an electrically conducting micropolar fluid impinging normally on a heated surface in the presence of a uniform magnetic field, has been carried out by (Ashraf and Ashraf, 2011). Very recently, (Ashraf and Ahmad, 2012) considered the radiation effects on MHD axisymmetric stagnation point flow towards a heated shrinking sheet.

In this paper, we have investigated the problem of boundary layer stagnation point flow of a micropolar fluid towards a heated surface, taking the viscous dissipation into account which was neglected by the above mentioned researchers. Our study has revealed that the viscous dissipation may cause reverse flow of heat near the surface which may be supported by the increasing values of the Prandtl number. Thus, the surface may start further heating instead of cooling as a result of fluid flow. We therefore conclude that the viscous dissipation should therefore be taken into consideration while studying the boundary layer stagnation point flows.

Problem Formulation

We consider two dimensional steady incompressible stagnation point flow of a micropolar fluid impinging normally on a flat surface, in the presence of viscous dissipation. Let (u, v) be the velocity components in the Cartesian co-ordinates (x, y). Following (Lok, *et al.,* 2007), the governing equations of motion with boundary layer approximations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\left(\frac{\mu+k}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+\frac{k}{\rho}\frac{\partial \phi}{\partial y}, \tag{2}$$

$$\rho j\left(u\frac{\partial \phi}{\partial x}+v\frac{\partial \phi}{\partial y}\right)=\gamma\frac{\partial^2 \phi}{\partial y^2}-k\left(2\phi+\frac{\partial u}{\partial y}\right). \tag{3}$$

Here ϕ is the component of the microrotation vector normal to the xy-plane, ρ is the density, k is the vortex viscosity, γ is the spin gradient viscosity, j is the microinertia density and p is the pressure. All the physical quantities ρ, μ, k, γ and j are assumed to be constants. The equation for temperature distribution for the present boundary value problem in the presence of viscous dissipation can be written as

$$\rho\, c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\kappa_0\frac{\partial^2 T}{\partial^2 y}+\mu\left(\frac{\partial u}{\partial y}\right)^2, \tag{4}$$

where T is the temperature, k_0 is the thermal conductivity and c_p is the specific heat capacity at constant pressure of the fluid.

The boundary conditions for the present problem are

$$u(x,0)=0,\ v(x,0)=0, u(x,\infty)=U=ax, \phi(x,0)=0,\ \phi(x,\infty)=0, T(x,0)=T_w,\ T(x,\infty)=T_\infty. \tag{5}$$

We have to solve the Eqs. (1)-(4) subject to the boundary conditions given in (5). For this we use the following similarity transformations compatible with the continuity equation (1).

$$\left.\begin{aligned}&\eta=\sqrt{\frac{a}{\upsilon}}y,\ p(x,\infty)=p_0-\frac{\rho a^2}{2}(x^2+y^2),\\&u(x,y)=axf'(\eta),\ v(x,y)=-\sqrt{a\upsilon}f(\eta),\\&\phi(x,y)=-a\sqrt{\frac{a}{\upsilon}}xg,\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}.\end{aligned}\right] \tag{6}$$

After some simplifications we have the following system

$$(1+c_1)f'''-c_1g'+1=f'^2-ff'', \tag{7}$$

$$c_2g''+c_1c_3(f''-2g)=f'g-fg', \tag{8}$$

$$\theta''+p_rf\theta'+p_rEc\ f''^2=0. \tag{9}$$

Here $c_1=\frac{\kappa}{\mu},\ c_2=\frac{\mu}{\rho ja},\ c_3=\frac{\gamma}{j\mu},$

$Ec=\frac{U^2}{c_p(T_w-T_\infty)}$ and $p_r=\frac{\mu c_p}{\kappa_0}$

vortex viscosity parameter, microinertia density parameter, spin gradient viscosity parameter, Eckert number and Prandtl number, respectively.

Boundary conditions (5), in view of Eq. (6), can be written as

$$f(0)=0, f'(0)=0, f'(\infty)=1, g(0)=0, g(\infty)=0, \theta(0)=1, \theta(\infty)=0. \tag{10}$$

Numerical Solution

For the numerical simulation of flow problems in a semi-infinite domain (like the present one), usually there are two techniques, namely domain transformation and domain truncation. In first technique, the semi-infinite domain $[0,\infty)$ is transformed to the finite one $[0,1)$ by using the transformation of the type:

$$\xi=\left(1-\frac{1}{\eta+1}\right).$$

This finite domain is then used as the computational domain. In the other method, the semi-infinite domain $[0,\infty)$ is replaced by a finite domain $[0,R]$. The length R of the finite domain is to be chosen in such a way that an increase in R does not have any remarkable influence on the solution. This makes the profiles compatible to their asymptotic behavior, as remarked by (Pantokratoras, 2009). Moreover, the boundary conditions at infinity are enforced at $\eta=R$. It is also known that R depends upon the parameters of the problem and therefore it may be different for the different sets of the parameters.

In the present problem, we have $f(\infty)=\infty$ due to the boundary condition $f'(\infty)=1$. This does not allow us to employ the domain transformation technique, as the numerical simulation up to infinity is impossible. We have therefore used the domain truncation technique in this paper. We reduce the order of Eq. (7) by one, using the substitution $q = f'$ such that the boundary value problem comprising Eqs. (7) - (9) and the boundary conditions given in Eq. (10) becomes as follows:

$$q = f' = \frac{df}{d\eta}, \tag{11}$$

$$(1+c_1)q'' - c_1 g' + 1 = q^2 - fq', \tag{12}$$

$$c_2 g'' + c_1 c_3 (q' - 2g) - qg + fg' = 0, \tag{13}$$

$$\theta'' + p_r f\theta' + p_r\, Ec\, q'^2 = 0. \tag{14}$$

Subject to the boundary conditions

$$f(0)=0, q(0)=0, q(\infty)=1, g(0)=0, g(\infty)=0, \theta(0)=1, \theta(\infty)=0 \tag{15}$$

For numerical solution of the present problem we first discretize the domain $[0,R]$ uniformly with step h. Eqs. (12) - (14) are discretized at a typical grid point $\eta=\eta_n$ of the interval $[0,R]$ by employing central difference approximations for the derivatives. The system of resultant finite difference equations is solved iteratively by SOR method (Hildebrand, 1978), subject to the appropriate boundary conditions given in Eq. (15). After every iteration, Eq. (11) is numerically integrated using Simpson's rule (Gerald, 1974). In order to accelerate the iterative procedure and to improve the accuracy of the solution, we use the solution procedure, which is mainly based on algorithm described by (Syed, *et al.,* 1997). Our recent work (Ali, *et al.,* 2012) is an excellent reference for the details of this computational procedure. One can even find the pseudocodes for the above-mentioned procedure.

The iterative process is stopped when the criterion is satisfied.

$$Max(\|g^{(k+1)} - g^{(k)}\|_2, \|q^{(k+1)} - q^{(k)}\|_{L_2}, \|f^{(k+1)} - f^{(k)}\|_2, \|\theta^{(k+1)} - \theta^{(k)}\|_2) < TOL_{iter}, \tag{16}$$

We have taken at least 10^{-12} for TOL_{iter} during the execution of the self developed computer program in MATLAB.

Results and Discussion

In this section, we present our numerical results in tabular and graphical forms so that the important features of the solution for a range of values of the parameters affecting the flow and heat transfer characteristics may be interpreted. The results are calculated for three grid sizes

$$h, \frac{h}{2}, \frac{h}{4}$$

and then are extrapolated using Richardson's extrapolation (Deuflhard, 1983). This is done for the validity of our numerical computations and to improve the accuracy of the solution. In Table 1, the comparison of numerical values of normal velocity $f(\eta)$ for three grid sizes and its extrapolated values are given.

Table 1: Dimensionless Normal Velocity $f(\eta)$ on three Grid Levels and Extrapolated Values for $n_\infty = 9$, $c_1 = 2$, $c_2 = 0.4$, $c_3 = 0.5$, $Ec = 2$ and $Pr = 0.5$.

η	$f(\eta)$			
	$h = 0.09$	$h = 0.045$	$h = 0.0225$	Extrapolated value
0	0	0	0	0
1.8	0.8731975	0.8733133	0.8733424	0.8733520
3.6	2.5494309	2.5495316	2.5495569	2.5495653
5.4	4.3411765	4.3412629	4.3412847	4.3412919
7.2	6.1409138	6.1409988	6.1410202	6.1410273
9	7.9409104	7.9409952	7.9410166	7.9410237

Excellent comparisons validate our numerical computations and the use of extrapolation for higher order accuracy. For a range of values of the Eckert number Ec, the micropolar parameters (that is, the vortex viscosity parameter c_1, the microinertia density parameter c_2, and the spin gradient viscosity parameter c_3) and Prandtl number Pr, we present the shear stresses, heat transfer rate, velocity and thermal boundary layer thickness, and the velocity and microrotation fields. The values of the micropolar parameters c_1, c_2 & c_3 are given in Table 2. The case 1 corresponds to the Newtonian fluids.

In order to study their effect on the flow behavior, they are arbitrarily chosen as done customarily in the literature works of (Guram and Anwar, 1981), (Takhar, *et al.,* 2000), and (Ashraf, *et al.,* 2009). It is clear that the variations in Ec and Pr do not affect the shear & couple stresses due to decoupled momentum and temperature equations. With fixed $c_1 = 2$, $c_2 = 0.4$ and $c_3 = 0.5$, Table 3 shows that an increase in Pr increases $\theta'(0)$, for different values of Ec.

Table 2: Values of the Micopolar Parameters used in the Present Study

Cases	C_1	C_2	C_3
01	00	00	00
02	02	0.4	0.5
03	04	0.5	0.6
04	06	0.6	0.7
05	08	0.7	0.8

Table 3. Heat Transfer Rate on the Surface for $n_\infty = 10$, $c_1 = 2$, $c_2 = 0.4$, $c_3 = 0.5$ and Various Values of Pr and Ec

Pr	$f(\eta)$			
	Ec=0	***Ec=2***	***Ec=4***	***Ec=6***
0.5	-0.3883998	-0.0562507	0.2758984	0.6080474
1.0	-0.5032734	0.1137997	0.7308727	1.3479457
1.5	-0.5834355	0.2974247	0.1782850	2.0591453
02	-0.6470221	0.4835001	0.6140225	2.7445447

It is worthy to mention that the sign of $\theta'(0)$ changes as Eckert number increases. Physically, this means a reversal in the direction of heat transfer on the surface. That is, heat has started flowing from the fluid to the surface which is an important point to note. Thus, we draw a very important conclusion that the viscous dissipation may cause a change in the direction of heat transfer which can be magnified by the Prandtl number. Therefore, it should be taken into consideration while studying the boundary layer stagnation point flows.

The effect of Ec on $\theta'(0)$ is presented in Table 4, for different sets of values of the micropolar parameters, with fixed $Pr = 0.5$. It is clear that an increase in the values of the micropolar parameters not only decreases the heat transfer rate on the surface but it may also prevent the thermal reversal caused by the viscous dissipation.

Table 4: Heat Transfer rate on the Surface for n_∞ = 10, Pr = 0.5, with Different Ec and Various Cases of the Micropolar Parameters

Cases	$\theta'(0)$			
	Ec=0	***Ec=2***	***Ec=4***	***Ec=6***
01	-0.4333642	0.1671152	0.7675946	1.3680739
02	-0.3883998	-0.0562507	0.2758984	0.6080474
03	-0.3691447	-0.10810017	0.1529444	0.4139890
04	-0.3568529	-0.132957707	0.0909376	0.3148328
05	-0.34771501	-0.148421392	0.0508722	0.2501659

With fixed *Ec* = 3, *Pr* = 0.5, Table 5 shows that an increase in the micropolar parameters results in the reduction in shear stress $f''(0)$ whereas an opposite effect can be observed for couple stress $g'(0)$. On the other hand, we observe that the sign of $\theta'(0)$ changes from positive to negative as the micropolar parameters increase, which means that the micropolar structure of the fluids opposes the thermal reversal due to viscous dissipation.

Table 5: Shear & Couple Stresses and Heat Transfer Rate on the Surface for n_∞ = 10, Pr = 0.5, Ec = 3 and Various Cases of the Micropolar Parameters

Cases	$f''(0)$	$\theta'(0)$	$g'(0)$
01	1.2325837	0.4673549	0
02	0.6629470	0.1098238	0.4529835
03	0.4780878	0.02242213	0.6097215
04	0.3856406	-0.02101007	0.6797921
05	0.3298172	-0.04877458	0.7183901

Now we give the interpretation of our graphical results. The effect of the micropolar parameters c_1, c_2 & c_3 for fixed values of *Ec* and *Pr* is shown in Figs. 1-4. An increase in these parameters decreases both the normal and streamvise velocity profiles while increasing the velocity boundary layer thickness, as shown in Figs. 1& 2. Fig. 3 indicates that the microrotation profiles rise with the increasing values of the micropolar parameters. An increase in the micropolar parameters decreases the temperature profiles near the surface where as an opposite effect can be seen away from the surface, as shown in Fig. 4. Thermal boundary layer thickness also increases with an increase in the micropolar parameters.

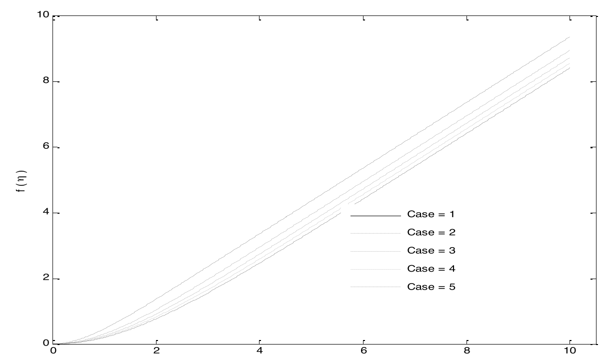

Figure 1: Normal Velocity Profiles for *Pr* =1,*Ec*=3 and Various Cases of the Micropolar Parameters

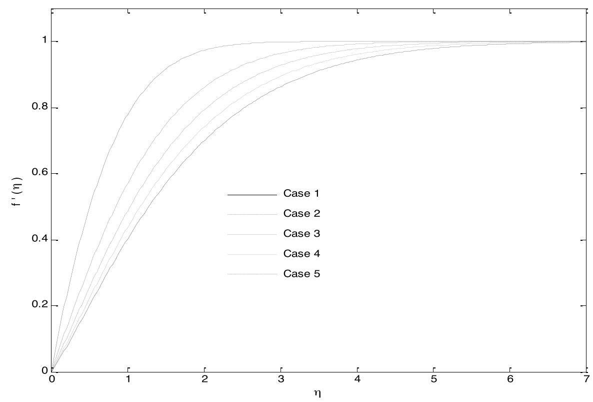

Figure 2: Streamvise Velocity Profiles for *Pr*=1, *Ec*= 3 and Various Cases of the Micropolar Parameters

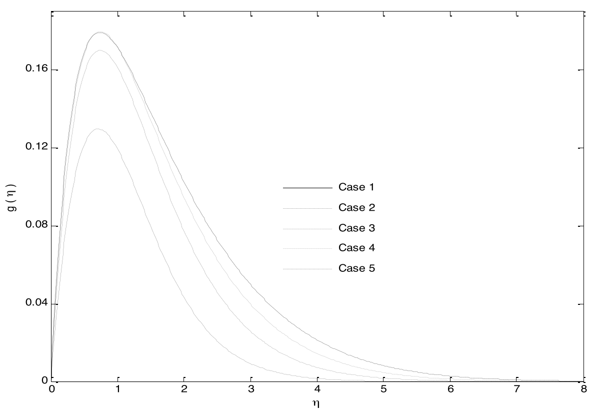

Figure 3: Microrotation Profiles for *Pr* = 1, *Ec*=3 and Various Cases of the Micropolar Parameters

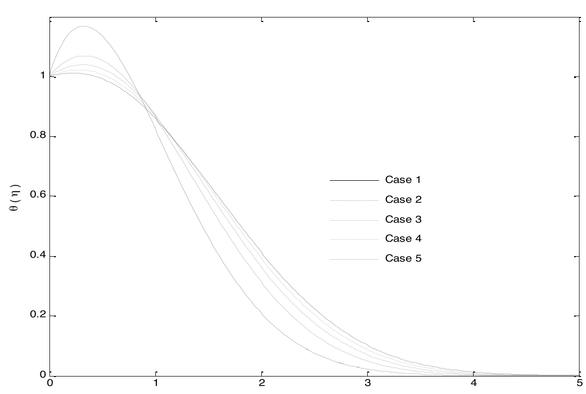

Figure 4: Temperature Profiles for *Pr* = 1, *Ec* =3 and Various Cases of the Micropolar Parameters

The effect of *Ec* on temperature profiles for different cases of the values of the micropolar parameters with fixed *Pr*, is shown in the Figs. 5-9. It is clear that the viscous dissipation may cause the thermal reversal near the surface while increasing the thermal boundary layer.

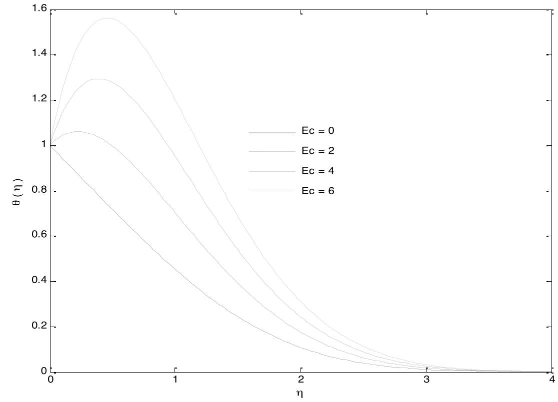

Figure 5: Temperature Profiles for *Pr* = 1, $Pr = 1,\ c_1 = 0, c_2 = 0, c_3 = 0$ and Various Values of Eckert Number

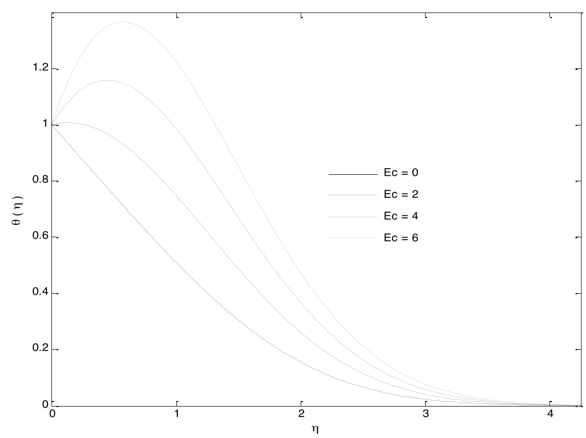

Figure 6: Temperature Profiles for *Pr* = 1, $Pr = 1,\ c_1 = 2, c_2 = 0.4, c_3 = 0.5$ and Various Values of Eckert Number

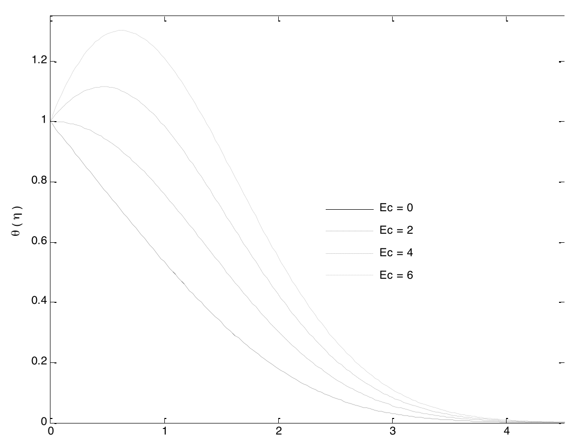

Figure 7. Temperature Profiles for *Pr* = 1, $Pr = 1,\ c_1 = 4, c_2 = 0.5, c_3 = 0.6$ and Various Values of Eckert Number

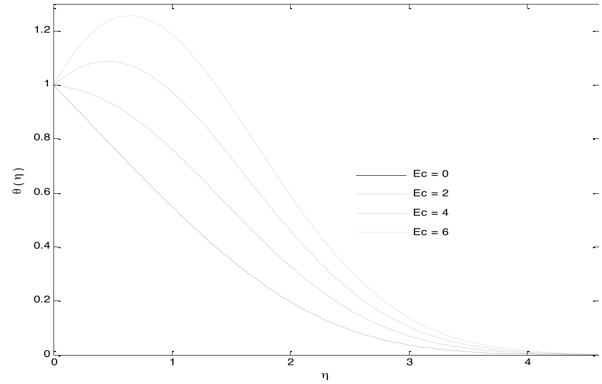

Figure 8: Temperature Profiles for Pr = 1, $c_1=6, c_2=0.6, c_3=0.7$ and Various Values of Eckert Number

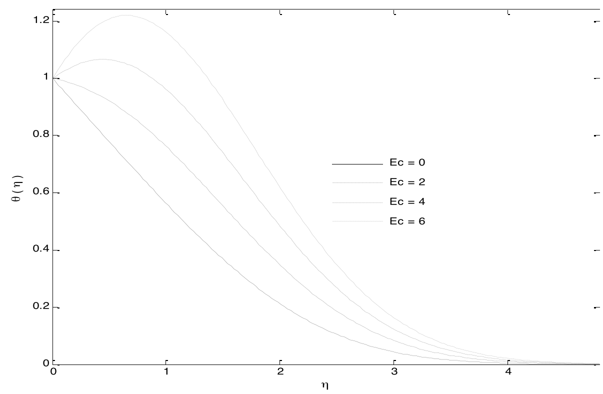

Figure 9: Temperature Profiles for Pr = 1, $c_1=8, c_2=0.7, c_3=0.8$ and Various Values of Eckert Number

Figs. 10-13 demonstrate the influence of the micropolar parameters on the temperature profiles for different values of Ec with fixed Pr. The micropolar parameters always tend to resist the change in the direction of heat transfer near the surface while increasing the thermal boundary layer thickness.

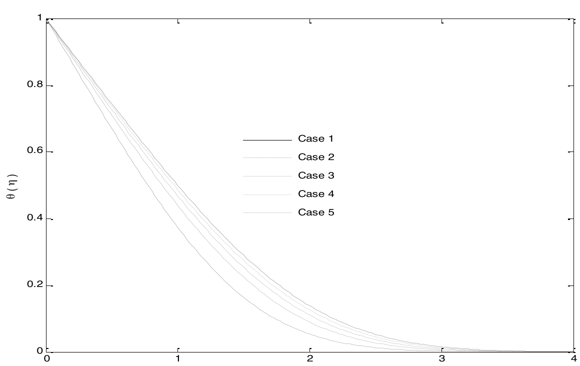

Figure 10. Temperature Profiles for Pr =1.5, Ec=0 and Various Cases of the Micropolar Parameters

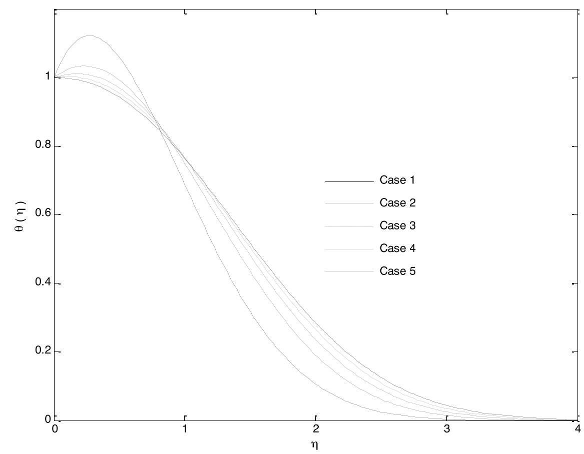

Figure 11. Temperature Profiles for Pr = 1.5, Ec = 2 and Various Cases of the Micropolar Parameters

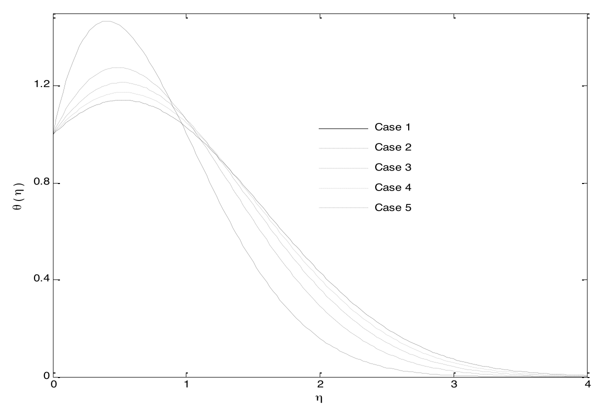

Figure 12. Temperature Profiles for Pr = 1.5, Ec = 4 and Various Cases of the Micropolar Parameters

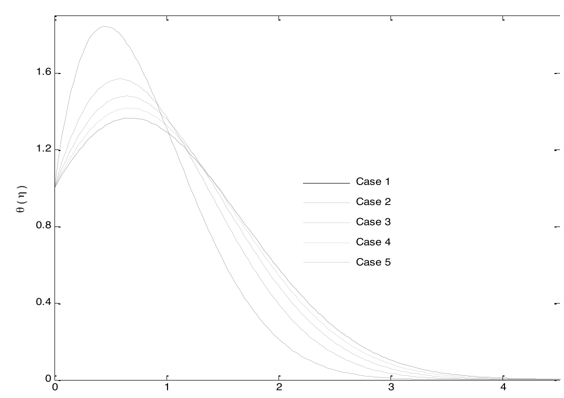

Figure 13. Temperature Profiles for Pr = 1.5, Ec = 6 and Various Cases of the Micropolar Parameters

For different values of *Ec*, the influence of *Pr* on the temperature profiles is presented in Fig. 14-17, with fixed c_1, c_2 and c_3. In the absence of viscous dissipation, an increase in *Pr* does not alter the direction of heat transfer as demonstrated by the Fig.14. On the other hand Figs.15-17 show that increasing *Pr* not only decreases the thermal boundary layer but also facilitates the thermal reversal near the surface while lowering the temperature profiles away from the surface. Physically, if *Pr* increases, the thermal diffusivity is decreased which leads to the reduction in energy ability that reduces the thermal boundary layer.

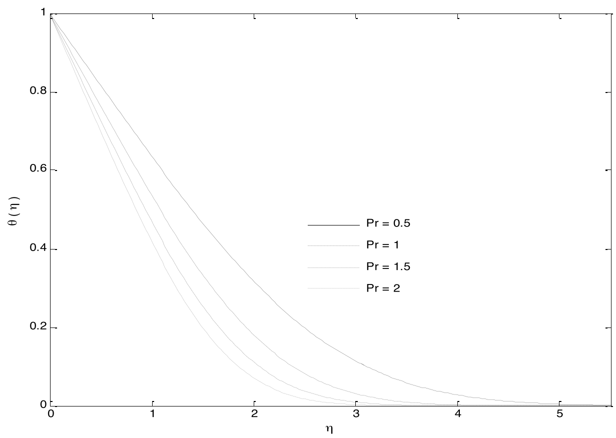

Figure 14: Temperature Profiles for *Ec=0* $c_1=6, c_2=0.6, c_3=0.7$ and Various Values of *Pr*

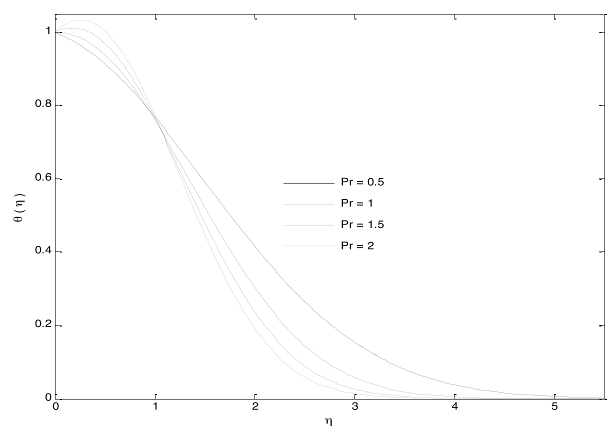

Figure 15: Temperature Profiles for *Ec* =2, $c_1=6, c_2=0.6, c_3=0.7$ and Various Values of *Pr*

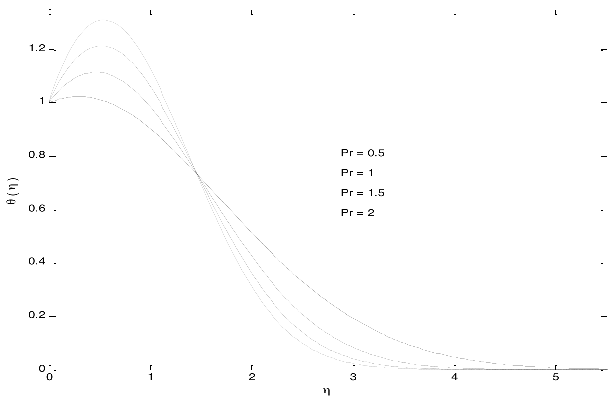

Figure 16: Temperature Profiles for *Ec* =4, $c_1=6, c_2=0.6, c_3=0.7$ and Various Values of *Pr*

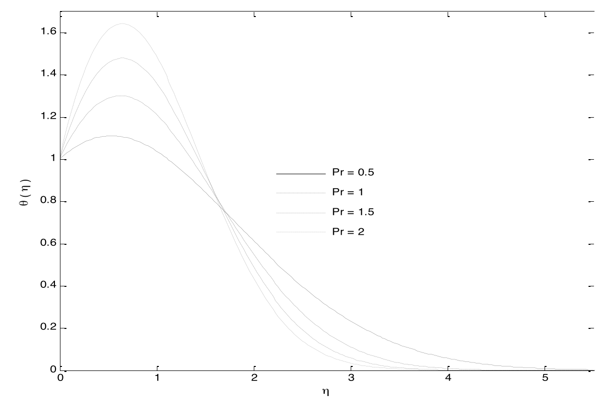

Figure 17: Temperature Profiles for Ec=6 $c_1=6, c_2=0.6, c_3=0.7$ and Various Values of *Pr*

Conclusions

A numerical study of boundary layer stagnation point flow with heat transfer effects of a micropolar fluid to wards a heated surface under the influence of viscous dissipation is presented for some values of the governing parameters. The following conclusions can be made: The micropolar fluids reduce the shear stresses and the heat transfer rate from the surface more than is done by the Newtonian fluids. Viscous dissipation may change the direction of heat transfer near the surface which is encouraged by the increasing values of Prandtl number. The velocity and thermal boundary layer thicknesses increase with an increase in the values of the micropolar parameters. Thermal boundary layer thickness is decreased by the increasing Prandtl number where as an opposite effect is observed for Eckert number. Micropolar structure of the fluid tends to decrease the thermal reversal caused by the viscous dissipation.

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