

Probability of the Size Distribution in Crushing

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ABSTRACT. The prediction of size distribution curves of jaw crushers product is one of the most important parameters in controlling its use.

Experimental and analytical studies are carried out on cubic samples of marble and limestone. The study shows that the size distribution curves in crushing obey the Gamma distribution. Definite relationships are obtained between either average diameter of the product (\bar{X}) or δ (standard deviation) and the set of the jaw crusher (S). The experimental values of the parameters of Gamma distribution (α_{act} , β_{act}) are compared with the calculated ones obtained from the above relationships. It was found that the greater the set of the jaw crusher, the greater the deviation.

The study of the size distribution in crushing under some variable parameters of the jaw crusher is one of the most important problems in the field to regulate the product of crushing. The application of the theory of probability in the study of the effect of the time of grinding on the size distribution curves in some Egyptian rocks and ores are carried out by Ahmed *et al.* (1982). From this study, it was concluded that for smaller times of grinding the size distribution obeys the Gamma distribution, and for greater times of grinding the distribution is normal. It was also concluded that the theory of probability can be applied to draw the size distribution predicted at any required time of grinding.

Ahmed *et al.* (1983) tried to apply the theory of probability in grinding by ball mills to study the effect of the rock hardness on the probability of the size distribution, and on the statistical parameters of the size distribution. It was concluded

that the Gamma distribution fits well with the empirical size distribution. The relation between either \bar{X} or $\hat{\sigma}$ and hardness (H) and ore to ball ratio (R) is expressed as follows:

$$\bar{X} = [1/(0.144 R + 0.77)] + [1/(0.007 R - 1.51)H] \dots\dots\dots (1)$$

$$\hat{\sigma} = [(1.03 + 0.009 R)] + [1/(0.906 R - 5.12)H] \dots\dots\dots (2)$$

In recent years, many trials are carried out to introduce a semi-empirical model to predict the size distribution of a product from a specified feed. Epstein (1948) suggests the use of two functions, namely the selection function and the breakage function. The selection function is a statement of the probability of breakage of a specific particle during a specific time interval. The selection function is related to a particle size and the intensity with which the energy is applied to a particle. The breakage function is a statement of the size distribution of particles produced on breakage of a single representative feed particle.

In recent years, much experimental evidence has been produced showing that grinding equations based on the above concepts may be used to predict, with some accuracy, the development of product size distribution with time in batch grinding, (*e.g.*, Austin and Klimpel 1964, Klimpel 1964) providing an adequate simulation of Heywood's classical grinding experiment. The equations can also be adapted to predict products from continuous operations through either predominantly empirical approach (*e.g.*, Lynch *et al.* 1967) or more theoretical treatment incorporating the dynamics of the system (*e.g.*, Kelsall and Reid 1965).

In the present paper, a study of the effect of the sizes produced from crushing is carried out. The cubic shaped particles of different sizes ($3 \times 3 \times 3$), ($5 \times 5 \times 5$), ($7 \times 7 \times 7$) and ($10 \times 10 \times 10$) cm^3 are fed to the jaw crusher under variable sets. The product of each experiment is analyzed on 11 sieves. Tested rocks are marble and limestone. Statistical analysis (Rigov 1973, Irwin and John 1977, Dlin 1958, Seymour 1974, and Speigel 1972) showed that the Gamma distribution fits well with the experimental data. We carried out 50 experiments on the two tested types of rocks.

Nomenclature

- \bar{X} = average diameter for the size distribution of the product of crushing, cm;
 $\hat{\sigma}$ = standard deviation of the distribution in the product, cm;
 α, β = parameters of the Gamma distribution
 $\alpha = [(\bar{X}/\hat{\sigma})^2 - 1], \beta = \bar{X}/(\alpha + 1)$
S = set of the jaw crusher, cm;
 $\alpha_{\text{act}}, \beta_{\text{act}}$ = actual obtained parameters of Gamma distribution, from the analysis of the experimental data;

$\alpha_{cal}, \beta_{cal}$ = calculated parameters of the Gamma distributions from the recommended mathematical relationships;

R = ore to ball ratio;

H = rock of ore hardness (Rockwell hardness number).

Experimental Data and Analysis

The parameters of the size distribution are determined for each experiment. The analysis showed that Gamma distribution fits the experimental data better than the normal distribution. The results of analysis for marble are given in Table 1.

Table 1. Statistical parameters of the experimental data for marble.

Size, cm ³	Set, cm	\bar{X} cm	$\hat{\sigma}$ cm	α	β
3 × 3 × 3	0.5	0.4337	0.3370	0.4200	0.306
	1.0	0.9960	0.6820	1.1320	0.470
	1.5	1.3720	0.9000	1.3240	0.590
	2.0	2.6180	1.7010	1.3700	1.105
	2.5	3.4870	1.6490	3.4710	0.780
5 × 5 × 5	0.5	0.451	0.342	0.7350	0.260
	1.0	1.203	1.019	0.3920	0.864
	1.5	1.964	1.437	0.8670	1.051
	2.0	3.095	1.750	2.1260	0.990
	2.5	3.401	1.730	2.8650	0.880
	3.0	2.944	1.644	2.2070	0.918
	3.5	4.408	1.670	3.1650	0.818
	4.0	3.788	1.622	4.3860	0.700
	4.5	3.997	1.520	5.9150	0.578
	5.0	4.164	1.419	7.6020	0.484
7 × 7 × 7	0.5	0.398	0.285	0.954	0.200
	1.0	1.019	0.889	0.314	0.775
	1.5	1.575	1.223	0.659	0.949
	2.0	2.383	1.581	1.272	1.049
	2.5	2.717	1.559	2.037	0.895
10 × 10 × 10	0.5	0.371	0.313	0.410	0.260
	1.0	0.856	0.550	1.422	0.353
	1.5	1.452	1.164	0.560	0.933
	2.0	1.996	1.511	0.794	1.112
	2.5	2.159	1.507	1.053	1.052

With regard to mathematical relationships between \bar{X} and the set of the jaw crusher, mathematical analysis showed that the initial size of the feed largely affects the mathematical form of the above relationship. The results of mathematical analysis and deviations are presented in Table 2.

Mathematical relationships between σ and the set of the jaw crusher are presented in Table 3.

The relations between either \bar{X} or $\hat{\sigma}$ and the set of jaw crusher (S) are shown in Fig. 1 and 2. From these figures, it is clear that the recommended mathematical relationships shown in Tables 2 and 3 best express the relations between the studied parameters. Larger deviations are remarked for $\hat{\sigma}$, while smaller deviations are noticed for \bar{X} .

The parameters of the Gamma distribution, α_{cal} and β_{cal} , calculated from the relationships shown in Tables 2 and 3, compared with that obtained from the analysis of experimental results (shown in Table 1) are given in Table 4.

Table 2. Best mathematical relationships for $\bar{X} = f(S)$, for marble and the average deviations.

Size, cm ³	Best mathematical relationship	Average dev. % ±
3 × 3 × 3	$(1/\bar{X}) = -0.202 + (1.252/S)$	6.73
5 × 5 × 5	$\bar{X} = 1.2079 \log S + 4.6029$	9.11
7 × 7 × 7	$\bar{X} = 1.200 S - 0.182$	3.69
10 × 10 × 10	$\bar{X} = S/(1.434 - 0.207 S)$	8.44

Table 3. Best mathematical relationships for $\hat{\sigma} = \phi(S)$, and average deviation for marble.

Size, cm ³	Best mathematical relationship	Average dev. % ±
3 × 3 × 3	$(1/\hat{\sigma}) = 0.171 + (1.177/S)$	14.10
5 × 5 × 5	$\hat{\sigma} = 1.841 - (0.726/S)$	9.42
7 × 7 × 7	$\hat{\sigma} = 1.063 S^{0.844}$	18.66
10 × 10 × 10	$1/\hat{\sigma} = (1.726/S) - 0.177$	9.00

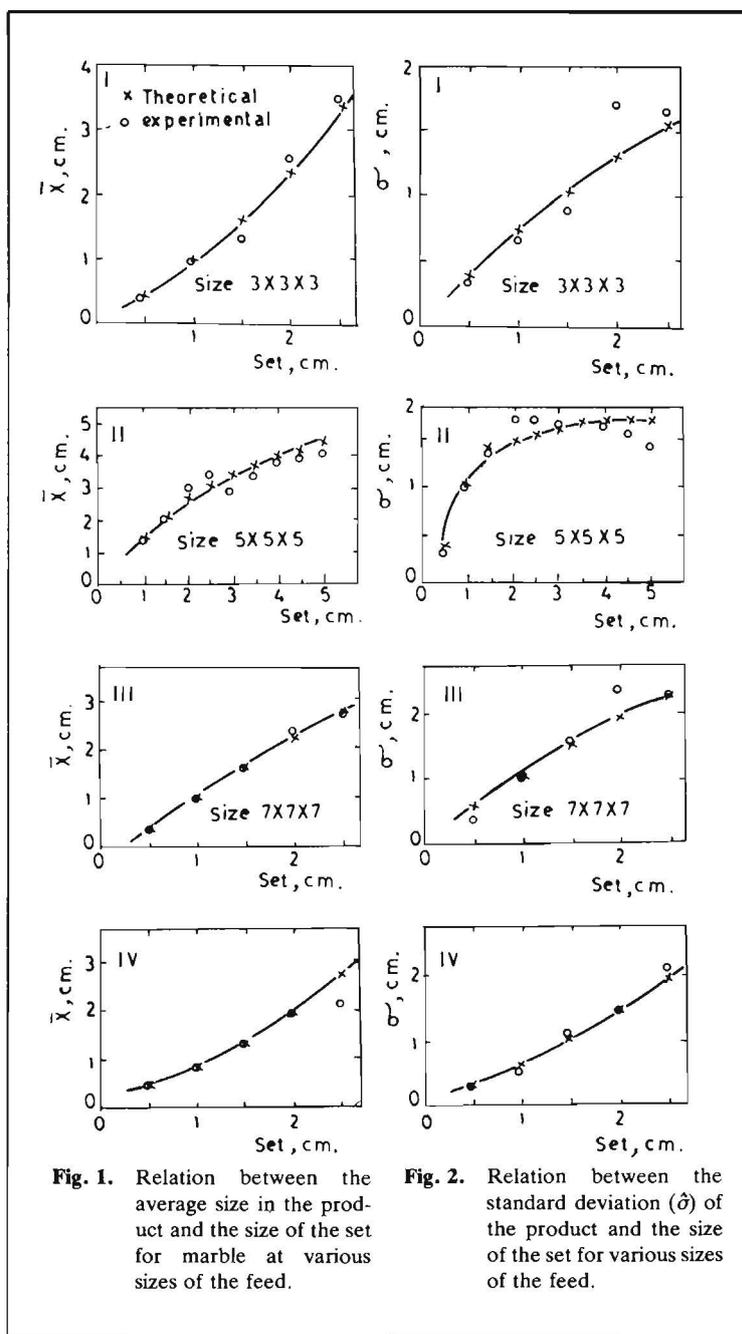


Table 4. Actual and calculated parameters of the Gamma distribution for various initial sizes for marble.

Size, cm ³	Set, cm	\bar{X}_{cal} cm	$\hat{\sigma}_{cal}$ cm	α_{cal}	α_{act}	β_{cal}	β_{act}
3 × 3 × 3	0.5	0.434	0.396	0.200	0.420	0.364	0.306
	1.0	0.952	0.741	0.650	1.132	0.577	0.470
	1.5	1.579	1.046	1.278	1.324	0.693	0.590
	2.0	2.357	1.317	2.203	1.307	0.714	1.105
	2.5	3.346	1.558	3.612	3.471	0.725	0.780
5 × 5 × 5	1.0	1.208	1.115	0.170	0.392	1.028	0.864
	1.5	2.018	1.357	1.211	0.867	0.913	1.051
	2.0	2.590	1.478	2.070	2.126	0.843	0.990
	2.5	3.040	1.550	2.847	2.865	0.790	0.880
	3.0	3.404	1.600	3.526	2.207	0.752	0.918
	3.5	3.712	1.633	4.167	3.165	0.718	0.818
	4.0	3.979	1.659	4.752	4.386	0.692	0.700
	4.5	4.215	1.679	5.302	5.915	0.669	0.578
7 × 7 × 7	0.5	0.418	0.592	-0.500	0.954	0.836	0.200
	1.0	1.017	1.062	-0.080	0.314	1.105	0.775
	1.5	1.618	1.496	0.170	0.659	1.383	0.949
	2.0	2.218	1.907	0.350	1.272	1.643	1.049
	2.5	2.818	2.303	0.500	2.037	1.879	0.895
10 × 10 × 10	0.5	0.376	0.305	0.520	0.410	0.250	0.260
	1.0	0.815	0.646	0.590	1.422	0.510	0.353
	1.5	1.335	1.027	0.690	0.560	0.790	0.933
	2.0	1.960	1.458	0.810	0.794	1.080	1.112
	2.5	2.727	1.948	0.960	1.053	1.390	1.052

The size distribution curves for marble, calculated from the values of α_{cal} and β_{cal} (shown in table 4) for the sizes of (3 × 3 × 3) and (5 × 5 × 5) compared with the experimental size distribution curves are shown in Fig. 3. From these figures, it seems that experimental and theoretical size distributions coincide reasonably well (e.g., for the size 3 × 3 × 3 at sets 1 and 1.5 cm). Small deviations are remarked in other figures except for the set (2.5 cm) at the size 3 × 3 × 3, which shows larger deviations. Generally, we can say that the recommended mathematical relationships are used with success to draw the predicted size distribution of the product of crushing for a given initial size and adjusted set.

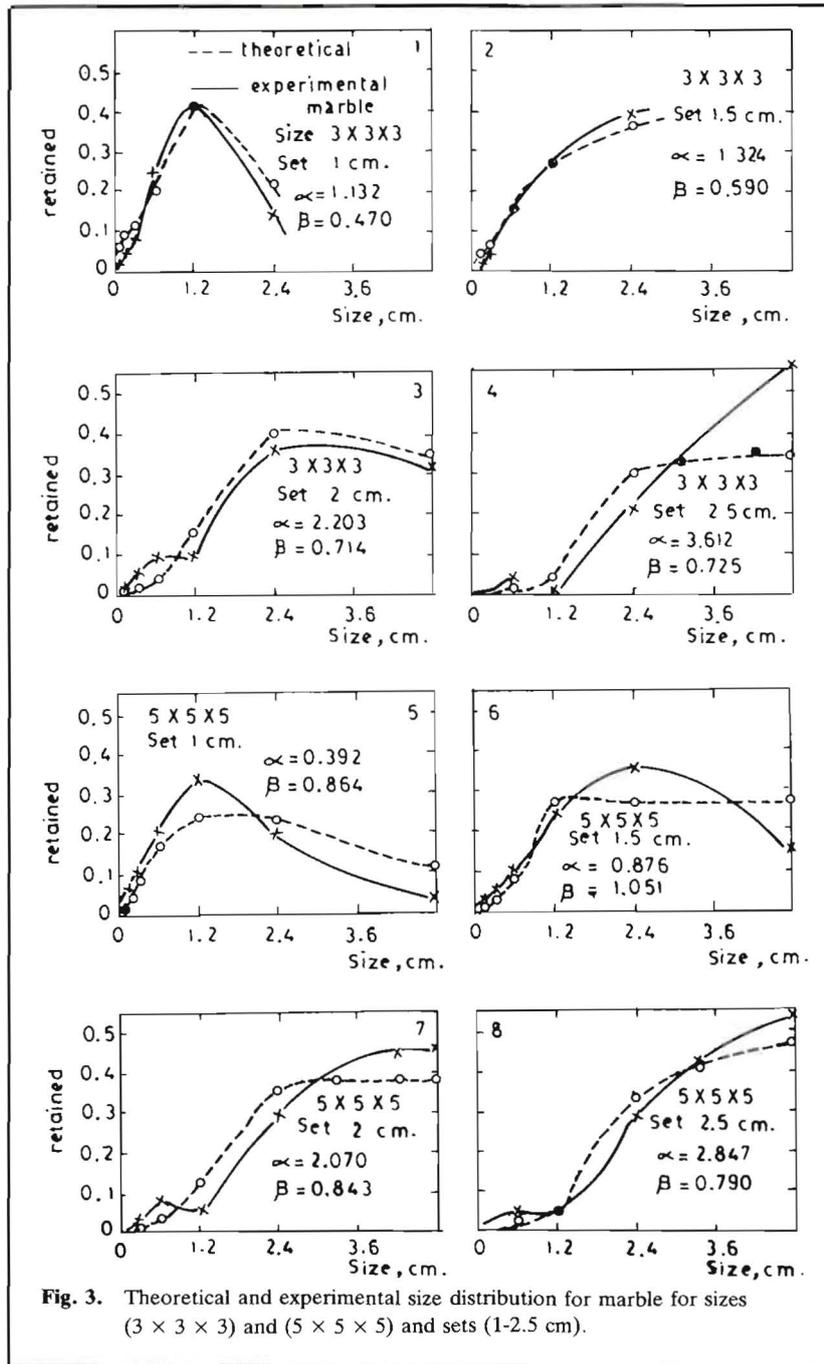


Fig. 3. Theoretical and experimental size distribution for marble for sizes (3 × 3 × 3) and (5 × 5 × 5) and sets (1-2.5 cm).

Table 5. Statistical parameters of the experimental data for limestone.

Size, cm ³	Set, cm	\bar{X} cm	$\hat{\sigma}$ cm	α	β
3 × 3 × 3	0.5	0.354	0.274	0.671	0.212
	1.0	1.109	0.700	1.534	0.438
	1.5	1.651	1.181	0.954	0.845
	2.0	2.409	1.307	2.396	0.709
	2.5	2.457	1.622	1.296	1.070
5 × 5 × 5	0.5	0.340	0.281	0.460	0.230
	1.0	0.834	0.637	0.715	0.486
	1.5	1.352	0.932	1.105	0.642
	2.0	2.005	1.461	0.883	1.065
	2.5	2.787	1.748	1.541	1.097
	3.0	2.715	1.650	1.708	1.003
	3.5	2.765	1.650	1.793	0.900
	4.0	3.597	1.735	3.300	0.840
4.5	3.997	1.520	5.915	0.578	
7 × 7 × 7	0.5	0.388	0.295	0.728	0.223
	1.0	0.731	0.531	0.891	0.386
	1.5	1.545	1.383	0.250	1.238
	2.0	2.188	1.643	0.773	1.234
	2.5	2.442	1.268	2.709	0.658
10 × 10 × 10	0.5	0.304	0.255	0.420	0.214
	1.0	0.870	0.926	-0.120	0.987
	1.5	1.606	1.178	0.858	0.864
	2.0	1.996	1.408	1.009	0.994
	2.5	2.278	1.674	0.850	1.231

Table 6. Best mathematical relationships for $\bar{X} = f(S)$, for limestone and the average deviations.

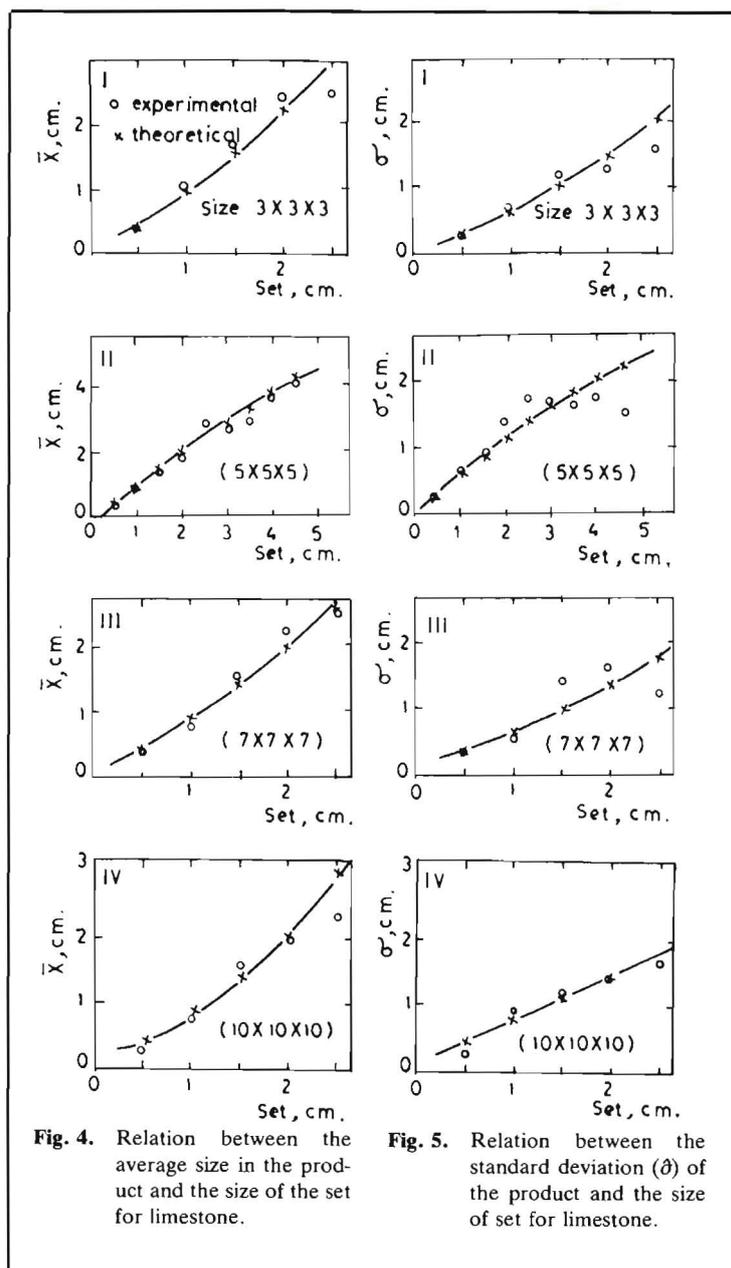
Size, cm ³	Best mathematical relationship	Average dev. % ±
3 × 3 × 3	$\bar{X} = 0.944 S^{1.235}$	12.13
5 × 5 × 5	$\bar{X} = 0.820 S^{1.1}$	7.94
7 × 7 × 7	$\bar{X} = 0.860 S^{1.214}$	9.40
10 × 10 × 10	$\bar{X} = 0.812 S^{1.285}$	9.49

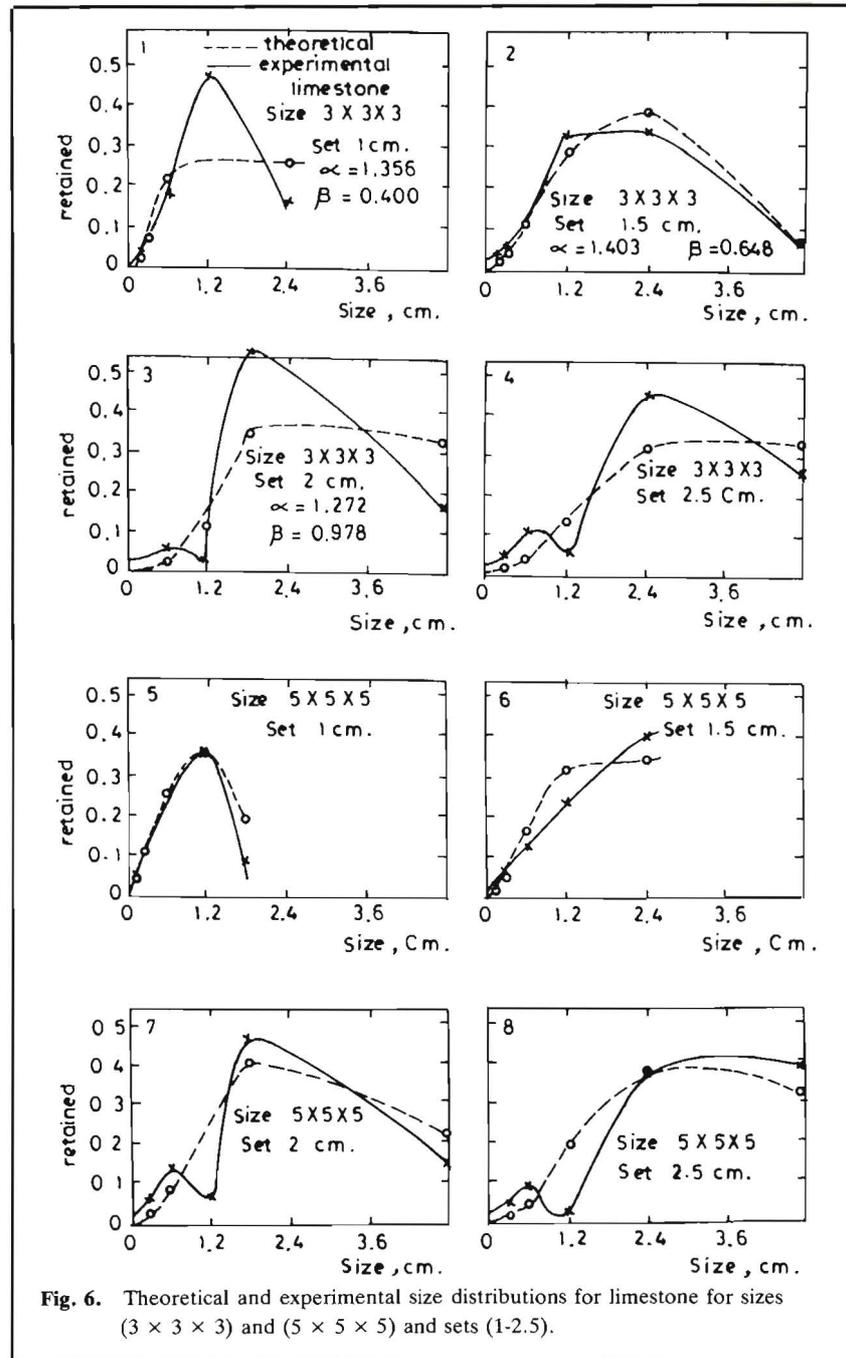
Table 7. Best mathematical relationships for $\hat{\sigma} = \hat{\sigma}(S)$, for limestone and the average deviations.

Size, cm ³	Best mathematical relationship	Average dev. % ±
3 × 3 × 3	$(1/\hat{\sigma}) = -0.269 + (1.895/S)$	13.90
5 × 5 × 5	$(1/\hat{\sigma}) = 0.089 + (1.610/S)$	16.24
7 × 7 × 7	$(1/\hat{\sigma}) = -0.126 + (1.757/S)$	20.44
10 × 10 × 10	$(1/\hat{\sigma}) = -0.448 + (2.058/S)$	9.82

Table 8. Actual and calculated parameters of the Gamma distribution for limestone.

Size, cm ³	Set	\bar{X}_{cal}	$\hat{\sigma}_{cal}$	α_{cal}	α_{act}	β_{cal}	β_{act}
1	2	3	4	5	6	7	8
3 × 3 × 3	0.5	0.401	0.284	0.994	0.671	0.201	0.212
	1.0	0.944	0.615	1.356	1.543	0.400	0.438
	1.5	1.558	1.005	1.403	0.954	0.648	0.845
	2.0	2.222	1.474	1.272	2.396	0.978	0.709
	2.5	2.927	2.045	1.049	1.296	1.429	1.070
5 × 5 × 5	0.5	0.383	0.302	0.608	0.460	0.238	0.230
	1.0	0.820	0.589	0.938	0.715	0.423	0.486
	1.5	1.282	0.860	1.222	1.105	0.577	0.642
	2.0	1.760	1.119	1.474	0.883	0.711	1.065
	2.5	2.249	1.365	1.715	1.541	0.828	1.097
	3.0	2.749	1.600	1.952	1.708	0.931	1.003
	3.5	3.256	1.820	2.200	1.793	1.017	0.990
	4.0	3.772	2.035	2.436	3.300	1.097	0.840
4.5	4.293	2.240	2.673	5.915	1.169	0.578	
7 × 7 × 7	0.5	0.371	0.295	0.582	0.728	0.235	0.223
	1.0	0.860	0.613	0.968	0.891	0.437	0.386
	1.5	1.407	0.957	1.162	0.250	0.650	1.238
	2.0	1.995	1.330	1.250	0.773	0.887	1.234
	2.5	2.615	1.735	1.272	2.709	1.151	0.658
10 × 10 × 10	0.5	0.333	0.273	0.488	0.420	0.223	0.214
	1.0	0.812	0.620	0.715	-0.120	0.473	0.478
	1.5	1.368	1.082	0.598	0.858	0.856	0.864
	2.0	1.979	1.721	0.322	1.009	1.494	0.994
	2.5	2.637	2.663	-0.019	0.850	2.689	1.231





A similar study is carried out on a limestone. The analysis of the experimental data gives the following results shown in Table 5.

An effort is made to get mathematical relationships between \bar{X} and the set of the jaw crusher (S), that is a general form given by $\bar{X} = f(S)$. Calculated constants of the recommended relationships and the average deviations are given in Table 6.

The relationships between the standard deviations ($\hat{\sigma}$) and the set (S) are presented in Table 7.

The relations between the set of the jaw crusher (S) and either \bar{X} or $\hat{\sigma}$ is represented in Fig. 4 and 5.

The parameters of the Gamma distribution α_{cal} and β_{cal} , calculated from the relationships shown in Tables 6 and 7, compared with that obtained from the analysis of the experimental results (shown in Table 5) are given in Table 8.

From Table 8, it appears that the maximum deviation between the calculated parameters of Gamma distribution (α_{cal} and β_{cal}) and the actual obtained values from experimental data (α_{act} and β_{act}) is noticed at higher values of the set. Hence, the higher the set of jaw crusher, the greater will be the deviations from the normal conditions. And, finally we obtain a wide range of size distributions. For smaller set of the jaw crusher, if finer the product then the theoretical and experimental size distributions will coincide. Experimental and theoretical size distributions for limestone for sizes ($3 \times 3 \times 3$ and $5 \times 5 \times 5$) and sets (1-2.5 cm) are shown in Fig. 6. From these figures, one can deduce that drawing the predicted size distribution by these recommended equations does not give greater errors, especially for smaller sets.

Conclusions and Recommendations

From the above development, the following conclusions are recommended:

1. The Gamma distribution is a good distribution to illustrate size distribution in crushing.
2. The mathematical relationships between either \bar{X} (average diameter of the product) or $\hat{\sigma}$ (standard deviation of the product) and the set (S) of the jaw crusher for marble and limestone, respectively, are derived. The results are shown in Tables 2 and 3 for marble and 6 and 7 for limestone.
3. A comparative study between the calculated parameters of the size distribution (Gamma distribution) α_{cal} and β_{cal} , and the actual obtained from the analysis of the experimental data (α_{act} and β_{act}) showed that maximum deviations from the experimental data occur at higher sets of the jaw crusher.
4. The recommended relationships can be used without significant errors to illustrate the size distribution curves for the product of crushing in jaw crushers.

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احتمالات التوزيع الحجمي لناتج الطحن الأولي

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في الوقت الحاضر يبدو من الأهمية بمكان إمكانية التنبؤ بمنحنيات التوزيع الحجمي لناتج عمليات الطحن الأولى وذلك للتحكم في متغيرات مكنة الطحن الأولى وكذلك في متغيرات الحجم الابتدائي . وقد أثبتت هذه الدراسة التي أجريت على مكعبات مختلفة الأحجام من الرخام ، والحجر الجيري أن توزيع جاما هو من أنسب التوزيعات الإحصائية التي تعبر عن احتمالات التوزيع الحجمي لناتج الطحن الأولى في الطواحين ذات الفكين .

وقد تم في هذا البحث إجراء تجارب معملية وتحليلات نظرية لناتج الطحن في العينات المدروسة ، وقد تم إيجاد المعادلات الرياضية التي تربط بين كل من الحجم المتوسط والناتج والانحراف المعياري له مع حجم فتحة مكنة الطحن الأولى ، وقد حُسبت متغيرات التوزيع الإحصائي (ألفا وبيتا) مرتين أولاً من تحليل نتائج التجارب المعملية والأخرى من المعادلات المقترحة ، وقورنت النتائج . وقد أثبتت هذه الدراسة أن الفرق بين القيمة المحسوبة والمقاسة تزداد بزيادة فتحة مكنة الطحن الأولى .