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The Explicit Solution of Reverse Problem for Finite Vertical Cylinder of Gravity Prospecting

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ABSTRACT The downward continuation method has been used to obtain a new approach for rapid gravity interpretation of Bouguer anomalies attributed to finite vertical cylinder. The problem has been solved mathematically together with a theoretical test. Accordingly, master curves have been prepared to facilitate the application of the method. The procedure to calculate the depths to the upper and lower surfaces, mass of unit length and radius of the causative body has been outlined. An interpreted example of gravity anomaly of Kharga Oasis area, Western Desert of Egypt, is given.

One can determine a body's attractive force (or a component of it) at any desired point in space either by direct application of Newton's law or by differentiation of its potential field. The direct objective of gravity prospecting has one solution only and is quite easily solved for different geometrical forms (Dobrin 1976). But, one has also to solve the reverse problem to find the characteristics of the masses that have the field anomaly. The problem does not always have a single solution. It is not always possible to give exact formulae for its solution, and one usually has to approach it by way of selection and successive approximation.

From a geological view point, an interesting case is that of a vertical cylinder existing below the surface. This is often convenient form for computing gravity anomalies from salt domes and volcanic plugs. A case of great practical interest is that of a cylinder with its top buried at a given depth below the surface.

The method of continuation, mainly the upward, has been used for interpreting gravity and magnetic anomalies caused by spherical bodies by Mohan *et al.* (1971) followed by El Hussaini (1978) and El Hussaini and Abd El All (1983) who solved the problem of the gravity anomalies due to infinite cylindrical masses and for the finite horizontal case, respectively. The solutions were obtained by using the downward continuation technique. The present work is concerned with giving a suitable solution of the reverse problem for the finite vertical cylinder, also by using the same method.

The solution is derived mathematically and can be easily employed to determine the various parameters of the attractive body such as, depths to upper and lower surfaces, mass of unit length and radius at known density contrast. This approach gives a new and accurate treatment of calculations.

Theory

Many expressions have been derived for the vertical component of the gravity field attributed to vertical cylinder of limit length, such as those given by Sazhina and Grushinsky (1971), Telford *et al.* (1978) and Pedersen (1978). The difference between them is in the method of derivation and the included parameters. For simplicity, the formula related to the first author has been used for the present mathematical treatment.

The gravity effect at a point p(x,0) on the surface a distance x off the axis of a vertical cylinder with lower and upper surfaces z_1 and z_2 below the surface, respectively, is given according to Sazhina and Grushinsky (1971) by:

$$g_{x,0} = f\lambda\left[\frac{1}{\sqrt{x^2 + z_1^2}} - \frac{1}{\sqrt{x^2 + z_2^2}}\right] \tag{1}$$

where λ is the mass of unit length, f is the universal gravitational constant. The gravity effect vertically above the cylinder can be derived from equation (1) by putting $x = 0$ and then:

$$g_{0,0} = f\lambda\left[\frac{1}{z_1} - \frac{1}{z_2}\right] \tag{2}$$

The downward continuation of the field at depth h below the plane of observation is given by:

$$g_{0,h} = f\lambda\left[\frac{1}{z_1 - h} - \frac{1}{z_2 - h}\right] \tag{3}$$

From equations (2) and (3) it can be concluded that:

$$g_{0,0}/g_{0,h} = \frac{(z_1 - h)(z_2 - h)}{z_1 z_2} = \frac{z_1 z_2 - h(z_1 + z_2) + h^2}{z_1 z_2} \tag{4}$$

Considering that $z_2 = nz_1$ where $n > 1$, then equation (4) leads to:

$$g_{0.0}/g_{0.h} = \frac{nz_1^2 - z_1h(n+1) + h^2}{nz_1^2} \tag{5}$$

$$= \frac{h^2}{nz_1^2} - \frac{h(n+1)}{nz_1} + 1 \tag{6}$$

Verification of the Obtained Formula

To verify relation (6), assume that the ratios $g_{0.0}/g_{0.h_1} = a$ and $g_{0.0}/g_{0.h_2} = b$ for two different levels h_1 and h_2 where $h_2 > h_1$ respectively, we get from equation (6) the following relations

$$(a-1)nz_1^2 + h_1(n+1)z_1 - h_1^2 = 0 \tag{7}$$

$$(b-1)nz_1^2 + h_2(n+1)z_1 - h_2^2 = 0 \tag{8}$$

Multiplying (7) and $(b - 1)$ and (8) by $(a - 1)$ and subtracting one gets:

$$(n+1)z_1 = \frac{h_1^2(b-1) - h_2^2(a-1)}{h_1(b-1) - h_2(a-1)} = A \tag{9}$$

Multiplying (7) and h_2 and (8) by h_1 and subtracting one obtains:

$$nz_1^2 = \frac{h_1h_2(h_2 - h_1)}{h_1(b-1) - h_2(a-1)} = B \tag{10}$$

where A and B are greater than zero.
Eliminating z_1 between (9) and (10) it leads to:

$$n^2B + n(2B - A^2) + B = 0 \tag{11}$$

The solution of equation (11) is given by:

$$n = \frac{1}{2B}\left[A^2 - 2B \pm A\sqrt{A^2 - 4B}\right]$$

It is clear from the last relation that n is a real value only when $A^2 > 4B$, *i.e.* when:

$$(n+1)^2 z_1^2 > 4n\, z_1^2$$

therefore $(n - 1)^2 > 0$

i.e. $n > 1$

which confirms the hypothesis.

Also, from (9) and (10):

$$\frac{A}{B} = \frac{h_1^2(b-1) - h_2^2(a-1)}{h_1 h_2(h_2 - h_1)}$$

i.e. $-a + 1 > b - 1$

then $a + b < 2$

which is right because: $0 < b < a < 1$.

The previous discussion illustrates that the mathematical derivation of the new method is completely correct.

Preparation of Master Curves

Usually master curves facilitate the interpretation of practical data and are more easier for application than the mathematical relations.

Accordingly, master curves corresponding to relation (5) after some modifications are prepared. Formula (5) can be written as:

$$g_{0,0}/g_{0,h} = 1 - \left(\frac{n+1}{n}\right)\frac{h}{z_1} + \frac{1}{n}\left(\frac{h}{z_1}\right)^2$$

$$= 1 - \left(1 + \frac{1}{n}\right)\frac{h}{z_1} + \frac{1}{n}\left(\frac{h}{z_1}\right)^2 \tag{12}$$

Substituting $\frac{1}{n} = E$ and $\frac{h}{z_1} = M$ in relation (12), where $0 < E < 1$ and $M > 0$, it leads to:

$$g_{0,0}/g_{0,h} = 1 - (1 + E)M + EM^2 \tag{13}$$

Master curves belonging to relation (13) are prepared for various values of E as a function of n on semi logarithmic paper as shown in Fig. 1. The obtained calculations are given in Table 1.

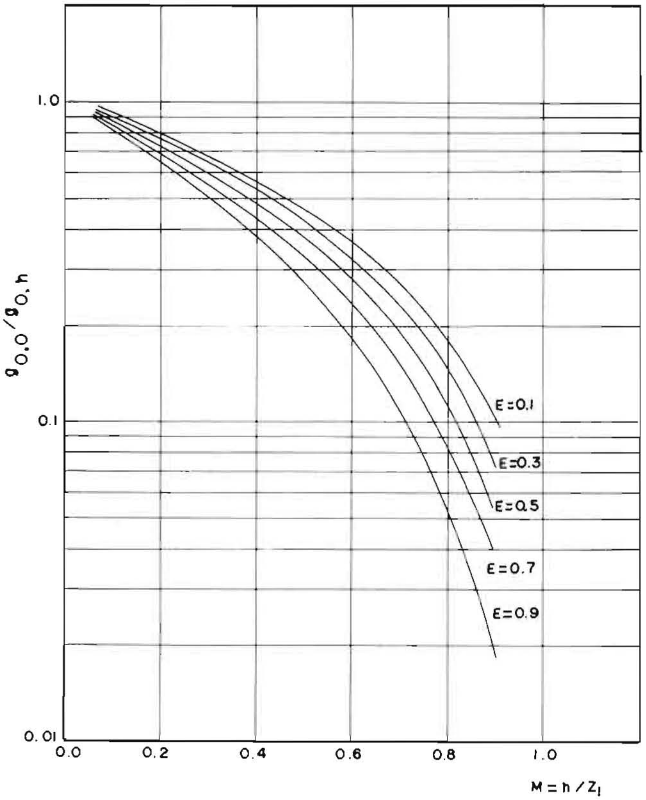

Fig. 1. Master curves for interpreting gravity anomaly due to finite vertical cylinder.

Table 1. Calculation of the obtained master curves for different values of E.

$g_{0,0}/g_{0,h}$					
M	**E = 0.1**	**E = 0.3**	**E = 0.5**	**E = 0.7**	**E = 0.9**
0.05	0.995	0.936	0.928	0.917	0.905
0.10	0.891	0.870	0.855	0.837	0.819
0.20	0.784	0.757	0.720	0.688	0.656
0.30	0.679	0.637	0.595	0.553	0.511
0.40	0.576	0.528	0.480	0.432	0.384
0.50	0.475	0.425	0.375	0.325	0.275
0.60	0.376	0.328	0.280	0.232	0.164
0.70	0.279	0.237	0.195	0.153	0.111
0.80	0.184	0.153	0.120	0.088	0.056
0.90	0.091	0.075	0.055	0.037	0.019

Technique and Method of Application

To calculate the different parameters of a finite vertical cylinder we follow the following steps:

1. The gravity anomaly that may represent a vertical cylinder must be approximately circular one to be suitable for interpretation by the new method. The position of the zero base line must be estimated with as much as care as possible to determine the maximum gravity effect $g_{0,0}$.

2. The values g_{0,h_1}, g_{0,h_2}, g_{0,h_3} which represent the downward continuation values at three different levels h_1, h_2 and h_3, respectively, are calculated with the help of any method such as that of Peters (1949), Constantinescu and Botezatu (1961) or Roy (1966) and then the values $(g_{0,0}/g_{0,h_1})$, $(g_{0,0}/g_{0,h_2})$ and $(g_{0,0}/g_{0,h_3})$ can be directly computed.

3. Three horizontal lines on the master curves shown in Fig. 1 are drawn at the mentioned values.

4. The corresponding values of $E\left(E = \frac{1}{n}\right)$ and $M\left(M = \frac{h}{z_1}\right)$ are directly obtained. Therefore, it becomes easily to conform sets of differnt depths z_1 with their corresponding E's. It is done by multiplying the first set by the known value h_1, the second by h_2 and the third by h_3. Then we obtain three sets of E and z_1.

5. The relation between E and z_1 is plotted as shown in Fig. 4 for the interpreted example given in Fig. 3.

6. Theoretically, the relation between E and M for three hypothetical values of $g_{0,0}/g_{0,h}$ is shown in Fig. 2. The representing curves are straight lines for all

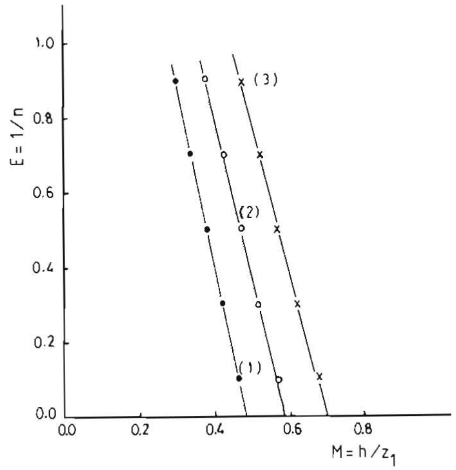

Fig. 2. Theoretical relation between E and M for $g_{0,0}/g_{0,h}$ = 0.5 (1), 0.4 (2) and 0.3 (3).

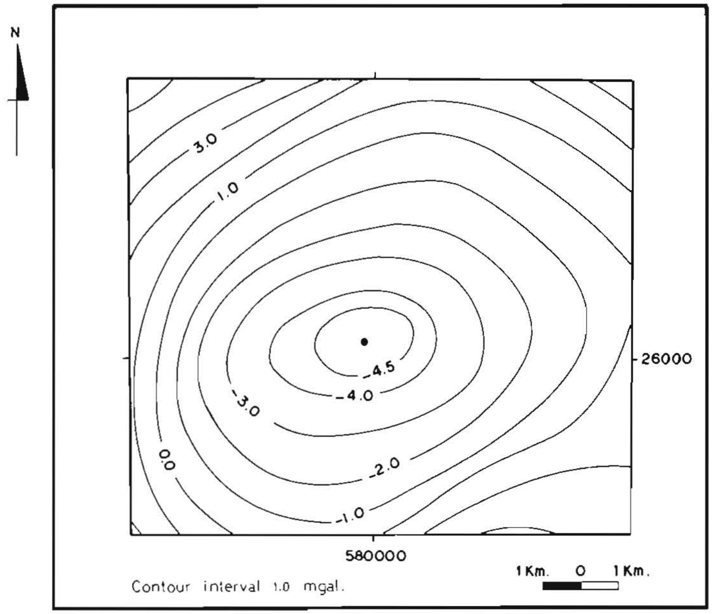

Fig. 3. Bouguer anomaly of Kharga Oasis area, Western Desert of Egypt.

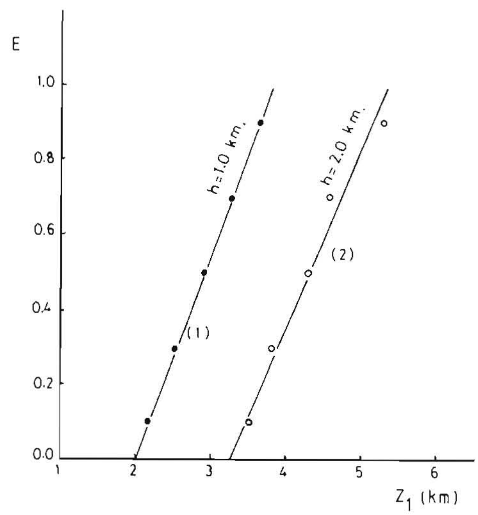

Fig. 4. Relation between E and z_1 for the interpreted anomaly given in Fig. 3.

different levels below the surface. With the knowledge of the h values, the curves representing the relation between E and z_1 can be easily drawn. These also will be straight lines. The reciprocal of the slope of such lines is the depth of the lower surface z_2.

7. The intersection of these lines with the x-axis represents the summation of the depth to the upper surface and the chosen h.

8. It is noticed that the different parameters can be calculated from one $E - z_1$ curve. For more accurate results, it is better to apply the method to a number of curves and the mean values are obtained.

Interpreted Example

A Bouguer anomaly of Kharga Oasis area, Western Desert of Egypt, has been selected for application of the new method. This anomaly is approximately circular and symmetric.

By using Constantinescu's method (1961) the downward continuation values are calculated at levels 1.0 and 2.0 km below the surface. $g_{0,0} = -4.75$, $g_{0,1} = -8.80$ and $g_{0,2} = -11.47$ m. gals, therefore $g_{0,0}/g_{0,1} = 0.54$ and $g_{0,0}/g_{0,2} = 0.41$. Drawing horizontal lines at these values on the master curves Fig. 1, the values of E and M are then obtained for the two levels. The curves representing the relation between E and z_1 are constructed according to the results obtained in Table 2, as shown in Fig. 4.

The intersection of the lines with x-axis is denoted, by $z_{in} = z_1 +$ the level of continuation.

From the diagram for the level $h = 1$

$2.05 = z_1 + 1$ Therefore, $z = 1.05$ km

Table 2. The obtained results for the interpreted example.

h = 1 km			**h = 2 km**		
E	**M**	**z_1**	**E**	**M**	**z_1**
0.9	0.27	3.70	0.9	0.37	5.26
0.7	0.30	3.33	0.7	0.43	4.65
0.5	0.34	2.94	0.5	0.47	4.25
0.3	0.39	2.56	0.3	0.53	3.83
0.1	0.44	2.27	0.1	0.57	3.51

and for the level h = 2

$$3.25 = z_1 + 2$$

$$z_1 = 1.25 \text{ km}$$

Therefore, the mean value of $z_1 = 1.15$ km.

The slope of the $E - z_1$ line $= \frac{1/n}{z_1} = \frac{1}{n \times z_1} = \frac{1}{z_2}$, *i.e.* the reciprocal of the slope of the $E - z_1$ line equals to the depth to the lower surface, therefore from the $E - z_1$ line No. 1 (h = 1 km) $z_2 = 3.57$ km and from the $E - z_2$ line No. 2 (h = 2 km) $z_2 = 3.64$ km. The mean value of $z_2 = 3.61$ km. The density contrast in this area is calculated by Abd El All (1982) from the drill hole data as 0.42 g/cc. The mass of unit length can be calculated from the formula:

$$\lambda = \frac{g_{0.0}}{f\left(\frac{1}{z_1} - \frac{1}{z_2}\right)} = \frac{4.75 \times 10^{-3}}{6.67 \times (0.8696 - 0.2770) \times 10^{-13}} = 120 \text{ kg/km}$$

The radius of the cylinder is given by:

$$R = \sqrt{\lambda/\pi\sigma} = 10^5\sqrt{\frac{1.2}{3.14 \times 0.42}} = 0.95 \text{ km}$$

El-Hussaini *et al.* (1978) concluded that the Bouguer anomalies of the Kharga Oasis area are mainly due to anomalous bodies laying within the basement rather than bodies laying on the basement surface. The present results coincide with their conclusion because the depth of the basement in this area ranges between 400-1000 m as it is obtained from the hole drill data (El-Samni and El Kashef 1965).

Conclusion

The present method provides an accurate interpretation of gravity anomaly over vertical finite cylinder by utilizing the downward continuation method. The mathematical expression has been converted into a set of master curves to be easier for interpretation. Procedure to obtain the various parameters (depths to upper and lower surfaces, radius and mass of unit length) has been outlined, together with an actual interpreted example.

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References

Abd El All, E.M. (1982) *Interpretation of Gravity Data of Kharga Oasis Area, Western Desert, Egypt,* M.Sc. Thesis, Dept. of Geology, Assiut University, Egypt.

Constantinescu, L. and **Botezatu, R.** (1961) Contributü la interpretarea fizica a anomalüer cimpuriler potentiale, *Probleme* Geofiz. R.P.R.I., I.

Dobrin, M.B. (1976) *Introduction to Geophysical Prospecting*, *3rd* ed., McGraw-Hill Book Co. 630 p.

El-Hussaini, A. (1978) Nomograms of interpreting gravity anomalies over spherical and cylindrical bodies, *Bull. Fac. Sci., Assiut Univ.*, **7** (2): 71-80.

El-Hussaini, A., Riad, S. and **Abd El All, E.** (1978) Tectonic trends in the Kharga Oasis area, Western Desert, Egypt, interpreted from gravity data, *Bull. Fac. Sci., Assiut Univ.* **7** (3): 149-159.

El-Hussaini, A. and **Abd El All, E.** (1983) Nomograms of interpreting gravity anomalies above a finite horizontal cylinder, *Sci. Bull., Univ. Qattar* (in press).

El Samni, A., and **El Kashef, A.F.** (1965) *Ground Water Potentialities of the New Valley*, Report no. 1 of the Arab Technical Committee, Cairo, Egypt.

Mohan, V., Drolier, R.K., Singh, R. and **Rathor, H.S.** (1971) Interpretation of gravity and magnetic anomalies caused due to spherical bodies, *Pageoph* **90** (7): 9-14.

Pedersen, L.B. (1978) A statistical analysis of potential fields using a vertical circular cylinder and dike, *Geophysics* **43**: 943-953.

Peters, L.J. (1949) The direct approach to magnetic interpretation and its practical application, *Geophysics* **14**: 290-319.

Roy, A. (1966) The method of continuation in mining geophysical interpretation, *Geoexploration* **14**: 65-81.

Sazhina, N. and **Grushinsky, N.** (1971) *Gravity Prospecting*, Mir Publ., U.S.S., Moscow, 491 p.

Telford, W.M., Geldart, L.P. and **Keys, D.A.** (1978) *Applied Geophysics*, Cambridge, 860 p.

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الحـل الو اضح للمشكلة العكسية لاسطو انة رأسية محدودة الطول فى الكشف بالطرق التثاقلية

علية محمد حماد الحسينى

قسم الجيولوجيا - كلية العلوم - جامعة أسيوط - مصر

يشتمـل هذا البحث على طريقـة جديـدة للتفسـير السـريع لقيـاسـات الجاذبية الناجمة عن اسطوانة رأسية محدودة الطول باستخدام طريقة الاستمرار السفلى .

ولهـذا الغـرض اشتقت بعض المعـادلات الـريـاضيـة لحسـاب البـا رامـترات المختلفة للجسم الجاذب علاوة على اختبـار نظـرى على هذه المعادلات . هذا وقد أُعدت مجموعة منحنيات مميزة لتسهيل الحسابات .

والطريقة التى تم الحصول عليها سهلة التطبيق للغاية وقد طبقت على مثال حقيقى بمنطقه الواحات الخارجة بمصر .