Arab Gulf J. scient. Res. **3** (1), pp. 409-419 (1985)

Effect of the Energy Consumed in Grinding and the Initial Size of the Feed on the Percentiles of the Product of the Grinding Process

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ABSTRACT. A set of 43 experiments are carried out on equal-sized samples of 200 g weight from numulitic-limestones crushed and screened to obtain a uniform size. These samples were ground in a vertical stamp mill at variable energy levels (0.7 to 3.5 KwH/t). The average initial size of the samples varied as 0.565, 0.45, 0.358, 0.283, 0.225 and 0.18 cm, respectively.

The percentiles of the product in each experiment are calculated.

Mathematical analysis is used in the present work to obtain some relationships between the percentiles (d_x) = (d_{20}, d_{30}, d_{40}, ... and d_{90}) and the initial size of the feed s_i rather than the energy consumed in the process of grinding E.

Recommended relationships can be used practically for regulating either the initial size feeding to the stamp mill or the energy required to get the required percentiles in the product.

The percentiles of the product are the screen aperatures which pass 20, 30, ... and 90 percent of the product. These percentiles are expressed generally as (d_x).

A set of 43 experiments were carried out on samples of numulitic-limestones. The mineral was fed first to a jaw crusher, its set was adjusted at 10 mm. The product of the jaw crusher was fed into roll crusher of set adjusted at 5 mm. The product from the roll crusher was screened to the following sizes −0.63 + 0.5, −0.5 + 0.4, −0.4 + 0.315, −0.315 + 0.25, −0.25 + 0.2, −0.2 + 0.16 cm to prepare the needed head sample. The head sample of each size was divided into small samples of 200 g weight.

Each sample was fed into the mortar of the stamp mill and ground at a definite number of drops. The product was screened on screens of sizes 0.63, 0.5, 0.4, 0.315, 0.25, 0.2, 0.16, 0.125, 0.1, 0.08, 0.063, 0.05, 0.04, 0.0315, 0.025, 0.020, 0.016, 0.0125, 0.009, 0.008, 0.0071 and 0.0056 cm, respectively.

The energy consumed in the process of grinding was calculated by making use of the formula suggested by Mehrim (1970)

$$E = 2.7235 \text{ WL} \times 10^{-6} \text{ KwH} \tag{1}$$

where:

W – crushing weight;

L – height at which the stamp falls on the ore for one drop.

Statistical analysis is used to get reliable correlations between the percentiles of the product (d_x) and the initial size of the feed (s_i) rather than the energy consumed in the process of grinding (E).

The deciles of the product of the jaw crusher affected by the reduction ratio was carried out by Ibrahim and Ibrahim (1984). From this study, it was deduced that the initial size feeding to the jaw crusher in the crushing process plays a negligible role on the deciles of the product.

In the present work, it is deduced that the initial size of the feed to the stamp mill plays a remarkable role on the parameters of the size distribution. It seems that there is a contradiction between these two obtained results, this phenomenon may be due to the fact that the crushing process consumes very little time compared with the grinding process.

Nomenclature

s_i	– initial size of the feed, cm;
d_x	– percentiles of the product d_{20}, d_{30}, ... and d_{90} cm;
E	– energy consumed in the grinding process, KwH/t;
a & b	– arbitrary constants, determined from the analysis of the experimental data;
A, B, A_1, B_1	– constants for the rock, these depend on the initial size of the feed (s_i); *i.e.* $A(s_i)$, $B(s_i)$, $A_1(s_i)$, $B_1(s_i)$;
$d_{x(cal)}$	– calculated values of the percentiles;
$d_{x(act)}$	– actual obtained values of the percentiles.

Analysis of the Obtained Data by Making Use of the Methods of Dlin (1958), Speigel (1972) and Rigov (1973)

The percentiles of the product (d_{20}, d_{30}, ... & d_{90}) for the data obtained from the seive analysis of the product was evaluated for each experiment at a definite initial size (s_i). The results of evaluation are shown in Table 1.

Table 1. Calculated percentiles of the product (d_x)

E, kwH/t	s_i cm	d_{20} cm	d_{30} cm	d_{40} cm	d_{50} cm	d_{60} cm	d_{70} cm	d_{80} cm	d_{90} cm
0.642	0.565	0.140	0.275	0.360	0.417	0.460	0.503	0.544	0.586
0.946	0.565	0.112	0.214	0.327	0.384	0.428	0.469	0.514	0.572
1.250	0.565	0.075	0.183	0.325	0.422	0.458	0.482	0.515	0.572
1.554	0.565	0.046	0.110	0.200	0.310	0.370	0.424	0.475	0.544
1.866	0.565	0.040	0.090	0.155	0.274	0.355	0.410	0.562	0.525
2.179	0.565	0.030	0.070	0.150	0.250	0.345	0.412	0.462	0.525
2.669	0.565	0.022	0.055	0.125	0.207	0.325	0.390	0.450	0.515
3.019	0.565	0.018	0.049	0.105	0.209	0.325	0.390	0.450	0.515
0.660	0.450	0.140	0.267	0.400	0.416	0.432	0.450	0.467	0.483
0.974	0.450	0.092	0.185	0.300	0.337	0.362	0.387	0.418	0.457
1.290	0.450	0.055	0.122	0.210	0.320	0.340	0.367	0.394	0.446
1.602	0.450	0.042	0.099	0.180	0.270	0.328	0.360	0.395	0.447
1.915	0.450	0.030	0.075	0.153	0.235	0.320	0.354	0.392	0.445
2.237	0.450	0.035	0.080	0.165	0.270	0.396	0.433	0.468	0.510
2.519	0.450	0.023	0.048	0.100	0.182	0.290	0.400	0.440	0.480
3.165	0.450	0.017	0.040	0.087	0.108	0.250	0.327	0.365	0.410
0.681	0.358	0.110	0.180	0.248	0.279	0.308	0.320	0.355	0.377
1.001	0.358	0.062	0.142	0.200	0.260	0.330	0.342	0.354	0.377
1.322	0.358	0.040	0.100	0.157	0.225	0.277	0.315	0.342	0.370
1.642	0.358	0.032	0.080	0.145	0.212	0.272	0.313	0.342	0.370
1.947	0.358	0.025	0.060	0.128	0.200	0.258	0.295	0.330	0.363
2.283	0.358	0.022	0.055	0.118	0.178	0.250	0.293	0.333	0.366
2.604	0.358	0.018	0.044	0.092	0.150	0.226	0.275	0.315	0.357
3.308	0.358	0.018	0.034	0.068	0.120	0.175	0.250	0.300	0.350
0.695	0.283	0.046	0.140	0.182	0.212	0.232	0.252	0.273	0.294
1.030	0.283	0.050	0.120	0.160	0.200	0.222	0.243	0.267	0.292
1.689	0.283	0.029	0.070	0.120	0.170	0.210	0.236	0.262	0.289
2.019	0.283	0.017	0.045	0.140	0.180	0.220	0.243	0.258	0.287
2.367	0.283	0.020	0.050	0.100	0.150	0.193	0.223	0.252	0.284
2.700	0.283	0.014	0.032	0.066	0.127	0.170	0.214	0.245	0.280
3.364	0.283	0.011	0.027	0.057	0.105	0.158	0.203	0.233	0.270
0.709	0.225	0.054	0.110	0.142	0.165	0.182	0.200	0.216	0.232
1.042	0.225	0.033	0.080	0.127	0.150	0.170	0.190	0.210	0.230
1.381	0.225	0.030	0.070	0.120	0.147	0.169	0.187	0.207	0.228
1.709	0.225	0.022	0.040	0.108	0.138	0.160	0.180	0.200	0.226
2.067	0.225	0.018	0.027	0.075	0.125	0.150	0.172	0.195	0.222
2.731	0.225	0.014	0.033	0.065	0.112	0.145	0.168	0.190	0.220

Table 1. Calculated percentiles of the product (d_x) *(continued)*.

1 E, kwH/t	2 s_i cm	3 d_{20} cm	4 d_{30} cm	5 d_{40} cm	6 d_{50} cm	7 d_{60} cm	8 d_{70} cm	9 d_{80} cm	10 d_{90} cm
0.720	0.180	0.059	0.100	0.130	0.140	0.151	0.159	0.173	0.189
1.055	0.180	0.040	0.080	0.125	0.135	0.147	0.156	0.170	0.185
1.397	0.180	0.027	0.062	0.097	0.128	0.139	0.150	0.160	0.182
1.736	0.180	0.020	0.044	0.082	0.117	0.134	0.145	0.155	0.175
2.083	0.180	0.020	0.040	0.075	0.112	0.132	0.142	0.154	0.170
2.784	0.180	0.010	0.025	0.045	0.072	0.107	0.132	0.145	0.159

Making use of the data shown in Table 1 for s_i = 0.565 cm, and using the recommended method of analysis by Demedovich *et al.* (1967), the selection of the best formula is carried out by a procedure shown in Table 2.

Table 2. Selection for the best relationship between the percentiles of the product (d_x) and the energy consumed in the process of grinding.

	$\bar{x}_s$		$\bar{y}_s$		$\hat{y}_s^x$	$\hat{y}_s - \bar{y}_s$	Equation
	formula	cal	formula	cal			
I	$(x_1 + x_n)/2$	1.831	$(y_1 + y_2)/2$	0.079	0.040	0.039	$y = ax + b$
II	$\sqrt{x_1 x_n}$	1.392	$\sqrt{y_1 y_n}$	0.050	0.058	0.008	$y = axb$
III	$(x_1 + x_n)/2$	1.831	$\sqrt{y_1 y_n}$	0.050	0.040	0.010	$y = ab^x$
IV	$2x_1x_n/(x_1 + x_n)$	1.059	$(y_1 + y_n)/2$	0.079	0.073	0.006	$y = a + (b/x)^x$
V	$(x_1 + x_n)/2$	1.831	$2y_1y_n/(y_1 + y_n)$	0.032	0.040	0.008	$y = 1/(ax + b)$
VI	$2x_1x_n/(x_1 + x_n)$	1.059	$2y_1y_n/y_1 + y_n$	0.032	0.073	0.041	$y = x/(ax + b)$
VII	$\sqrt{x_1 x_n}$	1.392	$(y_1 + y_n)/2$	0.079	0.054	0.025	$y = a \log x + b$

$\hat{y}_s^x$ – obtained by the interpolation of the given experimental data shown in Table 1.

As shown in Table 2, the best relationship which gives minimum deviations for $(\hat{y}_s - \bar{y}_s)$ is expressed generally by the following relationship:

$$y = a + (b/x) \tag{2}$$

where:

y – represents percentiles of the product d_x (d_{20}, d_{30}, ... and d_{90});
x – represents the energy consumed in the grinding process;
a, b – arbitrary constants determined from the analysis of the experimental data.

For d_{20} at an initial size s_i = 0.565 cm, the following relationship is recommended

$$d_{20} = -0.017 + (0.104/E) \quad (3)$$

$$a = -0.017, \quad b = 0.104$$

The average deviation obtained from equation (3) = 6.04%. The obtained values of the constants a & b for various initial sizes are shown in Table 3.

Calculated values of d_x (d_{20}, d_{30}, ... and d_{90}) versus energy consumed in the grinding process are represented in Fig. 1-6 for various initial sizes (0.565 − 0.180), respectively.

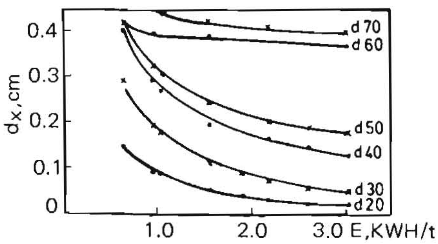

Fig. 1. The relation between (d_x) and energy (E) for initial size S_i = 0.565 cm.

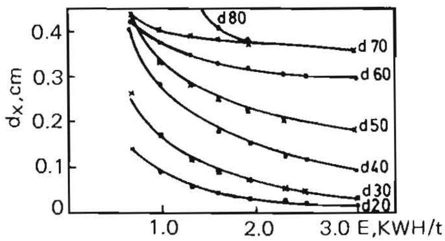

Fig. 2. The relation between (d_x) and energy (E) for an initial size of S_i = 0.45 cm.

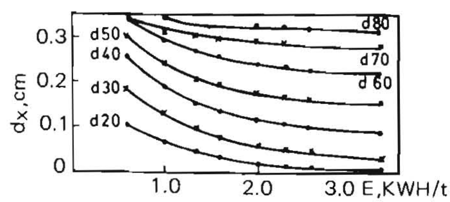

Fig. 3. The relation between (d_x) and energy (E) for an initial size of S_i = 0.358 cm.

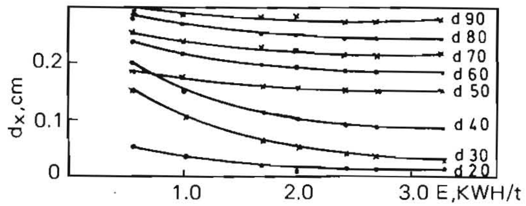

Fig. 4. The relation between (d_x) and energy (E) for an initial size of S_i = 0.283 cm.

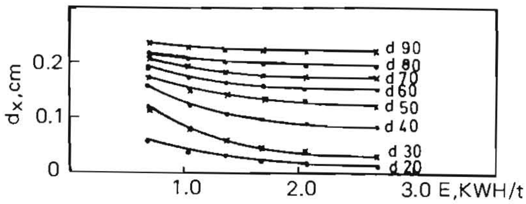

Fig. 5. The relation between (d_x) and energy (E) for an initial size of S_i = 0.225 cm.

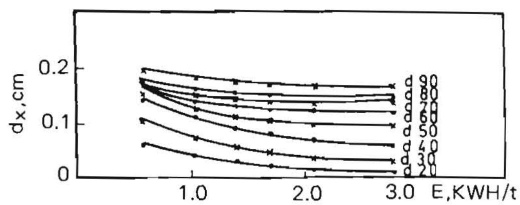

Fig. 6. The relation between (d_x) and energy (E) for an initial size of S_i = 0.180 cm.

From these figures, it is clear that there are some relationships between the obtained percentiles d_x and the initial size of the feed s_i. To get a general relationship between d_x and either s_i or E, an effort was made to get some definite relationships between either the constant a or b with the initial size of the feed s_i. Numerical analytical method similar to that illustrated in Table 2 showed that the best relationships between either a or b shown in Table 3 and the initial size of the feed s_i can be expressed as follows:

$$a = A + B(s_i) \tag{4}$$

$$b = A_1(s_i)^{B_1} \tag{5}$$

The obtained constants A, B, A_1 & B_1 for the above relationships (4 & 5) are shown in Table 4.

Table 3. Obtained constants (a & b) for various initial sizes (s_i).

d_x	$s_i = 0.565$ cm		$s_i = 0.45$ cm		$s_i = 0.358$ cm		$s_i = 0.283$ cm		$s_i = 0.225$ cm		$s_i = 0.180$ cm	
	b	**a**	**b**	**a**	**b**	**a**	**b**	**a**	**b**	**a**	**b**	**a**
20	0.104	−0.017	0.104	−0.019	0.080	−0.014	0.034	0.004	0.038	0.000	0.047	−0.005
30	0.196	−0.014	−0.191	−0.027	0.130	−0.002	0.105	0.000	0.082	−0.002	0.073	0.004
40	0.229	0.048	0.258	0.018	0.146	0.045	0.100	0.052	0.071	0.053	0.078	0.034
50	0.192	0.117	0.214	0.116	0.126	0.117	0.034	0.141	0.046	0.105	0.052	0.079
60	0.040	0.353	0.110	0.262	0.100	0.193	0.053	0.166	0.034	0.137	0.035	0.109
70	0.097	0.363	0.059	0.343	0.055	0.263	0.034	0.208	0.030	0.160	0.024	0.130
80	0.066	0.448	0.149	0.312	0.040	0.306	0.028	0.237	0.024	0.185	0.026	0.140
90	0.065	0.496	0.019	0.446	0.014	0.353	0.015	0.275	0.011	0.218	0.026	0.157

Table 4. Obtained constants (A, B, A_1 & B_1) for the parameters (a & b)

d_x	A	B	A_1	B_1
20	0.009	−0.052	0.188	0.982
30	0.016	−0.066	0.361	0.960
40	0.048	−0.018	0.515	1.204
50	0.086	0.077	0.508	1.547
60	−0.009	0.617	0.143	0.705
70	0.022	0.648	0.172	1.183
80	0.017	0.741	0.073	0.632
90	0.017	0.894	0.030	0.808

The relationships between either the parameter b or a and the initial size s_i are shown in Fig. 7 and 8.

Replacing the values of a & b in equation (2) with that represented by equations (4 & 5) are represented in Table 3; the following useful relationships are deduced:

$$d_{20} = 0.009 - 0.052\ s_i + 0.188(s_i)^{0.982}/E \quad (6)$$

$$d_{30} = 0.016 - 0.066\ s_i + 0.361(s_i)^{0.960}/E \quad (7)$$

$$d_{40} = 0.043 + 0.010\ s_i + 0.515(s_i)^{1.204}/E \quad (8)$$

$$d_{50} = 0.086 + 0.077\ s_i + 0.508(s_i)^{1.547}/E \quad (9)$$

$$d_{60} = 0.009 + 0.617\ s_i + 0.143(s_i)^{0.705}/E \quad (10)$$

$$d_{70} = 0.022 + 0.648\ s_i + 0.172(s_i)^{1.183}/E \quad (11)$$

$$d_{80} = 0.017 + 0.741\ s_i + 0.073(s_i)^{0.632}/E \quad (12)$$

$$d_{90} = 0.017 + 0.894\ s_i + 0.030(s_i)^{0.808}/E \quad (13)$$

By making use of these relationships (6-13), one can deduce that d_{80} and d_{90} do not depend largely on the energy consumed (E). To check the validity of these obtained relationships, random values of E and s_i are selected. Equation 6 is used for calculating $d_{20.cal}$, which are compared to the actual values shown in Table 1, average deviation (dev %) are shown in the last column. This study is carried for d_{20}, d_{30} and d_{70} as an example. The results of comparison are shown in Table 5.

From this investigation, it is clear that the obtained relationships give the expected percentiles within a reasonable error, which rarely exceeds 15%.

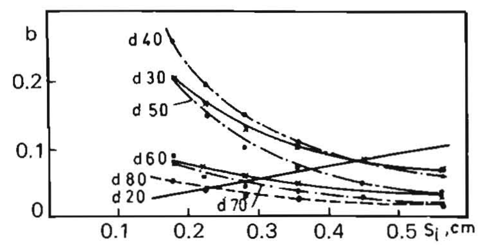

Fig. 7. The relation between the parameter (b) and initial size of (S_i) for various values of (d_x).

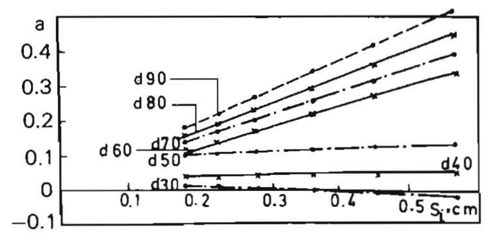

Fig. 8. The relation between the parameter (a) and initial size of (S_i) for various values of (d_x).

Conclusions and Recommendations

1. The relationships between the percentiles of the product d_x (d_{20}, d_{30}, ... and d_{90}) and the initial size of the feed s_i rather than the energy consumed in the process of grinding E are deduced and expressed mathematically by the following general relationship

$$y = a + (b/x)$$

obtained values of a and b for various initial sizes are given in Table 3.

2. As shown in the obtained relationships given in Table 3, the effect of the initial size of the feed s_i can not be neglected in the grinding process as this process takes larger times than the crushing process which it was deduced that it is slightly affected by the initial size of the feed.

3. The values of d_{70}, d_{80}, and d_{90} are slightly affected by the energy consumed in the process of grinding.

4. Deduced relationships can be used for controlling either the initial size of the feed or the energy consumed in the process of grinding E to get a definite size of the percentiles d_x.

Table 5. Check for the validity of the obtained relationships (6, 7 & 11) for d_{20}, d_{30}, and d_{70}.

E, kwH/t	s_i, cm	$d_{20\ cal.}$	$d_{20\ act.}$	dev. %
0.642	0.565	0.146	0.140	4.84
2.179	0.565	0.029	0.030	3.76
0.974	0.450	0.073	0.092	20.65
1.915	0.450	0.031	0.030	3.33
1.001	0.358	0.059	0.062	5.04
1.689	0.283	0.026	0.029	9.46
2.067	0.225	0.018	0.018	0.00
		$d_{30\ cal.}$	$d_{30\ act.}$	
3.019	0.565	0.048	0.049	2.39
0.974	0.450	0.158	0.185	17.08
1.947	0.358	0.062	0.060	2.55
3.308	0.358	0.033	0.034	2.70
2.367	0.283	0.043	0.050	14.56
0.709	0.225	0.123	0.110	11.60
1.397	0.180	0.054	0.062	14.04
		$d_{70\ cal.}$	$d_{70\ act.}$	
2.179	0.565	0.429	0.412	2.96
1.290	0.450	0.365	0.367	0.42
1.001	0.358	0.305	0.342	10.83
2.700	0.283	0.220	0.214	2.66
1.709	0.225	0.204	0.180	13.59
1.736	0.180	0.152	0.145	4.60

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(Received 17/01/1984;
in revised form 08/05/1984)

تأثير الطاقة المستهلكة فى عملية الطحن والحجم الابتدائى للحبيبات المطحونة على مئينيات نواتج الطحن فى عمليات الطحن

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أسيوط - مصر

أجريت **٤٣** تجربة على عينات من الحجر الجيرى ذات أحجام ابتدائية (S_i) ثابتة (**٠,٥٦٥ ، ٠,٤٥ ، ٠,٣٥٨ ، ٠,٢٨٣ ، ٠,١٨٠** سم على التوالى).

وقد تم طحن العينات باستخدام طاحونة رأسية حيث تغيرت طاقة الطحن من ٧,٠-٥,٣ كيلووات ساعة/طن.

بتحليل نواتج الطحن وحساب مئينيات الناتج فى كل تجربة، ومع استخدام التحليل الإحصائى تم استنتاج معادلات عامة تربط بين مئينيات نواتج الطحن وكل من الحجم الابتدائى أى بالإضافة إلى طاقة الطحن.