Arab Gulf Journal of Scientific Research, 28 (3): 163- 169 (2010)

Research Paper Ref. No 2501

New Wavelet –Based Algorithm for Speckle Reduction in Ultrasound Images

خوارزم جديد لتنقية صور الأشعة فوق السمعية من التشويش باستخدام المويجات

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Abstract: This paper proposes an adaptive threshold estimation method for de-noising in wavelet domains merged with translation invariant de-noising. The sub-band shrink is computationally more efficient and adaptive because the parameters required for estimating the threshold depend on subband data. A new probability density function is proposed to model the statistics of wavelet coefficients. The subband threshold is derived using Bayesian estimation theory and the new pdf. Different shifts are used and applied to the noisy image in order to attain different estimates to the unknown image and then linearly average the estimates. In speckle images, the noise content is multiplicative. The proposed method is applied for speckle ultrasound images by using logarithmic transformation. Experimental results on several test images are compared with various de-noising techniques.

Keywords: *wavelettransform, image De-spechling, translation invariatiant, Bayesian, estimation theory, estimation, ultrasound images*.

المستخلص: هذا البحث يقترح طريقة تقدير التقلص التكيفى للترشيح فى مجال المويجات مدمجة بطريقة الترشيح باستخدام الازاحة الثابتة . حيث ان التقلص الرباط الثانوى يكون حسابيا كفوء و اكثر تكيفا لان البارامترات الازمة لتقدير التقلص تعتمد على بيانات الروابط الثانوية، و لذلك نقترح دالة كثافة احتمالية جديدة تستخدم لنمذجة احصائيات معاملات المويجات. تستخدم دالة الكثافة الاحتمالية المقترحة مع نظرية التقدير البيزي لاستنتاج طريقة لحساب التقلص الرباط الثانوى. ازاحات مختلفة تتم على الصورة المشوشة لكى تنجز تقديرات للصورة النقية ثم نحسب متوسط هذة التقديرات خطيا للحصول على نتيجة تقريبية للصورة النقية. فى الصورة الرقطة يكون التشويش مضروبا فى الصورة. لذلك نطبق الطريقة المقترحة على صور الاشعة الفوق سمعية الرقطة باستعمال التحويل الوغارتيمى. نتائج تجريبية تمت على عدة صور اختبارية و تم مقارنتها بعدة طرق ترشيح اخرى.

كلمات مدخلية : *تحويلات المويجات، تنقية الصور الأرقطة، الازاحة الثابتة، التقدير البيزى، صور الاشعة الفوق سمعية.*

INTRODUCTION

Coherent imaging systems such as ultrasound suffer from speckle noise, creating images that appear inferior to those generated by other medical imaging modalitie. Speckle is a random interference pattern caused by coherent radiation in a medium containing many sub-resolution scatters. Speckle has a negative impact on ultrasound images as the texture does not reflect the local echogenicity of the underlying scatters. The local brightness of speckle pattern, however, does reflect the local echogenicity of the underlying scatters. Clinically, speckle noise has been shown to reduce the ability to detect lesions by a factor of eight (Anderson*, et al*. 2000).

Speckle filtering of medical ultrasound images represents a critical pre-processing step, providing clinicians with enhanced diagnostic ability. Efficient speckle noise removal algorithms may also find applications in real time surgical guidance assemblies. However, it is vital that regions of interest are not compromised during speckle removal.

Several methods have been proposed for de-noising the signals. Wavelets transform has proven a successful tool for analysis of signals due to its good localization properties in time and frequency domains (Daubechies, 1992; Graps, 1995). In recent years, there has been a fair amount of research on wavelet thresholding and threshold selection for signal de-noising (Westrink, *et al*. 1987; Donoho, 1993; Donoho and Johnstone, 1995; Grace, *et al*. 2000; Mastriani and Giraldez, 2006;) because wavelet provides an appropriate basis for separating the noisy signal from the image signal. The motivation is that as the wavelet transform is good at energy compaction, the small coefficients are more likely due to noise with large coefficient due to important signal features. These small coefficients can be thresholded without affecting the significant features of the image.

Thresholding is a simple non-linear technique, which operates on one wavelet coefficient at a time. In its most basic form, each coefficient is thresholded by comparing against threshold. If the coefficient is smaller than the threshold, is set to zero; otherwise the threshold is kept or modified. Replacing the small noisy coefficients by zero and inverse wavelet transforms on the result may lead to reconstruction of the essential signal characteristics with less noise.

This paper proposes a near optimal threshold estimation technique for image de-noising which is subband dependent, i.e. the parameters for computing the threshold are estimated from observed data, one set for each sub-band. A new probability density function (pdf) is proposed to model the statistics of wavelet coefficients, and the new threshold derived using Bayesian theory. The proposed method is used to speckle filtering of Ultrasound images. A logarithm is taken of the speckle image, then the speckle multiplicative corruption of the original image becomes additive.

This papermay be outlined as follows: The de-noising process in the transform domains is discussed in Section 2. In Section 3, a new sub-band adaptive shrinkage function is developed for natural images. Section 4 introduces the concept of translation – invariant de-noising**.** The proposed algorithm for the de-noising technique is presented in Section 5. The results of the proposed algorithm are presented in Section 6. The paper closes with conclusions in Section 7.

Wavelet Thresholding

Let

$$f = \{_{ij}, i, j, = 1,2,........., M\} \quad (1)$$

Denote the $M \times M$ matrix of the original image to be recovered and is some integer power of 2. During transmission to signal f is corrupted by independent and identically distributed (i.i.d) zero mean, white Gaussian noise $\mathbf{n}ij$ with standard deviation σ i.e. $n_{ij} \sim N(0,\sigma^2)$.At the receiver end, the noisy observation

$$g_{ij} = f_{ij} + \sigma\ n_{ij} \quad (2)$$

is obtained. The goal is to estimate an $\hat{f}$ which minimizes the mean squared error (*MSE*),

$$MSE = \frac{1}{M^2}\sum_{i,j=1}^{M}(\hat{f}_{ij} - f_{ij})^2 \quad (3)$$

Let W and W^{-1} denote the two dimensional orthogonal discrete wavelet transform (DWT) matrix and its inverse respectively. Then $Y=Wg$ represents the matrix of wavelet coefficients of having four sub bands *(LL, LH, HL, HH)*. The sub-bands HH_k, HL_k, LH_k are called details, where is the scale varying from 1, 2,……, J and J is the total number of decomposition. The size of the subband at scale k is $M/_{2^k} \times M/_{2^k}$ The subband LL_J is the low-resolution residue.

The different methods for denoising differ only in the selection of the threshold. The basic procedure remains the same.

- Calculate the discrete wavelet transform of the image.
- Threshold the wavelet coefficients. (Threshold may be universal or subband adaptive)
- Compute the inverse wavelet transforms to get the denoise estimation $\hat{f}$.

Soft thresholding has been used over hard thresholding for the following reasons: Soft thresholding has been shown to achieve near mini-max rate over a large number of Besov spaces (Donoho, 1993). Moreover, it is also found to yield visually more pleasing images. Hard thresholding is found to introduce artifacts in the recovered images. The soft thresholding with threshold λ is defined as follows:

$$D(U, \lambda) = \text{sgn}(U) \bullet \max(0, |U| - \lambda) \quad (4)$$

A disadvantage of the *DWT* is that, in contrast to the *CWT*, this decimated representation is not invariant under translation. The lack of shift invariance makes it unsuitable for pattern recognition and also limits its performance in denoising. The latter is perhaps more clear from the viewpoint of the lack of redundancy. The redundancy of a representation, in general, helps to better estimate a signal from its noisy observation. In this respect, Translation – invariant was proposed as: one averages the de-noising results of several cyclically shifted image versions.

Estimation of the Threshold Using Bayesian Denoising Model

In, the wavelet domain, when an orthogonal wavelet transform is used, the problem can be formulated as

$$Y = X + V \quad (5)$$

where $Y=Wg$ denotes the matrix of wavelet coefficients of g . Similarly $X=Wf$ and $V=Wf$.

This section aims to estimate the desired signal $\hat{f}$ from the noisy observation. The maximum a-posteriori (MAP) estimator is used for this purpose. The classical MAP estimator for (5)

$$\hat{X}(Y) = \arg\max_X P_{X|Y}(X|Y) \quad (6)$$

Using Bayes rule, one gets

$$\begin{aligned} \hat{X}(Y) &= \arg\max_X \{P_{Y|X}(Y|X) \bullet P_X(X)\} \\ &= \arg\max_X \{P_V(Y - X) \bullet P_X(X)\} \end{aligned} \quad (7)$$

Therefore, these equations allow the writing of this estimation in terms of pdf of the noise (P_v) and pdf of the signal coefficient (P_x). From the assumption on the noise P_v, is zero mean Gaussian with variance (σ_n) ,i.e.,

$$P_V(V) = \frac{1}{\sigma_n \sqrt{2\pi}} e^{-\frac{V^2}{2\sigma_n^2}} \quad (8)$$

It has been observed that wavelet coefficients of natural images have highly non-Gaussian statistics (Sendur and Selesnick, 2002a; Sendur and Selesnick, 2002b). The pdf for wavelet coefficients is often modeled as a generalized (heavy-tailed) Gaussian (Sendur and Selesnick, 2002c)

$$P_X(X) = K(s, p) e^{-\left|\frac{X}{s}\right|^p} \quad (9)$$

where s,p are the parameters for this model, and $k(s,p)$ is the parameter-dependent normalization constant. In practice. Two problems generally arise with the Baysian approach when an accurate but complicated $P_x(X)$ is used:

1- It can be difficult to estimate the parameters of P_x for a specific image, especially from noisy data,
2- the estimators for these models may not have A simple closed form solution and can be difficult to obtain.

The solution to these problems usually requires numerical techniques. Equation (7) is also equivalent to

$$\hat{X}(Y) = \arg\max_X [\log(P_V(Y - X) + \log(P_X(X)] \quad (10)$$

As in (Prasad, *et al.* 2008; Sendur and Selesnick, 2002b), let us define

$$f(X) = \log(P_X(X)) \tag{11}$$

By using (8), (11), (10) becomes

$$\hat{X}(y) = \arg\max_X[-\frac{(Y-X)^2}{2\sigma_n^2} + f(X)] \tag{12}$$

This is equivalent to solving the following for $\hat{W}$ if $P_x(X)$ is assumed to be strictly convex and differentiable

$$\frac{Y-\hat{X}}{\sigma_n^2} + f'(\hat{X}) = 0 \tag{13}$$

$$P_X(X) = \frac{1}{\beta\sigma} e^{-\frac{\beta|X|}{\sigma}} \tag{14}$$

Where the scale parameter β is computed once for each scale using the following equation as in [4],

$$\beta = \sqrt{\log(\frac{L_k}{J})} \tag{15}$$

Where L_k is the length of the subband at k th scale, then

$$f(X) = -\log(\beta\sigma) - \frac{\beta|X|}{\sigma} \tag{16}$$

and the estimator will be

$$\hat{X}(Y) = sign(Y) \bullet [|Y| - \frac{\beta\sigma_n^2}{\sigma}]_+ \tag{17}$$

Here, $(h)_+$ is defined as

$$(h)_+ = \begin{cases} 0 \,;\, \text{if } h < 0 \\ g \,;\, \text{other wise} \end{cases} \tag{18}$$

Equation (18) is the soft shrinkage function.

Translation–Invariant D-enoising

Thresholding in the orthogonal wavelet domain has been observed to produce significantly noticeable artifacts such as Gibbs-like ringing around edges and specks in smooth regions. To ameliorate this unpleasant phenomenon, Coifman and Donoho(1995) proposed the translation invariant (TI) de-noising. The discussion in (Coifman and Donoho, 1995) is one–dimensional (1-D), but Ismail and Nabil proposed TI in 2-D (Ismail and Nabil, 2004). Let $Shift_{k,l}[g]$ denote the operation of circularly shifting the input image g by k indices in the vertical direction and l indices in the horizontal, and let $Unshift_{k,l}[g]$ be a similar operation but in the opposite direction. Also, let *Denoise* $[g, T]$ denote the operation of taking the *DWT* of the input image g, threshold it with a threshold T according to equation (17), then transform it back to the space domain. TI denoising then yields an output which is the average of the threshold copies over all possible shifts:

$$\hat{f} = \frac{1}{M^2} \sum_{k,l}^{M-1} Unshift_{k,l}[Denoise[Shift_{k,l}[g],T]] \tag{19}$$

The rationale is that since the orthogonal wavelet transform is a time-varying transform and thresholding the coefficients produces ringing-like phenomena, thresholding a shifted input would produce ringing at different locations, and averaging overall different shifts would yield an output with more attenuated artifacts than a signal copy alone.

Proposed Algorithm

First, the global description of the method for computing the subband threshold and removing speckle is introduced.

Estimation of Threshold Parameters

This section describes the method for computing the various parameters used to calculate the threshold value (T), adaptive to different sub-band characteristics:

$$T = \frac{\beta\sigma_n^2}{\hat{\sigma}_y} \tag{20}$$

Where the scale parameter β is computed one for each scale using the following equation:

$$\beta = \sqrt{\log(\frac{L_k}{J})} \tag{21}$$

where L_k is the length of the subband k th scale.

$\hat{\sigma}^2_n$ is the noise variance, which is estimated for sub-band H_1, using formula Sendur and Selesnick, 2002c)

$$\hat{\sigma}_n^2 = [\frac{median(|Y_{ij}|)}{0.6745}]^2, Y_{ij} \in \text{subband } HH_1 \tag{22}$$

and σ_y is the standard deviation of the sub-band consideration.

Image Denoising Algorithm

This section describes the proposed image denoising algorithm in the wavelet domain for recovering the original from the noisy one. The algorithm is very simple to implement and computationally more efficient. It has the following steps:

1- for $k = 1,\ldots\ldots, M, l = 1,\ldots\ldots,M$ do *unshift* $_{k,l}$ [Denoise[*Shift* $_{k,l}$[g],T]]

Where the steps of *Denoise[I, T]* are:

- perform multi-scale decomposition of the image corrupted by Gaussian noise using wavelet transform (the total number of subbands of different scales is J).
- estimate $\hat{\sigma}^2_n$ using equation (22)
- for each level, compute the scale parameter β using equation (21)
- for each subband (except the low-pass residual):
  - a- compute the standard deviation σy
  - b- compute threshold T using equation (20)
  - c- apply soft thresholding to the noisy coefficients

2- compute the mean average to reconstruct the denoised image using $\hat{f}$ equation (19).

Speckle Model

The de-noising framework described is based on the assumption that the distribution of the noise is additive zero mean Gaussian. In speckle image, the noise content is multiplicative and non-Gaussian. Such noise is generally more difficult to remove than additive noise because the intensity of the noise varies with the image intensity. A model of multiplicative noise is given by

$$g(i,j) = f(i,j)n(i,j) \tag{23}$$

Where the speckle $g(i,j)$ is the product of the original image $f\ (i,j)$ and the non-Gaussian noise $n(i,j)$.

In most applications involving multiplicative noise, the noise content ia assumed to be stationary with unitary mean and unknown noise variance σ^2. To obtain an additive noise model, a logarithmic transformation on the special image $g(i,j)$ must be applied. The noise component in $n(i,j)$ can be approximated as an additive zero mean Gaussian process as shown in the following equation.

$$\ln g(i,j) = \ln f(i,j) + \ln n(i,j) \tag{24}$$

The *DWT* is then applied to ln($g(i,j)$). After the inverse *DWT*, the processed image is subject to an exponential transformation to reverse the logarithmic operation.

EXPERIMENTAL RESULTS

The results of the proposed algorithm and compared to the results using popular threshold based de-noising methods are presented here. The experiments were performed on simulated 256×256 Ultrasound images. As a quantitative performance measure we the signal to noise ratio used was

$$SNR = 10\log_{10}\left({P_{signal}}/{P_{noise}} \right) \tag{25}$$

where the power is estimated by calculating the variance. We have also compared the methods by calculating peak-signal-to-noise ratio

$$PSNR = 10\log_{10}((255)^2 / MSE) \tag{26}$$

The Daubechies D_1, D_2, D_3, D_4 filter pairs are used in wavelet transform. Here, the results are only reported for the Haar wavelet D_1.

For performance computation of the proposed method and comparison with other methods, we have taken a noise-free Ultrasound image was taken and added to multiplicative speckle noise. The result was compared with those of other similar methods such as Multilevel soft-threshold based method (Khare and Tiwary, 2005), Universal threshold based denoising and Visushrink. The computation of results from the other methods was done to compare results on the same image on a similar scale. In all multilevel experiments wavelet decomposition was performed up to 4 levels.

(Figure 1) Simulation results using the proposed algorithm and several other methods

Figure (1) shows the expanded view of the same image of a de-noised image by multilevel thresholding, universal thresholding and the proposed algorithms.

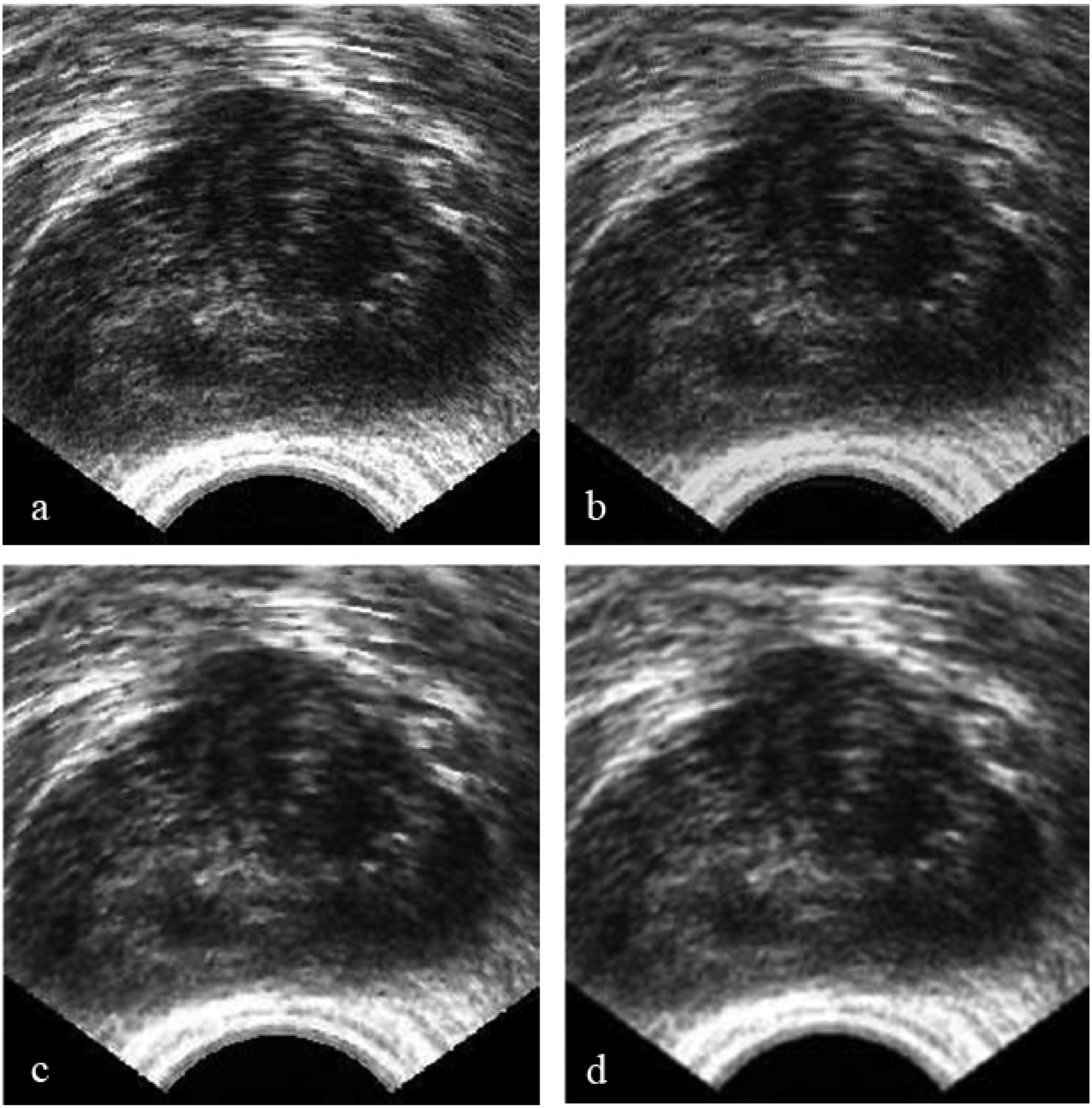

Fig. 1. (A-D) (a) Noisy image (SNR=47.8432, PSNR=32.1759).
(b) Denoised image using proposed method (SNR=59.4351, PSNR=42.7650).
(c) Multilevel threshold denoised image (SNR=51.0672, PSNR=34.6520).
(d)Universal threshold de-noised image (SNR=54.4763, PSNR=38.935).

The result in (Figure 1) shows that, the proposed method preserves detailed features and sharpens information to a greater extent compared to other methods. The superiority of the proposed method compared to previous efforts can be observed in smooth noise removal as well as in good preservation of sharp features. Although the Multilevel thresholding method also preserves sharp features, it has poor noise removal. Also, the Universal threshold is very poor for preserving sharp features compared to the present study. It should be noted that the SNR for the present study was found as 59.4351 which is greater compared to multilevel and universal thresholding, which implies good removal of noisy data.

CONCLUSION

In this study, simple and sub-band adaptive thresholding with transition invariant algorithm was proposed to address the issue of image recovery from its noisy counterpart. It is based on generalized Gaussian distribution modeling of sub-band coefficients. The method was found to be computationally efficient and significantly reduced the speckle while preserving the sharp features in the original images. Also, results obtained show the proposed method is useful in real time Ultrasound imaging enhancement. The comparisons against previous efforts show that present method is superior in extracting noisy data from the original image as well as in preserving the sharp features of the image.

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Ref. No. (2501)
Rec. 2/1/2009
In-revised form: 12/12/2010