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Economic Criteria for the Compensation of Reactive Power of Loads in Transmission and Distribution Networks

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ABSTRACT. The mathematical analysis for determining the optimum and economic compensation of the reactive power of loads in distribution networks is presented for both single and multi-feeder systems. This entails study of the optimum distribution of 11kV-rating capacitors in the network and is repeated for the case when the total installed capacity of capaictors is unknown. The economic criteria for the use of 0.4 kV capacitors is also considered and their need checked by a deduced formula. An algorithm for obtaining the economic distribution of capacitors is determined in order to compensate the reactive powers in transmission systems. It is based on both a hierarchy principle and partition technique with the equivalent circuit at different hierarchy levels. The parameters of the equivalent circuit depend on the economic characeristics of reactive power compensation. For simplicity, these characteristics may be approximated in the form of a second-order polynomial.

With the growth of generated powers in a large network, its economic operation becomes an important subject. This problem is incurred by the types of loads which are normally inductive. Inductive loads can be compensated on either a transmission or distribution level or both through static and synchronous capactors (Boening *et al.* 1982). This compensation is necessary in order to improve the power factors of different loads as well as to allow the node voltages to be modified. The compensation of reactive powers in a distribution system can be made more effective since the inductive effect of transmission lines can be excluded from the compensation process. Also, in this case, the power factors will be improved locally at each node. On the other hand, the static capacitors may be connected to nodes of either 11 or 0.4 kV or even at both sides. Although 0.4 kV capacitors have a long life, 11 kV capacitors are usually cheaper since their transient currents are manageable (Brown 1981). Also, in the presence of 0.4 kV capacitors with improved power factor, the 11 kV transformers can be loaded by more power although total energy loss in them may then be increased. This phenomenon may cause a moderate decrease in the required ratings of transformers as well as in their quantity (Shoults and Sun 1982).

For large reactive powers, the compensation of either loads or system components is sometimes achieved through compensators and synchronous capacitors in a transmission system. The improvement in power factors at the nodes of distribution systems may deviate from that required due to the change in the types of loads at different nodes of a system. From this point of view, economic compensation can also be achieved through some nodes of a transmission system and the economic criteria for such application must be studied. The static capacitors may be connected to some nodes on the 11 kV side in order to distribute economically the reactive powers in a distribution network. This economic distribution depends on the type of loads at these nodes. Such compensation may be applied on either the 0.4 kV or 11 kV side of the load and it is necessary to compare the two cases. Constant transformer rating and cross-sectional area of line capacitors are assumed.

Distribution Systems

For a certain distribution of reactive powers with constant node voltage, the required generated reactive powers by capacitors and power loss in a network should be determined (Hamed *et al.* 1985). For simplicity, the daily load curve must be sectionalized into many periods each with a constant mean value of reactive power. The economic criteria of a certain part of a given load curve for the compensation of reactive power can be derived (Hamed *et al.* 1984) from the total cost

$$F = (a + d)\,K + e\,(E + E_c) \tag{1}$$

where

a efficiency factor,
d depreciation and maintenance coefficient,
K total cost of capacitors,
e price of energy loss,
E_c energy loss in capacitors, and
E energy loss in the system.

In this case, the total energy loss in a network becomes the sum of the losses obtained for all intervals.

The two cases of single and multi-feeder networks are now investigated. The problem of economic distribution of capacitors in a distribution system (Fig. 1)

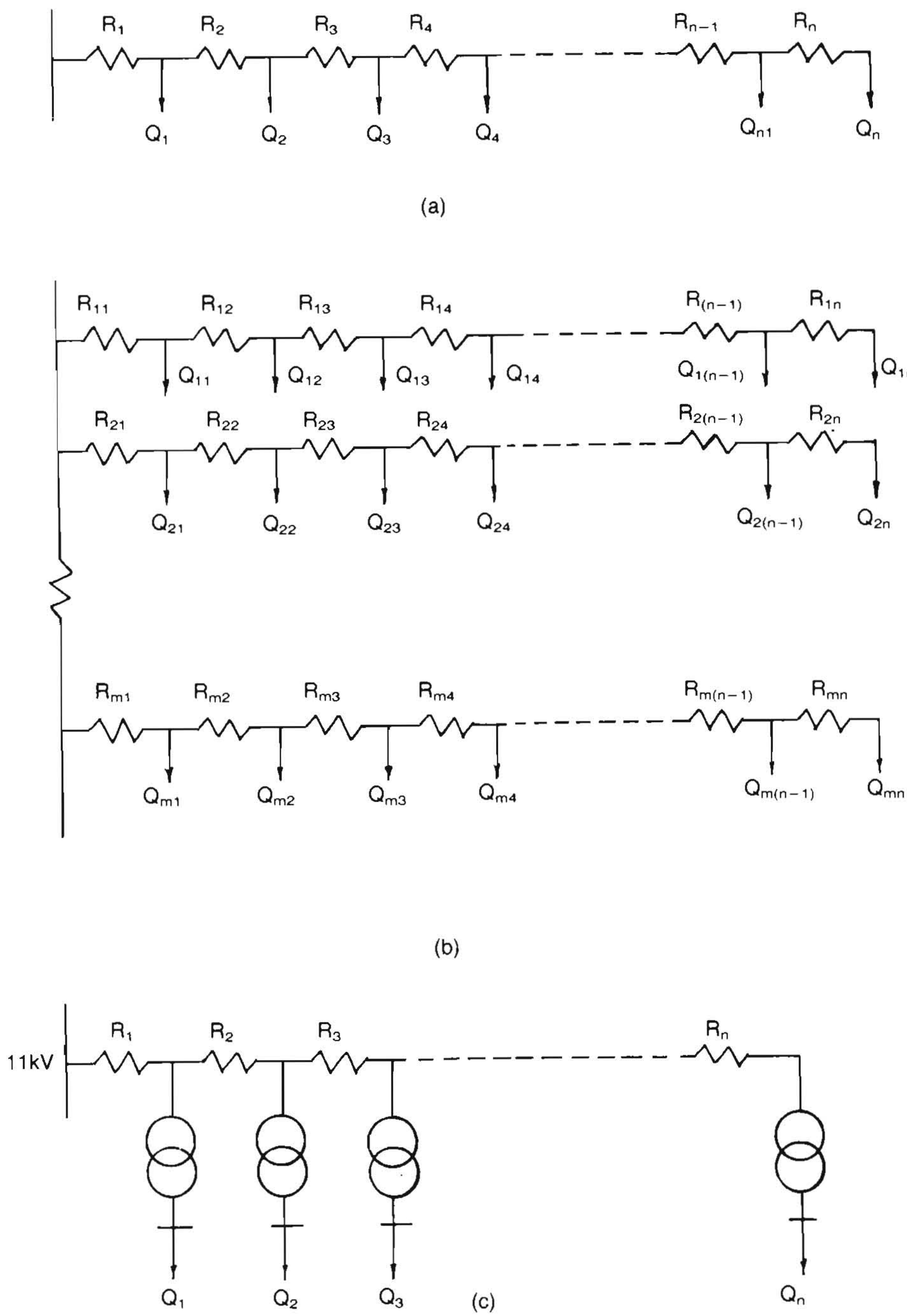

Fig. 1. Different circuits for study of reactive power compensation

appears to be more difficult since it must be solved for each i-th feeder alone and then the solution for the system of r feeders as a whole. The total rating of capacitors Q_{ct} may be estimated by

$$Q_{ct} = \sum_{i=1}^{r} \sum_{j=1}^{n_i} Q_{cij} \tag{2}$$

where

- j number of the transformer at the j-th node,
- n_i number of nodes in the i-th feeder, and
- Q_{cij} capacitor rating at the j-th node of the i-th feeder.

It is necessary to derive the economic criteria for the distribution of capacitors at nodes of a single feeder with n transformers. The total capacity Q_{ct} of capacitors as well as the voltage V_0 at the nodes of a system for a certain period t are given. The cost of either capacitors or power loss in them will be considered as a constant value which does not depend on the final distribution of these capacitors on the different nodes of a feeder (see Fig. 1). The economic distribution of capacitors can be obtained from the energy loss in the elements of a network as a function of capacitor rating Q_{cij} and reactive power Q_{ij} at i-th node of the j-th feeder. Using the method of Lagrange undefined factors, the energy loss in elements of a network M as a function of Lagrangian L can be deduced (Hamed *et al.* 1984, El-Hawary and Wellon 1982). This may be formulated for a certain j-th feeder as

$$\begin{pmatrix} R_{1j} & R_{1j} & R_{1j} & R_{1j} \\ R_{1j} & \sum_{i=1}^{2} R_{ij} & \sum_{i=1}^{2} R_{ij} & \sum_{i=1}^{2} R_{ij} \\ \vdots & \vdots & \vdots & \vdots \\ R_{1j} & \sum_{i=1}^{2} R_{ij} & \sum_{i=1}^{m} R_{ij} & \sum_{i=1}^{m} R_{ij} \\ \vdots & \vdots & \vdots & \vdots \\ R_{1j} & \sum_{i=1}^{2} R_{ij} & \sum_{i=1}^{n} R_{ij} & \sum_{i=1}^{n} R_{ij} \end{pmatrix} \begin{pmatrix} Q_{ij} \\ Q_{c2j} \\ \vdots \\ Q_{cmj} \\ \vdots \\ Q_{cnj} \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{1} \sum_{9=i}^{n} R_{ij} \quad Q_{sj}\text{-}LV_0^2/2t \\ \sum_{i=1}^{2} \sum_{8=i}^{n} R_{ij} \quad Q_{sj}\text{-}LV_0^2/2t \\ \vdots \\ \sum_{i=1}^{m} \sum_{S=i}^{n} R_{ij} \quad Q_{sj}\text{-}LV_0^2/2t \\ \\ \sum_{i=1}^{n} \sum_{S=i}^{n} R_{ij} \quad Q_{sj}\text{-}LV_0^2/2t \end{pmatrix} \tag{3}$$

where R_{ij} is the loss factor (?) at the i-th node of the j-th feeder.

For simplicity, the solution of eq. (3) must be initiated at the last node in the form

$$\begin{bmatrix} R_{1j} & R_{1j} & R_{1j} \cdots R_{1j} & \cdots & R_{1j} \\ 0 & R_{2j} & R_{2j} & R_{2j} & R_{2j} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & R_{mj} & R_{mj} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & R_{nj} \end{bmatrix} \begin{bmatrix} Q_{d1j} \\ Q_{d2j} \\ \vdots \\ Q_{dmj} \\ \vdots \\ Q_{dnj} \end{bmatrix} = \begin{bmatrix} g_j \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{4}$$

where

$$Q_{dij} = Q_{ij} - Q_{cij}$$

$$\text{and } g_j = R_1 \left(\sum_{i=1}^{n} Q_{ij} - Q_{ctj} \right)$$

The solution must be repeated with a reduced number of variables until the condition, at a node m, where the value of computed reactive power of capacitor Q_{cmj} becomes positive. In this case, all negative values of calculated powers should be equated to zero. On the other hand, by solving eq. (3), the minimum function can be increased if, for the j-th feeder,

$$\frac{2t}{V_0^2} \sum_{i=j+1}^{m-1} \sum_{s=i}^{m-1} R_{ij} \; Q_{sj} > 0 \tag{5}$$

In this case, the Lagrangian can be estimated from

$$L_j = \frac{2t}{V_0^2} \left(\sum_{i=1}^{m} \sum_{s=i}^{n} R_i \, Q_{sj} - Q_{ctj} \sum_{i=1}^{m} R_{ij} \right)$$

Then, the objective function can be formulated as

$$M = \frac{t}{V_0^2} \sum_{i=1}^{r} \sum_{j=1}^{n_i}{}' R_{ij} \left[\sum_{s=j}^{n_i}{}' (Q_{is} - Q_{cis}) \right] + \left[\sum_{i=1}^{r} \sum_{j=1}^{n_i}{}' Q_{cij} - Q_{ct} \right] L \tag{6}$$

The coefficient g of eq. (4) can be deduced in the form

$$g = \sum_{i=1}^{r} \sum_{j=1}^{n_i}{}' (Q_{ij} - Q_{ct}) \Big/ \left[\sum_{i=1}^{r} (1/R_{i1}) \right] \tag{7}$$

Economic Criteria for the Use of 0.4 kV Capacitors

If the total installed capacity of capacitors is unknown, the cost can be derived as

$$F = (a+d)\ K_0 \sum_{j=i}^{r} \sum_{i=1}^{n} Q_{cij} + etp \sum_{j=1}^{r} \sum_{i=1}^{n} Q_{cij} + \frac{et}{V_0^2} \sum_{j=1}^{r} \sum_{i=1}^{n} R_{ij} \left[\sum_{s=i}^{n} (Q_{sj} - Q_{csj})\right]^2 \quad (8)$$

where

K_0 price of 11 kV capacitor, and

P specific loss of active power in 11 kV capacitors.

It is known (Hauth *et al.* 1982) that the reactive power of a load should be compensated from the nearest node. Beginning with the last node, eq. (4) for cost will be valid. The reactive power for each j-th feeder can be evaluated from

$$Q_{cij} = Q_{1j} - \frac{u}{A_{ij}},\ Q_{cij} = Q_{ij},\ i=2,3,\ldots,n,\ j=1,2,\ldots,r \quad (9)$$

where

$A_{ij} = 2\ et\ R_{ij} / V_0^2$, and $U = (a+d)\ K_0 + etp$

This computational process should be repeated until a certain node m, where the negative value of Q_{cm} disappears as before and then the reactive powers can be determined in the form

$$Q_{cm} = Q_m - \left[u - \sum_{i=1}^{m-1} \sum_{s=1}^{m-1} A_i\ Q_s\right] / \sum_{i=1}^{m} A_i \geqq 0 \quad (10)$$

and

$$Q_{ci} = Q_i,\ i = m+1,\ m+2,\ \ldots,\ n$$

The above analysis is also accurate for a positive cost rise. The deduced values of reactive powers must be standerized with all small values neglected. Assume that the capacitors are installed on both sides of transformers. The economic distribution for capacitors on the 11 kV side as well as on 0.4 kV nodes (Fig. 2) can be determined according to the total cost of these capacitors:

$$K = K_0 \sum_{i=1}^{n} Q_{ci} + K_0' \sum_{i=1}^{n} q_{ci} \quad (11)$$

where, K_0' and q_{ci} are the costs per KVAR of the installed capacity of 0.4 kV capacitors and the capacitors' reactive power at i-th nodes on the 0.4 kV side, respectively.

The total cost will be then expressed by

$$F = u \sum_{i=1}^{n} Q_{ci} + H \sum_{i=1}^{n} q_{ci} + \frac{et}{V_0^2} \left[\sum_{i=1}^{n} r_i\ (Q_i - q_{ci})^2\right.$$

$$+\sum_{i=1}^{n} R_i \{\sum_{s=i}^{n} (Q_s - Q_{cs} - q_{cs})\}^2] \quad (12)$$

with

$$H = (a + d) K_0' + etp'$$

where, P′ is a specific power loss in 0.4 kV capacitors.
If the capacitors are not installed at all 0.4 kV nodes, the gradient of cost will be a negative. This means that the cost of capacitors will be increased more rapidly than the reduction of energy loss in transformers as the power of the 0.4 kV capacitors is increased. On the other hand, the condition of optimum distribution of capacitors on the 11 kv side can be simplified in the form:

$$H - u - A_{tj} \quad Q_j + \sum_{i=j+1}^{m-1} \quad \sum_{s=i}^{m-1} A_i \quad Q_s + \frac{\sum_{i=j+1}^{m} A_i}{\sum_{i=1}^{m} Q_i}$$

$$[u - \sum_{i=1}^{m-1} \sum_{s=i}^{m-1} A_i \; Q_s] \geqq 0 \quad (13)$$

If the given condition is not checked for $j \geqq m$, or for eq. (13) even for nodes only at $j < m$, then the capacitors must be distributed on both sides. In this complicated case, the algorithm may be programmed. The economic criteria for the distribution of capacitors on the 11 kV side can be determined by the relation of the difference between costs of capacitors on the two sides ($d K_0 = K'_0 - K_0$) with respect to the reactive power. This is shown for various total capacitor ratings in Fig. 2. The calculations are considered for different mean values of reactive powers of loads at nominal rating of transformers. If the condition studies has a value of d K_0 above the specified values of the curve (see Fig. 2), only 11 kV capacitors should be connected. This may be defined as the economic characteristic for use of either 11 kV or 0.4 kV capacitors. For compensation of reactive power using 11 kV capacitors with a given total installed capacity, the objective function can be formulated as

$$M = u \sum_{i=1}^{n} Q_{ci} + H \sum_{i=1}^{n} q_{ci} + \frac{et}{V_0^2} [\sum_{i=1}^{n} r_i \; (Q_i - q_{ci})^2$$

$$+ \sum_{i=1}^{n} R_i \; \{\sum_{s=i}^{n} Q_s - Q_{cs} - q_{cs}\}^2 + L \sum_{i=1}^{n}$$

$$(Q_{ci} + q_{ci}) - Q_{ct}] \quad (14)$$

with

$$L = \sum_{i=1}^{m} \sum_{s=i}^{n} Q_i \; Q_s - Q_{ct} \sum_{i=1}^{m} A_i - u$$

In this case, eq. (6) is valid with the condition

$$\frac{\partial u}{\partial q_{cj}} = \begin{cases} H-u-(A_{tj}\ Q_j)_{max} \geqq 0,\ J \geqq m \\ \\ H-u-(A_{tj}Q_j)_{max}-Q_{ct} \sum\limits_{i=j+1}^{m} A_i + \sum\limits_{i=j+1}^{m} \sum\limits_{s=i}^{n} A_iQ_s \geqq 0, j<m \end{cases} \tag{15}$$

Installing the capacitors on the 11 kV side is not the most economic distribution so that the use of 0.4 kV capacitors should be considered. In this case, the cost gradient can be expressed as

$$\frac{\partial F}{\partial Q_{cn}} = u - \sum_{i=1}^{m} \sum_{s=i}^{n} A_i\ (Q_s - q_{cs}) \geqq 0$$

and

$$\frac{\partial F}{\partial q_{cn}} = H - \sum_{i=1}^{m} \sum_{s=i}^{n} A_i\ (Q_s - q_{cs}) - A_{tn}\ (Q_n - q_{cn}) = 0 \tag{16}$$

Consequently, the final condition for optimum distribution of capacitors on the 0.4 kV side as well as on the 11 kV side may be formulated as

$$\frac{\partial F}{\partial Q_{cn}} = u - H - A_{tn}\,(Q_n - q_{cn}) \geqq 0 \tag{17}$$

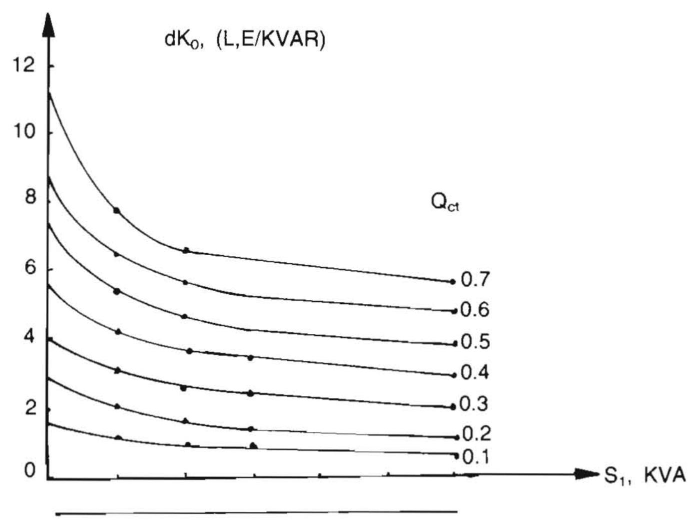

Fig. 2. The dependance of difference between costs of capacitors on both sides 11 and 0.4 kV on the reactive power

Transmission Systems

As the compensation of reactive power may be realized through capacitors in a distribution network, so it can also be achieved by installing synchronous capacitors at some nodes of a transmission system. The partition principle simplifies this problem on the basis of an equivalent circuit. By using the hierarchy principle, the network can be transformed into its equivalent at different hierarchy levels. A simple algorithm which gives the required equivalent circuit without the need for iterations at different levels of hierarchy is now necessary.

Equivalent Circuits

The economic criteria for the compensation of reactive power may be determined from the cost of necessary capacitors as well as the energy loss in a network. Applying the partition principle, the network can be transformed into significant and remainder parts (Fig. 3-a). As the sum of reactive powers of capacitors in a significant part is given, its local optimization can be justified. The results of a optimization do not depend on remainder part A if a constant voltage is considered. In this case, the distribution of active powers in a network will not be changed. This means that the total cost of part P can be expressed for different distributions of capaictors as a function of total rating of capacitors (see Fig. 3-b). The optimum compensation of reactive power of a network as a whole must be one of the calculated values of local optimizations F_p. For the determination of this optimum condition with a given capacitor rating, the solution must be checked also for remainder part A. Part P will be represented by a given total reactive power circuit (Fig. 3-b). Hence, the iteration algorithm should not be used. Instead of iterations, account must be taken of the optimum distributions for different cases. The three quadratic terms can be used, according to El-Hawary and Wellon (1982). The constants of the proposed expression can be obtained using the minimum quadratic method. The regime of part A is independent of equivalent part P. The cost of generated reactive power which is distributed in part P within its limits must be included. The error of such a concept appears to be caused by the approximation in the expression of the three quadratic terms. For best accuracy, a higher-order polynomial should be assumed.

Effect of Voltage Variation at Generating Nodes

The compensation of reactive power is directly related to the problem of voltage regulation so that the voltage variation at generating nodes must be included in the calculations of economic distribution of capacitors in a network (Mamandur 1982). Hence, the effective method for the determination of network equivalents including the effect of voltage variation at the generating buses must be analysed. The power loss will also be changed. The variation of voltage V_0 at generating nodes has a significant effect on the compensation characteristics of reactive powers in a network. (Mamandur 1982) as well as on the distribution of both active and reactive powers in a network. For accurate results, these variations must be considered. Thus, the total active power of loads P_1 as well as total reactive power of loads Q_1 and total cost F can be expressed in the form:

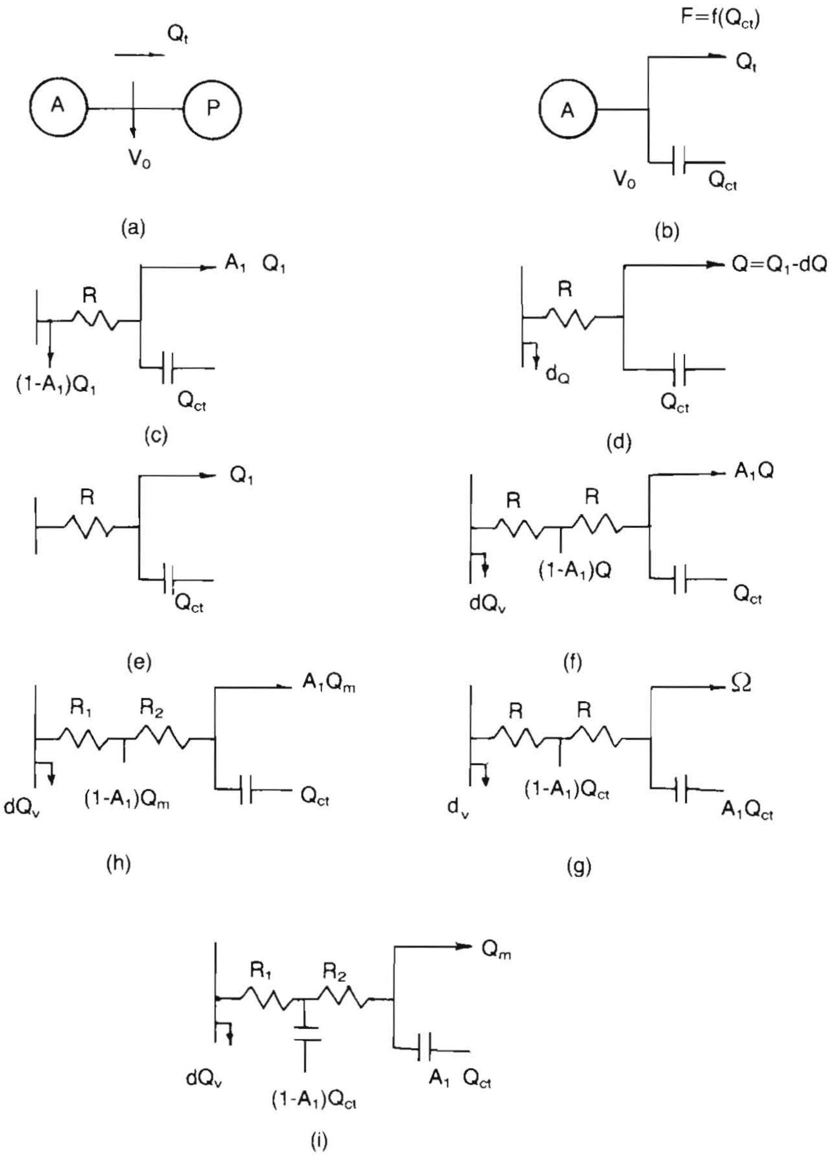

Fig. 3. Different equivalent circuits for reactive power compensation

$$F = A_1(V_0)\,Q_{ct}^2 + B_1(V_0)\,Q_{ct} + C_1(V_0)$$

or (18)

$$P_1 + jQ_1 = [A_2(V_0) + jA_3(V_0)]\,Q_{ct}^2 + [B_2(V_0) + jb_3(V_0)]\,Q_{ct} + [C_2(V_0) + jC_3(V_0)]$$

Equation (18) may be used for an "equivalented" section in order to compute the optimum compensation of reactive power in transmission systems. The dependency of either cost or load powers on total rating of capacitors at different values of voltage at generating nodes V_0 in a distribution network is as shown in Fig. 4. The approximated equations are used for the deviation in voltage V_d as a

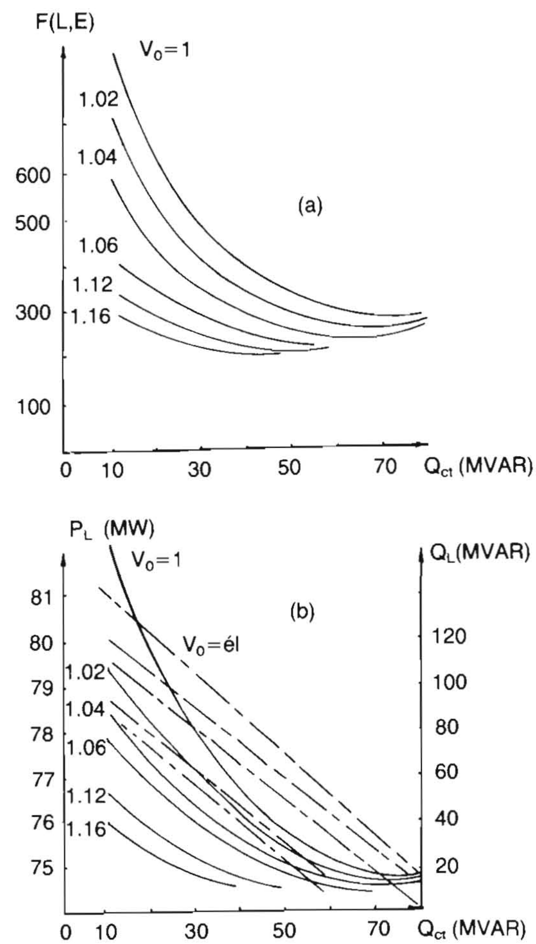

Fig. 4. The effect of total rating of capacitors on cost (F) and powers (P_l)

function of the nominal voltage V_n in the form

$$V_d = (V_0 - V_n) / V_n$$

The results of computed coefficients of polynomials are given in Fig. 5. The

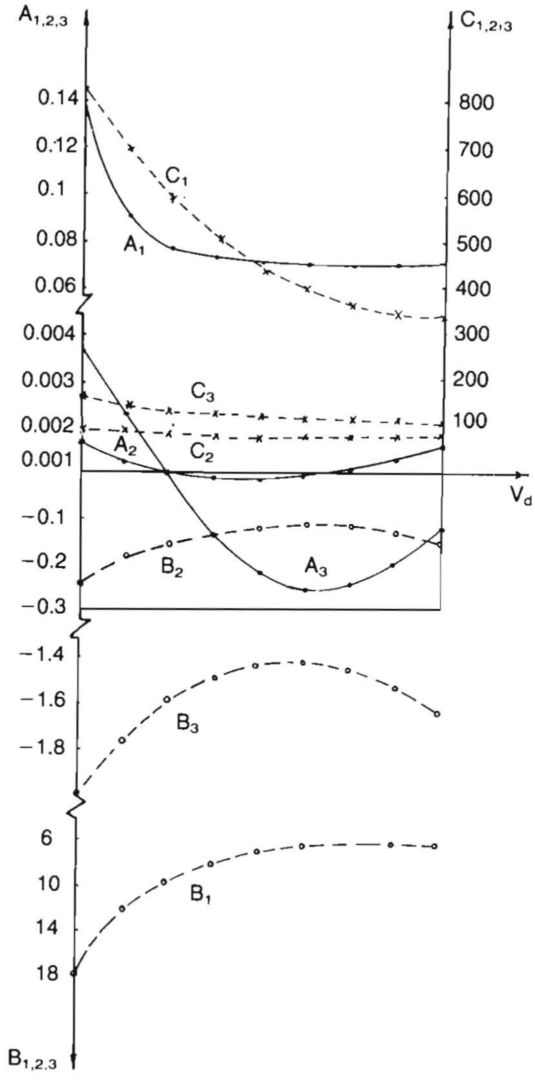

Fig. 5. The calculated coefficients

approximated relations using the minimum quadratic method may be simplified as

$$F = (0.07e^{-64V_d} + 0.07)\, Q_{ct}^2 - (11e^{-29V_d} + 6.5)\, Q_{ct} + (55e^{-17V_d} + 300)$$

$$P_1 = (0.12\, V_d^2 - 0.02\, V_d + 0.002)\, Q_{ct}^2 - (11\, V_d^2 - 2\, V_d + 0.235)\, Q_{ct} + 352\, V_d^2 - 93\, V_d + 84 \tag{19}$$

$$Q_1 = (0.5\, V_d^2 - 0.11\, V_d + 0.003)\, Q_{ct}^2 - (66.2\, V_d^2 - 13\, V_d + 2)\, Q_{ct} + 0.25\, V_d^2 - 688\, V_d + 155$$

The mean deviation in the computed values of coefficients are listed in Table 1:

Table 1. The mean deviation in the computed values of coefficients.

Coeffs.	A_1	B_1	C_1	A_2	B_2	C_2	A_3	B_3	C_3
MEAN DEV.	1.3×10^{-3}	0.56	2.7	7×10^{-5}	0.11	0.4	0.64×10^{-3}	0.08	2.9

From Fig. 5, it is seen that the maximum deviation in the values of coefficients appears at small values of voltage deviation. The calculated deviations in either cost or powers as a function of total rating of capacitors are drawn in Fig. 6. The

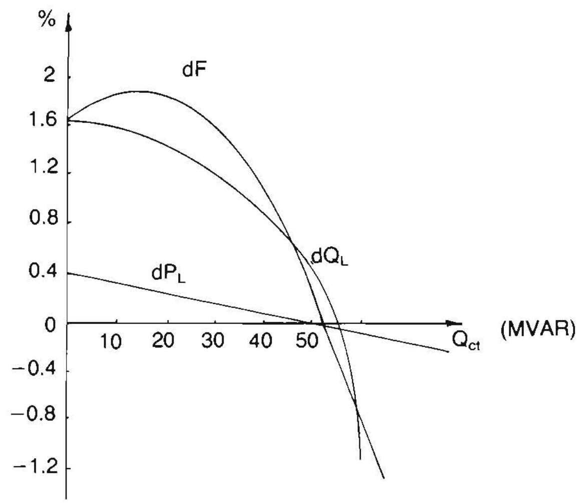

Fig. 6. The deviation with total rating of capacitors

evaluated power of loads and cost as a function of voltage V_0 at nodes of a network for total rating of capacitors of 50 and 20 MVAR are given in Fig. 7. It is seen that cost varies greatly with voltage at large reactive powers.

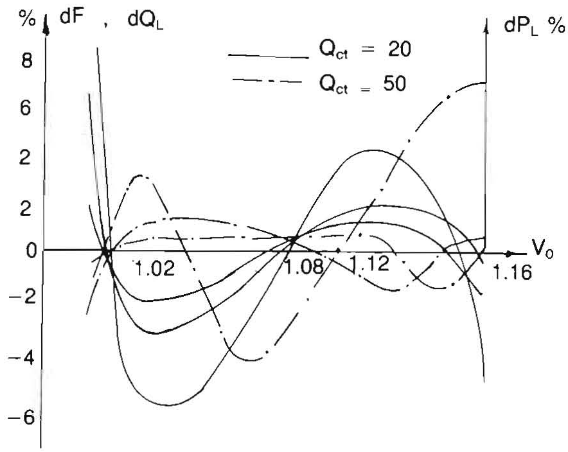

Fig. 7. The percentage deviation at 20 and 50 MWAR

The Mean Parameters of Equivalent Circuit

The physical translation of a network or one of its sections into an equivalent element eads to different distributions of capacitors. For this reason, a new equivalent translation on the basis of the economic characteristic for the compensation of reactive power should be considered. The expression of cost using three -term quadratics means that the equivalent circuit should have three main parameters. The cost is a quadratic function of the total rating of the capacitors while the total reactive power of the loads can be represented by AQ_1 (see Fig. 3-c). The reactive power (1-A) Q_1 can be inserted in the non-equivalent part of a network. The main parameters appear to be resistance R, coefficient A and constant component of cost F_0. Firstly, the total reactive power of loads Q_1 will be a constant value when the cost takes the form

$$F = \frac{eRt}{V_0^2}(Q_{ct}^2 + A^2Q_1^2) + (X - \frac{2AeRt}{V_0^2} Q_1)\, Q_{ct} + F_0 \tag{20}$$

with $X = d\, K_0 + E_c\, e\, t$

The main parmeters will be deduced in the form

$$R = A_1 V_0^2 / et, \quad A = (X - B_1) / 2\, A_1 Q_1$$
$$F_0 = C_1 - (X - B_1)^2 / 4\, A_1 \tag{21}$$

If the total reactive power of the loads is given by a load curve, the cost will be expressed by

$$F = \frac{eRt}{V_0^2} Q_{ct}^2 + (X - \frac{2AeR}{V_0^2} \sum_{n=1}^{m} Q_{1n}\, t_n) Q_{ct} + \frac{eA^2R}{V_0^2} \sum_{n=1}^{m} Q_{1n} t_n + F_0 \qquad (22)$$

$$t = \sum_{n=1}^{m} t_n$$

where m is the number of sections of a load curve.

The approximated coefficients A_1, B_1 and C_1 can be utilized for inclusion in the three main parameters as

$$R = A_1\, V_0^2/et, \quad A = (X-B_1)t/[2A_1 \sum_{n=1}^{m} Q_{1n}\, t_n]$$
$$F_0 = C_1 - t(X-B_1)^2 \sum_{n=1}^{m} Q_{1n} t_n/[4A_1(\sum_{n=1}^{m} Q_{1n}\, t_n)] \qquad (23)$$

If the load curve is equally sectionalized, the main parameters becomes

$$R = A_1\, V_0^2/et, \quad A = m\,(X-B_1)\; [2A_1 \sum_{n=1}^{m} Q_{1n}]$$
$$F_0 = C_1 - m(X-B_1)^2 \sum_{n=1}^{m} Q_{1n}^2 \,/\, [4A_1(\sum_{n=1}^{m} Q_{1n})^2] \qquad (24)$$

On the other hand, another equivalent circuit is shown in Fig. 3-d. A new parameter dQ, for the variation of reactive power now appears. It is used in order to represent the circuit of the remainder part of a network without changing it. Then, the three main parameters become R, dQ and F_0.

Considering that the total reactive power of the loads to be constant, the cost can be formulated as

$$F = (dK_0 + E_c\, et)\, Q_{ct} + \frac{(Q' - Q_{ct})}{V_0^2} R\, et + F_0 \qquad (25)$$

Te main parameters are expressed by

$$R = A_1\, A_0^2/et, \quad dQ = Q_1 - \frac{X-B_1}{2A_1}, \quad F_0 = C_1 - \frac{(X-B_1)^2}{4\, A_1} \qquad (26)$$

The parameter dQ should be considered as a constant value for each interval of the load curve. Hence, for the given equivalented part after the resistance, the fixed part of the total load power appears. As the reactive power Q of each period and the total rating Q_{ct} of capacitors are constants, the cost may be expressed as

$$F = X\, Q_{ct} + (Q - Q_{ct})^2\, eRt \,/\, V_0^2 + F_0 \qquad (27)$$

The expressions of main parameters are simpler for the circuit of Fig. 3-c than the circuit of Fig. 3-d.

Another variant for the equivalent circuit is shown in Fig. 3-e. The cost in this case takes the form:

$$F = X\ Q_{ct} + (Q_1 - Q_{ct})^2\ \frac{2Rt}{V_0^2} + F_0 \tag{28}$$

The main parameters for any period of load curve may be deduced in the form

$$R = \frac{V_0^2\ [n \sum_{i=1}^{n} (F_i - X\ Q_{cti}) \sum_{n=1}^{m} (Q_{1n} - Q_{ci})^2}{et\ \{n \sum_{i=1}^{n} (\sum_{n=1}^{m} (Q_{1n} - Q_{ci})^2]^2} \;\frac{- [\sum_{i=1}^{n} \sum_{n=1}^{m} (Q_{1n} - Q_{ci})^2 \sum_{i=1}^{n} (F_i - XQ_{cti})}{- [\sum_{i=1}^{n} \sum_{n=1}^{m} (Q_{1n} - Q_{ci})^2]^2} \tag{29}$$

$$F_0 = \frac{1}{n}\ [\sum_{i=1}^{n} (F_i - XQ_{cti}) - \frac{eRt}{mV_0^2} \sum_{i=1}^{n} \sum_{n=1}^{m} (Q_{1n} - Q_{cti})^2]$$

An extra resistance can be also inserted (see Figs. 3-f and 3-g). While the load is varying, the value of dQ will be constant. It can be replaced by the value Q and then the cost will be determined by

$$F = \frac{etR}{V_0^2}\ (2\ Q_{ct}^2 + (1+A^2)^2 Q) + (X - \frac{2etRQ}{V_0^2}\ (1+A)\ Q_{ct} \tag{30}$$

This means that the analysis is suitable for the case of unregulated systems. As a constant component appears in expression (30), the main parts are obtained as

$$R = \frac{A_1 V_0^2}{2et}\ ,\ Q_{1,2} = \frac{X - B_1}{2A} \pm [\frac{C_1}{A_1} - \frac{(X - B_1)^2}{R\ A_1^2}]^{\frac{1}{2}}, \tag{31}$$

$$A'_{1,2} = \frac{X - B_1}{A_1 Q_{1,2}} - 1$$

The evaluated costs for the first and second solution are

$$F_{1,2} = XQ_{ct} + [(Q_{1,2} - Q_{ct})^2 + (A'_{1,2} - Q_{1,2} - Q_{ct})^2]\ \frac{etR}{V_0^2} \tag{32}$$

This means that both circuits give the same results at

$$Q_1 = A'_2\, Q_2 \text{ and } Q_2 = A'_1\, Q_1 \tag{33}$$

As the undefined resistances R_1 and R_2 are inserted (Fig. 3-h or 3-i), the third parameter A_1 appears. The value of Q_m can be given as the maximum constant part of the load curve. If the total reactive power of the loads is known, the main parameters can then be determined by

$$R_2 = \frac{V_0^2 \, (X-2A_1 \, Q_m - B_1)^2}{4et[A_1 \, Q_m^2 + (B_1 - X) \, Q_m + C_1}, \quad R_1 = \frac{A_1 \, V_0^2}{et} - R_2$$

and (34)

$$A = \frac{(X-2A_1 \, Q_m - B_1)V_0^2}{2et \, R_2 \, Q_m} + 1$$

This also demonstrates that the circuit of Fig. 3-c is still the simplest so that it must be recommended for economic study of reactive power compensation in a large electrical networks. The sectionalization principle will simplifies the mathematical analysis.

Application

The mean value of reactive powers of a load for each transformer (see Fig. 1-c) is 315 KVAR. An 11 kV base voltage with specific power loss in capacitors of 4 W/KVAR will be considered. Using eq. (10), the relation between installed capacity of capacitors and parameter u, including the effect of specific power loss in capacitors (Fig. 8) can be determined. Fig. 8 shows that for a specific cost of capaictors greater than 6 L.E/KVAR, the compensation of reactive power by only 11 kV capacitors is not suitable. The installed capacity of capacitors for smaller values can be obtained from Fig. 3, as indicated by eq. (10). For a specific power loss in 11 kV capacitors per KVAR (Shoults and Sun 1982), the optimum distribution of reactive power is listed in Table 2.

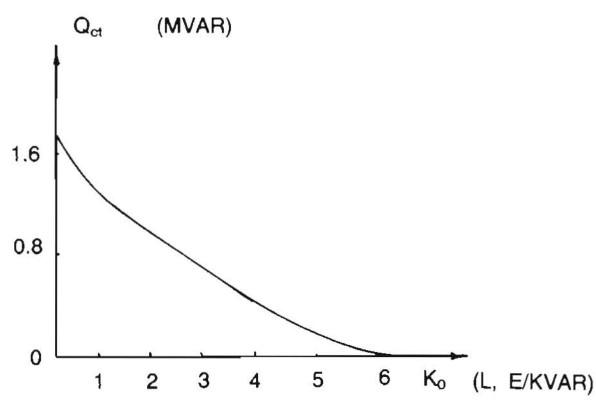

Fig. 8. The specific cost with total rating of capacitors

Table 2. The optimum distribution of reactive power at different nodes.

Node Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Condenser rating (KVAR)	0	0	0	0	0	90	315

Only 11 kV capacitors may be installed if dK_0 is greater than 3,8 with a ratio factor of 0.5. This means that for 11 kV capacitors, the cost is 6 while for 0.4 kV capacitors it will be more than 10. Hence, the curve of Fig. 2 may be recommended in order to find the economic criteria of reactive power compensation on the 11 kV side.

Conclusions

The economic characteristic on the basis of the hierarchy principle for an equivalented part of a network may be applied in order to find the optimum compensation of reactive power in either transmission or distribution networks. The sectionalization of a network into different hierarchy levels and solution the problem for different levels by equivalent character simplifies the mathematical analysis for economic compensation of reactive power in large networks. The proposed technique is recommended for use in the design of distribution networks especially in either unregulated or poorly regulated systems. The rating of an 11 kV capacitor at a node of a feeder can be evaluated as the mean value of the uncompensated reactive powers of loads behind the studied node for a certain period of load curve. The need for 0.4 kV capacitors can be easily checked with the help of the deduced formula.

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المعايير الاقتصادية لتعويض القدرة غير الفعالة للأحمال في شبكات النقل والتوزيع

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تناول البحث دراسة حالة المغذي المفرد والشبكة متعددة المغذيات لتعويض القدرة غير الفعالة باستخدام مكثفات عند الجهد الأعلى. وقد تم في البحث وضع طريقة عددية للمعيار الاقتصادي لتوزيع المكثفات بغرض تعويض القدرة غير الفعالة في الشبكة. كما تم بحث حالة وجود مكثفات عند جهد التوزيع ٤٠٠ فولت، واستنتج رياضيا معادلة لتحديد حالة الاحتياج وحالة عدم الاحتياج إلى مكثفات ٤٠٠ فولت.