Arab Gulf J. scient. Res. **4** (2), pp. 629-635 (1986)

How Many Nucleons are Involved in the Decay of A Δ (1232) in a Nucleus?

M. A. Abdulmomen and A. Y. Abul-Magd

Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia and
Faculty of Medicine and Allid Sciences, Physics Section,
King Abdulaziz University, Jeddah, Saudi Arabia.

Abstract. The effective number of nucleons with which a Δ (1232) shares its energy when it decays inside a nucleus is calculated by assuming that the Δ simultaneously interacts with all the surrounding nucleons within a certain range.

It is now becoming evident that nucleon isobars are formed inside nuclei during their interaction with high energy pions, nucleons and heavy ions. These isobars are normally short lived. For example, when a Δ (1232) isobar is formed, it decays in a time of 0.6×10^{-23} it has a decay width of 100 MeV. The process by which the Δ decays in nuclear matter is still an open question Hufner (1975), Brown and Weise (1975). Experiments on pion absorption in nuclei indicate that there are more than two nucleons involved in the decay of the Δ isobar. A brief account of these experiments is given in Sec. 1.

Additional evidence for the involvement of more than two nucleons in sharing the Δ decay energy comes from the analysis of the angular distribution of negative pions produced in 183 MeV/nucleon heavy-ion induced reaction Nagamiya *et al.* (1982) as well as the recoil properties on neutron-deficient fragments produced in the interaction of relativistic protons with nuclei Abdulmomen and Abul-Magd (1984).

In Section 2, we present a model in which the Δ decays in finite nuclei by sharing its excitation energy with its nearest neighbours which form a sphere of radius d. By averaging over the nuclear density and taking into account the nuclear surface, one obtains the dependence of the number of nucleons involved in the delta decay on the mass number of the host nucleus. In Section 3, we discuss the results.

Summary of Pion Absorption Experiments

The main features due to the Δ (1232) is strongly evident in all pion-nucleus scattering and absorption cross sections, even in the heaviest elements Hufner (1975) and Jackson (1968). It is generally thought that the absorption of pions in the energy range of the resonance (100-300 MeV) is due to the capture by a correlated pair nucleons according to the process

$$\pi + 2N \rightarrow \Delta N \rightarrow NN$$

while the residual nucleus remains as a spectator. The emitted nucleons in this process should be ejected at 180° with respect to each other. Nordberg *et al.* (1968) have found that, for nuclei in the region of lithium to oxygen, about 40% of all stopped π produce back-to-back correlated nucleons. Cheshire and Sobottka (1969, 1970) found that even within the spectrum of correlated nucleons, emitted after absorption by ^{12}C there is a large part which is not attributed to the quasi-free two nucleon absorption model. The importance of other mechanisms for pion absorption in nuclei has also been noted in early emulsion Redotov (1966) and bubble chamber Belotti *et al.* (1973) experiments. In Belotti's experiment, it is found that 24% of 130 MeV $\pi - ^{12}C$ absorption events result in the emission of three charged particles and 15% in more than three charged particles. It is hard to understand the emission of more than two particles with such a high probability with the quasi-free two particle absorption model. Because the mean free path of 100 MeV nucleons in nuclear matter is 4-9 fm, the probability that the correlated pair of nucleons knock more nucleons in their way out of a small nucleus like ^{12}C is small, even if pions are absorbed in the surface region. More recent measurements of inclusive proton spectra from pions incident on a variety of nuclei Jackson *et al.* (1977) show that the peak corresponding to the kinematics of the quasi-free two-nucleon absorption is only evident in the lightest nuclei. In heavier nuclei ($A \geq 12$) it is a minor feature of the proton energy spectrum and is barely evident in nuclei like Ta.

Recently McKeown *et al.* (1980) analysed energy spectra of protons emitted at different angles after the absorption of $\pi^{\pm}$ at 100, 160 and 220 MeV by different nuclei. They represented their spectra by rapidity distribution curves which appeared to be well described by isotropic emission of protons from a Lorentz frame. The latter is moving with a laboratory velocity equal to the velocity of the centre of mass of a pion plus N nucleons which absorb the energy and momentum of the pion, with $N=3\text{-}6$ for nuclei with $A=12\text{-}181$. They also found that the ratio of the total proton yield for π^+ to that of π^- averaged over three incident pion energies varies from $R=5\text{-}3$ which is consistant with absorption proceeding through N nucleons deduced from the rapidity analysis, but much less than $R=11$ expected from the two-nucleon mechanism. These results suggest that there is a different mechanisms other than the two-nucleon one. Mckeown *et al.* (1980) suggest that when a $\triangle$ is formed in nuclear matter, it prefers to decay via soft-pion exchange with several nearby nucleon rather than a hard-pion exchange with another nucleon. In the next section we explore the possibility and try to predict the number of nucleon involved in the decay deduced from the rapidity analysis of absorption in nuclei.

2. *A Model for Δ Decay:*

In this section, we evaluate the effective number N of nucleons that share the excitation energy of a $\triangle$ when it decays in a nucleus with mass number A, and compare it with the corresponding number obtained in the analysis of pion absorption at the $\triangle$ (3,3) resonance energy carried by McKeown *et al.* (1980). To do this we assume that the decaying delta shares its energy with its nearest neighbours. Thus, if the delta decays at a position described by a radius vector r, then after the decay the excitation energy carried by the delta will be distributed among nucleons contained within a sphere of radius d centred at r. The number of nucleons in this region is given by

$$n(r) = \int \theta\,(d - |r - r'|)\,\rho\,(r')\,dr' \tag{1}$$

where ρ (r) is the nuclear density and θ is the step function. At the nuclear centre, if $d << R$ where R is the nuclear radius, eq. (1) becomes

$$n(O) = (4\pi/3)\,d^3\,\rho(O) \tag{2}$$

We shall estimate the distance d by assuming that the central part of the nucleus can be approximately described within the lattice cell model of liquids Reed *et al.* (1973). As the name implies, the model assumes that the particles constituting the liquid spend most of their time near one location of the volume. This model serves as a satisfactory description for the liquid state when the density is not less than twice the critical density. This condition is satisfied for nuclear matter at the centre where the density is approximately 0.16 fm^{-3}, while the critical density is of the order of 0.07 fm^{-3} Curtin *et al.* (1983). The number of nearest neighbours of a particle in a lattice depends on its crystal structure. If the particles are regarded as closely packed (cp) spheres, then the crystal will have either a hexagonal or face-centred cubic structure. In this case the number of nearest neighbours is 12. Other possible less packed structures are the body-centred cubic (bcc) lattice and the simple cubic (sc) lattice with the number of nearest neighbours being 8 and 6 respectively Kittel (1971). The number of nucleons sharing the decay of the delta at the nuclear centre is taken to be the number of nearest neighbours plus one (the delta itself). Therefore,

$n(O) = 13$ for cp lattice,

$$\begin{aligned} &9 \text{ for bcc lattice,} \\ &7 \text{ for sc lattice.} \end{aligned} \qquad (3)$$

corresponding to values of 2.68 fm, 2.38 fm and 2.19 fm, respectively, for the distance d. The effective number of nucleons sharing the energy of decay anywhere in the nucleus is obtained by averaging n(r) with respect to the nuclear density:

$$N = (1/A) \int \int \theta(d\text{-}x)\, \rho(y+\tfrac{1}{2}x)\, \rho(y-\tfrac{1}{2}x)\, dx\, dy \qquad (4)$$

Assuming again that d << R, we simplify eq. (4) by expanding $\rho\ (y-\tfrac{1}{2}x)$ in powers of x and keeping only second order terms. We then obtain

$$N = n(O)\ [<\rho> + (d^2/10) < \frac{1}{y} \frac{d\rho}{dy} >]/\,\rho(O) \qquad (5)$$

where

$$<F> = (1/A) \int \rho(y)\ F(y)\ dy$$

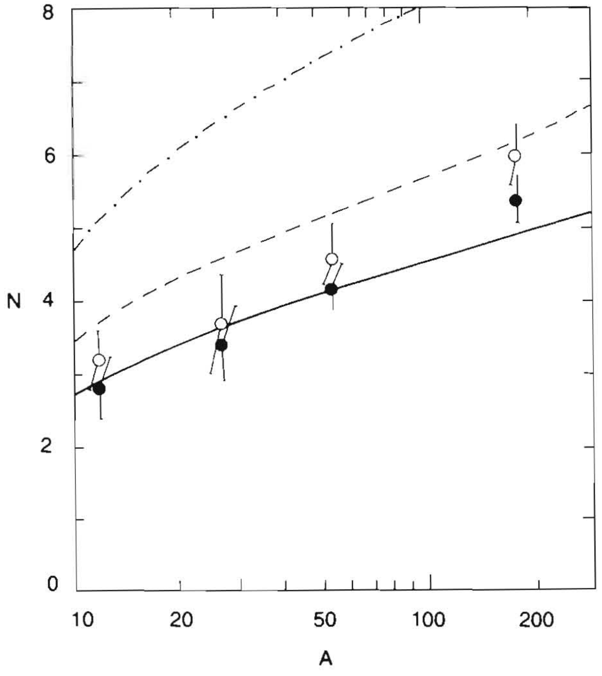

Fig. 1. The average number of nucleons sharing the excitation energy of a delta in a nucleus of mass number A calculated using eq. (5) assuming that the number of nearest neighbours at the nuclear center is 6 (solid curve), 8 (deashed curve) and 12 (dashed dotted curve). The experimental points are those of Mckeown *et al.* (1980) from the rapidity distribution of nucleons following the absorption of π^- (solid circles) and π^+ (open circles) at the Δ (3,3) resonance energy.

Equation (5) is used to calculate the effective number of nucleons which share the excitation energy of a delta in a nucleus as a function of the mass number of the nucleus. We take

$$\rho(r) = \rho(O) \; [\, 1+\exp \{(r-R)/a\} \,]^{-1}$$

with $r = 1.07 \; A^{\frac{1}{3}}$ fm, $a = 0.55$ fm and

$$\rho(O) = 3/(4 \; \pi \; R^3) \; [\, 1+ (\pi a/R)^2 \,]^{-1}$$

The results of calculation are plotted in figure 1. The solid curve corresponds to $n(O)=7$, the dashed to $n(O)=9$ and the dashed-dotted to $n(O)=13$. The experimental points of McKeown *et al.* (1980) are also plotted. The increase of N with A shown by the experimental data is correctly reproduced by the present calculation.

3. *Discussion:*

We have calculated the number of nucleons involved in decay in finite nuclei for different assumption about the number of nearest neighbours based on the lattice cell theory of liquids. We compared the results we have obtained for the number of nucleons sharing the delta decay energy with the numbers deduced from pion absorption analysis. Our predictions agree well with the experiment. We consider this agreement as an evidence that delta decay in nuclear matter is not a two body process but a many body process in which the delta shares its energy between its neighbours. In fact, the leading term in the expansion (5) of N is proportional to the mean nuclear density $< \rho >$ which monotonously increases with A.

We also see from the figure that the comparison of the experimental data with the theoretical curves does not support a model of nearest neighbours based on close packing ($n(O)=13$); rather it supports a lattice structure at the nuclear centre between the base-centred cubic and the simple cubic. As yet there is no solid foundation for the application of lattice cell models of liquids for nuclear matter; nevertheless the predictions generated here by such an application seem to be reasonable.

We finally note that eq. (4) can only provide an approximate calculation of the number of nucleon involved in pion absorption. The integrand should involve a term describing the dependence of the probability of pion absorption to form the delta on the distance from the nuclear centre, giving a preference for pion absorption at the nuclear surface. The effect of this term is small when the sum of pion absorption length and the distance the delta makes before it decays is large compared to the nuclear surface thickness. Unfortunately, this is not the case in the

(3,3) resonance region, where the mean free path of a pion is of the order of 1 fm for nuclear matter density (Hufner and Thies 1979), although it may be larger in the nuclear surface region.

References

Abdulmomen, M.A. and **Abdul-Magd, A.Y.** (1984) Nuclear recoil Velocity after interaction with high energy protons, *Nucl.Phys.A* **423:** pp 498-510.

Belotti, E., Cavalli, D. and **Matteuzzi, C.** (1973) Positive pion absorption by C nuclei at 130 MeV., *Nuovo Cim.* **18A:** 75-93.

Brown, G.E. and **Weise, W.** (1975) pion Scattering and Isobars in nuclei, *Phys. Rep.* **22:** 280-337.

Cheshire, D.L. and **Sobottka, S.E.** (1969) Pion capture in ^{12}C with emission of neutron pair, *Phys. Lett.* **30B:** 244-245.

Cheshire, D.L. and **Sobottka, S.E.** (1970) Final state interaction effects in neutron pair emission from pion caputre on ^{12}C, *Nucl. Phys. A* **146:** 129-135.

Curtin, M.W., Toki, H. and **Scott, D.K.** (1983) Liquidgas phase instabilities in nuclear system, *Phys. Lett.* **123B:** 289-292.

Hufner, J. (1975) Pions interact with nuclei, *Phys. Rep.* **21:** 1-79.

Hufner, J. and **Thies, M.** (1979) Pion nucleus scattering and absorption as a solution of the Boltzmann Equation, *Phys. Rev.* **C20:** 273-285.

Jackson, D.F. (1968) Nuclear Structure at Medium energy, *Adv. Phys.* **17:** 481-508.

Jackson, H.E., Kaufman, S.B., Mayer, L., Schutzmeister, J., Schiffer, P., Tabor, S.L., Vigdor, S.E., Burman, R.L., Gram, P.A.M., Redwine, R.P. and **Yates, M.A.** (1977) Energetic charged particle yields induced by pions on complex nuclei,*Phys. Rev.* **C16:** 730-740.

Kittel, C. (1971) *Introduction to solid state Physics,* (Wiley, New York).

Mckeown, R.D., Sanders, S.J., Schiffer, J.P., Jackson, H.E., Paul, M., Specht, J.P., Stephenson, E.J., Redwine, R.P., and **Segel, R.E.** (1980) How many nucleons are involved in pion absorption in nuclei?, *Phys. Rev. Lett.* **44:** 1033-1036.

Nagamiya, S., Hamagaki, H., Hecking, H., Hecking, P., Kadota, S., Lombard, R., Miake, Y., Moeller, E., Schnetzer, S., Steiner, H., Tanihata, I., Bohrmann, S. and **Knoll, J.** (1982) Production of negative pions with 183-MeV/nucleon Ne beams, *Phys. Rev. Lett.* **48:** 1780-1783.

Nordberg Jr, M.E., Kinsey, K.F. and **Burman, R.L.** (1968) Two-nucleon emission following absorption of stopped negative pions, *Phys. Rev.* **165:** 1096-1104.

Reed, T.M. and **Gubbins, K.E.** *Applied Statistical Mechanics.* (MeGraw Hill, New York, 1973).

Redotov, P.I. (1966) Absorption of stopped π-mesons by carbon nuclei, *Sov. J. Nucl. Phys.* **2:** 335.

(Received 12/05/1985;
in revised form 04/02/1986)

كم من النيوكلونات تشارك في اضمحلال جسيم Δ داخل النواة ؟

محمد أحمد عبد المؤمن و عادل يحيى أبو المجد

قسم الفيزياء - كلية العلوم - وقسم الفيزياء
كلية الطب - جامعة الملك عبد العزيز
جدة - المملكة العربية السعودية

في هذا البحث نحسب عدد النيوكلونات التي تشارك جسيم △ في طاقته عندما يضمحل داخل النواة. وذلك بفرض أن هذا الجسيم يتفاعل مع كل النيوكلونات المحيطة به في مدى معين.