

Design of Achromatic Aplanatic Lenses

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ABSTRACT. On the basis of Hopkins' method the design of achromatic objectives, erectors and eyepieces has been investigated. Many trials were done to control the aberration of the designed lenses. The optical glasses were chosen from those readily available in Egypt, so the designed lenses can be manufactured and inspected easily. Good agreement has been obtained between the calculated optical properties and those measured experimentally on the finished lenses.

Achromatic doublets and Cook triplets are used in a large number of optical instruments such as telescopes and photographic objectives. Design procedure for such lenses have been described by several authors, Sharma (1980), Sharma and Rama Gobal (1983), Brixner (1982) and Communay (1976). Conrady (1957) has given a detailed account of their design procedures.

On the basis of Hopkins' method (Conrady 1957), two optical designs are formulated in this article. One is an aplanatic achromatic doublet while the other is a three lens system of the triplet family. Both systems could be used as objectives, eyepieces and erectors of telescopic systems. The aberrations of these lenses are optimized to a minimum within the tolerance according to the Rayleigh limit.

Doublet Objective

Owing to the low aperture and angular field of doublets, only three aberrations are considered to be important, spherical aberration, coma and longitudinal chromatic aberration. Since the aperture stop is at the doublet,

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distortion and lateral chromatic aberration are corrected by the correction of longitudinal chromatic aberration.

The lens is usually a cemented achromatic aplanatic doublet and the lens holder itself is the aperture stop. The conditions for zero longitudinal chromatic aberration for the doublet, which consists of two lenses A and B are,

$$K_A = K \frac{V_A}{V_A - V_B} = (n_A - 1) (C_1 - C_2)$$

$$K_B = -K \frac{V_B}{V_A - V_B} = (n_B - 1) (C_2 - C_3)$$

where K is the power of the doublet, V is the number of the glass and n its refractive index. Knowing K , V_A , V_B , n_A and n_B , it is possible to calculate C_1 , C_2 and C_3 which are the curvatures of the lenses surfaces respectively. For sighting systems an objective of equivalent focal length 120.00 mm is suitable, meanwhile the optimized full aperture is 30.00 mm *i.e.* $f/4$. According to the characteristic curves of glass, two kinds of glass are chosen with their V numbers as widely apart as possible to minimize the curvatures. The available glass are crown (BSC 569561) and flint (EDF 699301). Both types are chosen according to their characteristic curves and are also readily available. Glass types may be varied to obtain more advantageous higher-order aberration residuals as well as for reduction of distortion, astigmatism and longitudinal chromatic aberration.

Following Hopkins' method of optical design for paraxial and finite ray tracing at different zones of the aperture, the following parameters will be used for the Seidel aberrations at the stop:

C_i	Curvature of the i^{th} surface,
d	axial separation between two surfaces,
n	refractive index,
SI	spherical aberration coefficient,
SII	coma coefficient,
SIII	astigmatism coefficient,
SIV	Petzval curvature coefficient,
SV	distortion coefficient,
C_L	coefficient of longitudinal chromatic aberration for paraxial ray tracing, and
C_T	coefficient of transverse chromatic aberration.

For Seidel aberrations not at the stop the coefficients are: S^*I , S^*II , S^*III , S^*IV , S^*V , C_L^* and C_T^* respectively.

More, $\delta\ell'$ is the longitudinal spherical aberration and $\Delta'T$ is the longitudinal curvature of the field. The quantity $\Sigma(d-D)\delta n$ is the longitudinal chromatic aberration of finite ray tracing.

In the design program, the degrees of freedom are the choice of curvature C_i , glass type V and thickness d_i of the lens. By coordinating them judiciously, it is possible to get the optimum conditions of design.

The central thickness of the convex lens is chosen to be a slightly more than the sum of the Sagittae of the two surfaces of the concave lens. Furthermore, the central thickness of the concave lens is chosen to be greater than one tenth of its diameter to avoid lens damage. Purely trigonometric methods are applied to get the lens shape which gives minimum spherical aberration, coma and longitudinal chromatic aberration at full aperture.

Taking $C_1 = 0.0107 \text{ mm}^{-1}$, $C_2 = -0.0215 \text{ mm}^{-1}$, $C_3 = -0.0073237 \text{ mm}^{-1}$, $d_1 = 6.00 \text{ mm}$ and $D_2 = 2.00 \text{ mm}$, this solution will give $SI = -0.0000214$, $SII = -0.0074763$, $C_L = -0.0001042$, $\delta\ell' = 0.0665 \text{ mm}$, coma = 0.0114897 mm and $\Sigma(d-D)\delta n = -0.0015256 \text{ mm}$. The results for different aperture zones are listed in Table 1.

Triplet Objective

A triplet objective is sometimes preferable to a doublet objective, since the triplet has more degrees of freedom *e.g.* shape factor and can distribute the power of one positive lens over two positive lenses. This will improve performance but with additional difficulties in design and manufacture. The previous doublet objective is split into a triplet (single lens and cemented doublet).

Taking into consideration that C_5 , of the last surface of the doublet has to be positive to correct S^*III , the two convex lenses in the triplet are made equiconvex for ease in manufacture, *i.e.* $C_1 = C_2$ and $C_3 = C_4$.

Starting with $C_1 = 0.00534 \text{ mm}^{-1}$, $C_2 = -0.00534 \text{ mm}^{-1}$, $C_3 = 0.0104 \text{ mm}^{-1}$, $C_4 = -0.0104 \text{ mm}^{-1}$, $C_5 = 0.00344 \text{ mm}^{-1}$, $d_1 = 3.00 \text{ mm}$, $d_2 = 1.00 \text{ mm}$, $d_3 = 4.00 \text{ mm}$ and $d_4 = 2.00 \text{ mm}$, the resulting aberrations are found to be:

$SI = -0.0048663$, $SII = -0.0101427$ and $C_L = 0.0021862 \text{ mm}$. If the entrance pupil is considered to be at the first surface of the triplet, then $\Sigma S^*III = 0.0083368$, $\Sigma S^*IV = 0.0057199$, $\Sigma S^*V = 0.0001556$ and $\Sigma C_T^* = -0.00039801$. Following paraxial and finite ray tracing technique at different zones of the aperture for different wavelengths $\lambda\lambda 587.56$, 486.13 and 656.28 nm respectively, it is found that the following results shown in Table 2 are acceptable.

Triplet Eyepiece

The eyepiece should cover a fairly wide field of view through a relatively small aperture, the exist pupil, which is outside the system. Because of the external aperture stop and wide field, coma, distortion, lateral chromatic aberration and field curvature have to be handled with great care. A suitable relative aperture of the eyepiece tends to hold spherical aberration and axial chromatic aberration at reasonable values. An eyepiece can be considered reasonably corrected for distortion if it has an error of not more than $\pm 3\%$ to $\pm 5\%$ for moderate field (Smith 1966).

Following the same procedure as in the objective, the final results of the eyepiece are $C_1 = C_2 = \pm 0.017333 \text{ mm}^{-1}$, $C_3 = C_4 = \pm 0.045333 \text{ mm}^{-1}$, $C_5 = 0.029208 \text{ mm}^{-1}$, $d_1 = 5.0 \text{ mm}$, $d_2 = 1.00 \text{ mm}$, $d_3 = 13.00 \text{ mm}$ and $d_4 = 10.00 \text{ mm}$. The aberrations are $SI = 0.000544$, $SII = -0.00564$, $C_L = 0.000099 \text{ mm}$, $\delta\ell' = -0.027583 \text{ mm}$, coma is 0.064407 mm and $\Sigma (d-D) \delta n = 0.000878 \text{ mm}$. All these results are within the tolerance of the desired system.

The Symmetrical Doublet Erector

If the total length of a telescope is considered to be about 250 mm and the equivalent focal length of the objective and eyepiece are 120 and 30 mm respectively, then the available space for the erector is 100 mm. The symmetrical doublet erector consists of two symmetrical achromatic doublets as shown in Fig. 1. This will make the design and the manufacture easier. Taking into consideration that its magnification is to be unity *i.e.* $m = 1$ then $-U = +U' = 0.125^\circ$. Starting with S (Fig. 1) equal to 100 mm, the focal length for each doublet will be about 45 mm. To prevent vignetting of the oblique pencils for the desired field of view, the diameter of the erector should be 14.0 mm. To reduce its diameter and consequently its aberration, S is chosen to be 60.0 mm and f for each doublet to be 25.0 mm. The same types of glass used in the objective will be used for the erecting system. Following the previously described procedure in designing the achromatic doublet, a purely trigonometrical method is applied and the resultant solution will be $C_1 = 0.0625 \text{ mm}^{-1}$, $C_2 = -0.0966 \text{ mm}^{-1}$, $C_3 = -0.0251929 \text{ mm}^{-1}$, $d_1 = 3.00 \text{ mm}$, $d_2 = 1.0 \text{ mm}$ and full aperture = 11.00 mm. From paraxial ray tracing, for the doublet it is found that $SI = -0.0007309$, $SII = -0.0000715$ and $C_L = 0.0000644 \text{ mm}$. For the whole erector (two doublets) SI and C_2 will be added while SII will be cancelled by symmetry. From finite ray tracing for the doublet, $\delta\ell' = 0.0640 \text{ mm}$, coma = 0.004723 mm and $\Sigma (d-D) \delta n = -0.0000607 \text{ mm}$. All these results are acceptable for the desired design, since the objective height ($h = 3.125 \text{ mm}$) is small and the aberrations at full aperture are within tolerance. Accordingly the aberrations for the other zones will be acceptable. Moreover, S^*III , S^*IV , S^*V and C^*_T can be calculated after knowing the positions of the stops of the whole system.

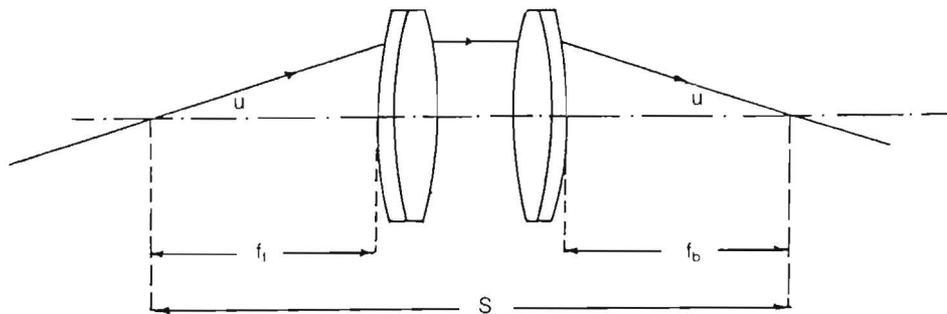


Fig. 1. Doublet Gauss erector.

Manufacture of the Objective and the Eyepiece

The required data for manufacturing a lens are the radii of curvature r_1 and r_2 , the central thickness d , its diameter D , the type of optical glass n and V , type of coating and finally the required tolerances. Also, centering error, surface quality, types of grinding, polishing and surface finish are the required data for inspection. The lens, during manufacturing passes through several stages. Tables 3 and 4 show all the experimental parameters which are used in manufacturing the objective and the eyepiece. The lenses are coated by a single layer of one quarter wavelength thickness of low index material *e.g.* magnesium fluoride ($n = 1.38$) by thermal deposition under vacuum. During evaporation the temperature of the glass component is raised to 300–500°C depending on the softening point of the glass. By this technique the coating surface will be hard and durable. Multiple antireflection coatings may also be used to decrease reflectivity. In this case, three or four alternate layers of titanium dioxide (high refractive index) and silicon monoxide (low refractive index) are deposited. Hence the reflectivity could be reduced to 0.25% for all types of glass. However these coatings are less durable and more expensive.

Within our limited facilities, only the doublet objective and eyepiece have been manufactured and tested. The experimental results of the manufactured objective and eyepiece are shown in Tables 5 and 6, respectively. An optical cement is used to cement lenses and special jigs are used to hold optical components in place while cementing. The lenses are cleaned after cementing, and centering error is checked with a centering tester with an accuracy ± 0.5 minute of arc. The radii of curvature are measured by a precision optical spherometer with accuracy of ± 0.0001 mm. Homogeneity, refractive index and bi-refringence of the material are checked interferometrically. The focal lengths of single and doublet lenses are measured with a focal collimator with an accuracy ± 0.001 mm.

Spherical aberration, coma, astigmatism, distortion and chromatic aberration are checked by the star test as given by Malacare (1978). The results for the mean spherical aberration and chromatic aberration are shown in Table 7.

Conclusions

The possibility of designing objectives, eyepieces and erectors for telescopic system and manufacturing objectives and eyepieces is verified as shown in the Tables, by applying Hopkins' method. From Tables 5 and 6, it can be seen that the equivalent focal length of the designed and manufacturing lenses are nearly identical. This is achieved by choosing the correct type of glass and accurate adjustment of curvatures of the optical tools and test plates.

In Table 7 it is clear that the measured aberrations are within the design tolerances. This was achieved by adjusting the sphericity, centering and thicknesses of the manufactured curved surfaces. The number V of the glass affects greatly the chromatic aberration.

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Table 1. The longitudinal spherical aberrations, coma and longitudinal chromatic aberrations for different zones of the aperture of doublet objective.

Zone	$\delta\ell'$ in mm	coma in mm	$\Sigma(d-D)\delta n$ in mm
Full aperture	0.0665	0.0114897	-0.0015256
0.866 aperture	0.12651	0.0073847	-0.0008512
0.707 aperture	0.1318	0.0041229	-0.0003768
0.500 aperture	0.08721	0.0017105	-0.00009895

Table 2. The longitudinal spherical aberrations, coma and longitudinal chromatic aberrations for different zones of the aperture of the triplet eyepiece.

Aperture	$\delta\ell'$ (mm)	Coma (mm)	$\Sigma(d-D)\delta n$ (mm)
Full aperture ($\lambda = 587.56$ nm)	0.00428	0.007984	0.0005322
0.707 aperture ($\lambda = 587.56$ nm)	0.00202	0.0039227	0.0004073
Full aperture ($\lambda = 486.13$ nm)	-0.0851	0.0095411	
0.707 aperture ($\lambda = 486.13$ nm)	-0.03994	0.004615	
Full aperture ($\lambda = 656.28$ nm)	0.03513	0.0074723	
0.707 aperture ($\lambda = 656.28$ nm)	0.0182	0.0036501	

Table 3. The experimental parameters of the manufactured objective.

Item		Convex lens	Meniscus lens
Glass thickness (X mm)		7.2 ± 0.1	5.7 ± 0.1
Square length (L mm)		35 ± 0.1	35 ± 0.1
Disc diameter (D mm)		33 ± 0.1	33 ± 0.1
Central thickness (d mm) during curve generation	C ₁	6.80 ± 0.1	2.95 ± 0.1
	C ₂	6.50 ± 0.1	2.60 ± 0.1
Angle θ using cutter with d = 20.0 mm and r = 3.0 mm	C ₁	5° 57' 2"	15° 17' 12"
	C ₂	11° 39' 8"	4° 6' 34"
Radius of curvature of truing and smoothing tools (mm)	C ₁	-93.457 ± 0.002	$+46.511 \pm 0.002$
	C ₂	-46.511 ± 0.002	-136.543 ± 0.002
Radius of curvature of the runner (mm)	C ₁	$+86.56 \pm 0.1$	-51.50 ± 0.1
	C ₂	$+40.01 \pm 0.1$	$+132.54 \pm 0.1$
Radius of curvature of polisher (mm)	C ₁	-95.45 ± 0.1	+44.51
	C ₂	-48.51 ± 0.1	-138.54
Radius of curvature of the test plates (mm)	C ₁	-93.457 ± 0.001	$+46.511 \pm 0.001$
	C ₂	-46.511 ± 0.001	-136.543 ± 0.001
No. of lenses per block	C ₁	25	7
	C ₂	7	53

Table 4. The experimental parameters of the manufactured eyepiece.

Item		Single lens	Convex lens (of doublet)	Concave lens (of doublet)
Glass thickness (X mm)		6.2 ± 0.1	14.2 ± 0.1	23.0 ± 0.1
Square length (L mm)		35 ± 0.1	35 ± 0.1	
Disc diameter (D mm)		33 ± 0.1	33 ± 0.1	33 ± 0.1
Central thickness (d mm) during curve generation	C ₁	5.80 ± 0.1	13.80 ± 0.1	14.80 ± 0.1
	C ₂	5.50 ± 0.1	13.50 ± 0.1	10.60 ± 0.1
Angle θ° using cutter with d = 20.0 mm and r = 3.0 mm	C ₁	$9^\circ 29' \text{---}''$	$23^\circ 31' 10''$	$31^\circ 38' 50''$
	C ₂	$9^\circ 29' \text{---}''$	$23^\circ 31' 10''$	$18^\circ 40' 16''$
Radius of curvature of trueing and smoothing tools (mm)	C ₁	-57.693 ± 0.002	-22.058 ± 0.002	$+22.058 \pm 0.002$
	C ₂	-57.693 ± 0.002	-22.058 ± 0.002	$+34.236 \pm 0.002$
Radius of curvature of the runner (mm)	C ₁	$+51.890 \pm 0.100$	$+8.250 \pm 0.100$	36.050 ± 0.100
	C ₂	$+51.890 \pm 0.100$	$+8.250 \pm 0.100$	47.770 ± 0.100
Radius of curvature of polisher (mm)	C ₁	-59.690 ± 0.100	-24.050 ± 0.100	$+20.050 \pm 0.100$
	C ₂	-59.690 ± 0.100	-24.050 ± 0.100	$+32.230 \pm 0.100$
Radius of curvature of the test plates (mm)	C ₁	-57.693 ± 0.001	-22.058 ± 0.001	$+22.058 \pm 0.001$
	C ₂	-57.693 ± 0.001	-22.058 ± 0.001	$+34.236 \pm 0.001$
No. of lenses per block	C ₁	11	1	1
	C ₂	11	1	3

Table 5. The experimental results of the manufactured objective.

Test	Convex lens		Meniscus lens		
	Designed	Manu- factured	Designed	Manu- factured	
Radius of curvature in mm	r_1	+93.457	+93.461	- 46.511	- 46.508
	r_2	+46.511	+46.509	+136.543	+136.540
Diameter in mm	30.0	29.99	30.00	29.99	
Central thickness in mm	6.0	6.012	2.00	2.008	
Equivalent focal length in mm	+55.460	+55.451	-101.861	-101.868	
Centering error	—	5"	—	4"	
Surface quality	1	—	good	—	good
	2	—	good	—	good
Reading of test plate	1	—	2 fringes	—	3 fringes
	2	—	2 fringes	—	2 fringes

Table 6. The experimental results of the manufactured eyepieces.

Test	Single (convex)		Convex of doublet		Concave of doublet		
	Designed	Manufac.	Designed	Manufac.	Designed	Manufac.	
Radius of curvature in mm	r_1	+57.693	+57.698	+22.058	+22.061	-22.058	-22.060
	r_2	+57.693	+57.697	+22.058	+22.060	-34.236	-34.240
Diameter in mm	30.00	29.98	30.00	29.97	30.00	29.97	
Central thickness in mm	5.00	5.02	13.00	12.96	10.00	9.95	
Equivalent focal length in mm	+51.524	+51.518	+21.710	+21.695	-17.887	-17.806	
Centering error	—	4	—	2"	—	2	
Surface quality	1	—	good	—	good	—	good
	2	—	good	—	good	—	good
Reading of test plate	1	—	3 fringes	—	1.5 fringes	—	2 fringes
	2	—	3 fringes	—	1.5 fringes	—	2 fringes

Table 7. The spherical and chromatic aberrations data of the manufactured objective and eyepiece.

Lens \ Test	Spherical Aberration		Chromatic Aberration	
	Calculated	Measured	Calculated	Measured
Objective	0.0665	0.0600	0.005	0.005
Eyepiece	-0.02758	-0.02500	-0.02158	-0.0200

تصميم عدسات مصححة

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