

# Elevation-Angle Variation of LEO Satellites over the Kingdom of Saudi Arabia

## تغير زوايا إرتفاع الأقمار الاصطناعية منخفضة المدار الأرضي المتجولة فوق المملكة العربية السعودية

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**ABSTRACT:** A new and simple method to calculate the time variation of the elevation angle for LEO satellites is presented in this article. The method is useful to predict the link availability of LEO satellites due to propagation impairments. The method has been applied as a realistic study of LEO satellite flying over different regions of the Kingdom of Saudi Arabia (KSA). It is found that the percentage of time during which this satellite is visible (P%) over KSA decreases with the increase of the elevation angle ( $\theta$ ). The variation of P% with  $\theta$  could be modelled either by an exponential function, or simply by a decreasing power's law relation.

**Keywords:** LEO Satellite, elevation angle, Ka-band, KSA.

**المستخلص:** يعرض هذا المقال طريقة جديدة و بسيطة لحساب تغير زوايا ارتفاع الأقمار الاصطناعية منخفضة المدار الأرضي. وهذه الطريقة مفيدة للتنبؤ بمدى بقاء إشارات هذا النوع من الأقمار الاصطناعية عند محطة الاستقبال الأرضية. وقد تم تطبيق هذه الطريقة لحسابات واقعية للأقمار الاصطناعية منخفضة المدار الأرضي العاملة فوق المناطق الجغرافية المختلفة للمملكة العربية السعودية. وتم التوصل في هذا البحث إلى أن النسبة المئوية من الوقت-التي يمكن خلاله أن تكون هذه الأقمار الاصطناعية ظاهرة لمحطة الاستقبال الأرضية- تقل مع ازدياد زاوية ميل هذه الأقمار. ويمكن تمثيل تغير تلك النسبة المئوية مع تغير زاوية ميل هذه الأقمار الاصطناعية، إما بدالة أسية أو تمثيلها بدالة تناقص القدرة.

**كلمات مدخلية:** الأقمار الاصطناعية منخفضة المدار الأرضي، زوايا الإرتفاع، نطاق ما فوق "كيه"، المملكة العربية السعودية.

## INTRODUCTION

The International Mobile Telecommunications-2000 (IMT-2000) (ITU, 2006) provides a framework for third generation (3G) -and beyond - wireless systems, which consolidate diverse and

incompatible mobile environments into seamless radio and network infrastructure (cellular, cordless, satellite, terrestrial network and user terminals) and rigorous Quality of Service (QoS). The strategic aim is to deliver high-bit rate (2 Mb/s in phase 1 and up to 1 GB/s for the

4G mobile) multimedia, data, voice and portable internet (ITU, 2005) using common spectrum up to 2.9 GHz, with worldwide roaming capability and long range exposure using a small pocket-terminal. It will offer ubiquitous global user, terminal and service mobility.

The importance of IMT-2000 becomes more evident by the recent move to next generation network (NGN) and more ubiquitous wireless coverage, especially with the significant growth of the ICT sector over the past decade, with spectacular success of mobile sector and rise of broadband. This move is necessary in order to achieve information society (ITU, 2007-a). Detailed specifications of the radio interfaces at IMT-2000 have been approved by the Radio Communication Assembly (ITU, 2000) and recently by the ITU New Initiatives Programme (ITU, 2007-b). Nevertheless, true global service availability of IMT-2000 is not possible without a satellite component, which is an integral part of IMT-2000 to complement the terrestrial segment and to ensure that users have real global coverage.

Coverage is the primary advantage of Geo-Stationary earth Orbit (GSO) satellite when compared with Low Earth Orbit (LEO) system. However, the round-trip delay of GSO satellite is much higher than for LEO system. Higher altitude also makes GSO satellites more expensive to launch than LEO system. LEO system is normally designed with low link margin. As a result, signal quality may suffer from propagation impairments, which increases with a decrease in the elevation angle. Due to their low altitude, low earth orbit LEO satellites have the advantage of lower propagation delay, smaller antennas and lower power requirement if compared with the GSO satellites.

Atmospheric impairments, such as troposphere scintillation, gaseous and rain attenuation are higher for the low elevation angle paths. Calculation of the LEO link availability requires the determination of the time variation of the elevation angle, which is a complicated process due to the non-stationary characteristic of both the LEO satellites and the earth rotation. For LEO satellite paths, the minimum and maximum elevation angles are not good enough for link availability calculations. Moreover, the

percentage of time that the LEO satellite is visible as a function of elevation angle is required as well (Farserotu, 2000). One approach is to consider the elevation angle as a random variable and then try to find its probability density function or its associated cumulative distribution function.

Along this line, it has been shown that the elevation angle variation is characterized by a single probability distribution function for Iridium and Globalstar satellites using an empirical model (Crowe and, Raines, 1999). This model involves extensive numerical fitting effort. The distribution of elevation angle for the two systems is given in terms of parameters that are found by regression analysis and the user latitude.

The visibility-time function has been presented for Orbocomm LEO satellites (Ali, *et al.* 1999) using an expression relating the visibility window duration at a ground terminal as a function of the maximum elevation angle (Ali, *et al.* 1998). However, the claimed simplicity of the proposed algorithm was based on approximating the ground trace of the satellite by a great-circular arc and assuming minimum elevation angle of  $10^\circ$ . The approximation of great-circular arc is valid only when the LEO is in close vicinity to the earth station and assuming a constant angular velocity of the satellite.

A similar iterative approach was used by Radizk and Maral (1995) to determine in-view duration. Though, the effect of moving ground station, as a result of rotating earth, was ignored. This approximation may result in an error that is in excess of 10% on in-view computation for altitudes of more than 2200 km. To enhance the accuracy, the first approximations of fixed earth needs correction by further iterations.

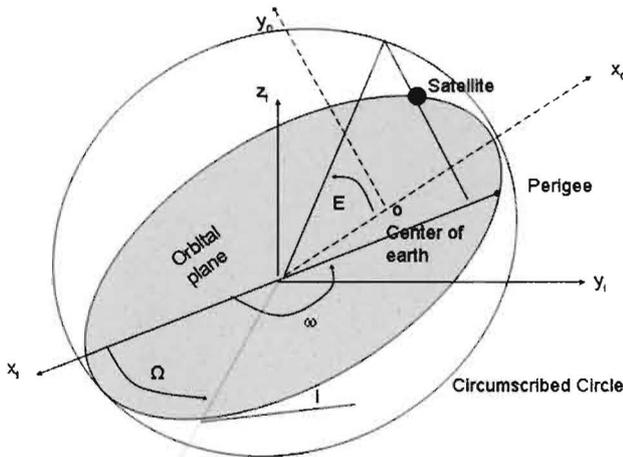
In this paper, a simple methodology is provided to calculate the percentage of time that a mobile LEO satellite is visible as a function of the elevation angle. The proposed procedure is simple and also avoids approximations and sources of errors applicable to analytical or empirical models described in the mentioned cited literature. The proposed procedure may be used to determine the effect of rain attenuation on the link availability of LEO satellites operating in the Ka-band over KSA. MATLAB language (The Math Works Inc.) is used to develop the software,

which can be run on any laptop or even handheld computers in the field, which makes it valuable for field engineers.

## METHODOLOGY

The proposed method of calculating the elevation angle of LEO Satellites requires the knowledge of the satellite constellation parameters. The orbit of the LEO satellites at certain time (t) in space is shown schematically in (Figure 1), and defined by the following six parameters (Pratt, *et al.* 2003):

- i inclination,
- $\Omega$  ascending node angle,
- e eccentricity,
- $\omega$  argument of Perigee,
- a semi major axis,
- M mean anomaly of angular velocity.



**Fig. 1.** LEO Satellite Parameters.

The radial coordinate  $r_0$  in the orbital plane is determined as:

$$r_0 = a(1 - e \cos E), \text{ where } E \text{ is the eccentric anomaly, which is related to } M \text{ as:}$$

$$M = E - e \sin E$$

$$\approx E \text{ for very small eccentricity (e)}$$

This approximation is normally accepted since for LEO satellite the orbit is very near to circular and  $e \leq 0.001$ . The Cartesian coordinates in its orbital plane are therefore:

$$x_0 = r_0 \cos v, y_0 = r_0 \sin v, z_0 = 0, \text{ where } v \text{ is the true anomaly and is related to } M \text{ as:}$$

$$v \approx M + [2e - (e^3/4)] \sin M$$

$$\text{and } r_0 = (R_e + H_s)(1 - e^2)/(1 + e \cos v).$$

where

E eccentric anomaly of angular velocity,

v true anomaly of angular velocity,

$H_s$  satellite altitude,

$H_r$  receiver altitude above sea level,

$R_e$  is the radius vector of the earth = 6378 km.

The transformation of the satellite position to Earth Centered Inertial (ECI) coordinate, which is illustrated in Figure (1), can be given as a composition of rotation about three axes, and thus can be represented by a  $3 \times 3$  matrix operating on a vector, defined as:

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = R_{eci} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$$

where the rotation matrix  $R_{eci}$  is given by:

$$R_{eci} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$a_{11} = \cos \omega \cos \Omega - \sin \omega \cos i \sin \Omega$$

$$a_{12} = -\sin \omega \cos \Omega - \cos \omega \cos i \sin \Omega$$

$$a_{13} = \sin \Omega \sin i$$

$$a_{21} = \cos \omega \sin \Omega + \sin \omega \cos i \cos \Omega$$

$$a_{22} = -\sin \omega \sin \Omega + \cos \omega \cos i \cos \Omega$$

$$a_{23} = -\cos \Omega \sin i$$

$$a_{31} = \sin \omega \sin i$$

$$a_{32} = \cos \omega \sin i$$

$$a_{33} = \cos i$$

Since the earth rotates in the x-y plane, the x-y coordinates of an observer on the earth's surface will vary with time. Therefore, more coordinate transformation is needed to determine the ECI position of the satellite relative to an observer on the rotating earth. To do this, the Vernal Equinox<sup>1</sup> ( $\gamma$ ) is introduced. The rotation angle between the Greenwich Meridian (zero longitude) and the Vernal Equinox ( $\gamma$ ), which is called the Greenwich Meridian Sidereal Time (GMST) is given by:

$$GMST = GMST_0 + W_e t$$

where

t is the time measured from midnight

$W_e = 7.292 \times 10^{-5}$  rad/s is the rate of rotation of

<sup>1</sup> Vernal Equinox is an imaginary point in space that lies along the line representing the intersection of the Earth's equatorial plane and the plane of the Earth's orbit around the Sun

the earth.

GMST<sub>0</sub> is GMST at midnight according to the Universal Time (UT1) of the start of day in question.

The value of GMST<sub>0</sub> can be obtained as (Pratt *et al.*, 2003)

$$GMST_0 = a + b + T_u + T_u^2 + d.T_u^3,$$

where

$$a = 24110^s.54841,$$

$$b = 8640184^s.812866,$$

$$c = 0^s.093104, d = 6.2 \times 10^{-6}.$$

T<sub>u</sub> = JD -du/36525 is the time elapsed measured in Julian centuries of 36525 days of universal time since Jan.1, 2000, 12<sup>h</sup>, where du = 451545.0 is the number of days of Universal Time since UT1 at noon time. JD is the Julian Date of the year plus the Julian Date of the calendar date in question, which can be obtained from reference (USGPO, 1988). Then, the transformation to rotating coordinate is defined by:

$$\begin{bmatrix} x_s \\ y_s \\ z_s \end{bmatrix} = R_r \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}$$

Where the rotation matrix R<sub>r</sub> is:

$$R_r = \begin{bmatrix} \cos(GMST) & \sin(GMST) & 0 \\ -\sin(GMST) & \cos(GMST) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The ECI position of an observer on earth station can be found as:

$$x_e = R_e \cos(L_e) \cdot \cos(\phi_e),$$

$$y_e = R_e \cos(L_e) \cdot \sin(\phi_e),$$

$$z_e = R_e \sin(L_e),$$

Where (L<sub>e</sub>) is the latitude and (φ<sub>e</sub>) is the longitude of the earth station respectively.

Then the elevation angle θ is given:

$$\theta = \sin^{-1} r.R_e / r R_e$$

Where r is the range vector between the satellite and earth station with norm r given by:

$$r = \sqrt{(x_s - x_e)^2 + (y_s - y_e)^2 + (z_s - z_e)^2}$$

The proposed method for calculating the elevation angle for a single Iridium satellite, as an

example, could be applied for the others with their corresponding TLE parameters (CSSI, 2007).

## RESULTS

Figure (2) shows the variation of the elevation angle (θ) of an Iridium satellite (ID8) with time (t) of the Julian day 00163 at midnight passing over Riyadh, KSA. This satellite at this time has the following TLE parameters: i = 86.4001°, Ω=35.5620°, e= 0.0002503, ω= 81.5759°, M = 278.5676°, revolution/day = 1434. Figure (3) shows the results of calculating the percent of time (P %), at which this satellite is visible as a function of elevation angle (θ) over Riyadh, which can be represented by a decreasing power relation as:

$$P\% = 122.1\theta^{-0.81}$$

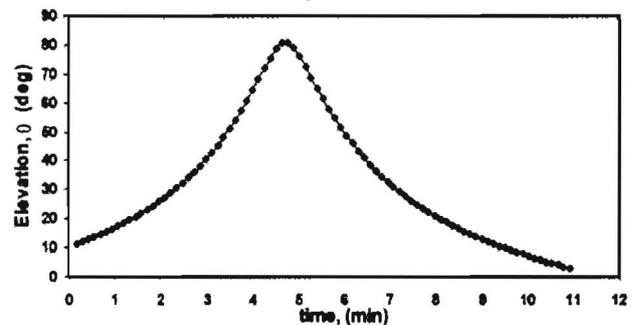


Fig. 2. Time-variation of Iridium LEO Satellite over Riyadh, KSA.

Alternatively, the distribution of P% of elevation angle (θ) can be described by exponential function as:

$$P\%(\theta) = \frac{1}{7.65} \exp(-(\theta-2.89)/30.5).$$

A similar exponential behaviour for the Iridium path elevation angle between the equator and 60° latitude has been cited (Crowe, and Raines, 1999).

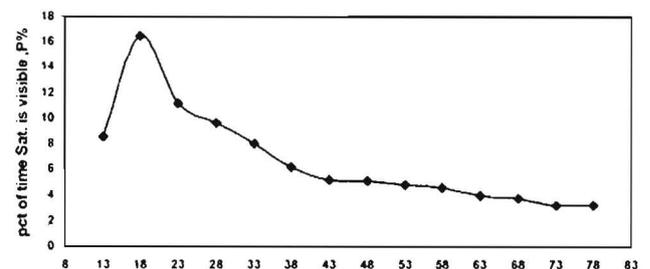


Fig. 3. Percent of Time Iridium is Visible over Riyadh, KSA.

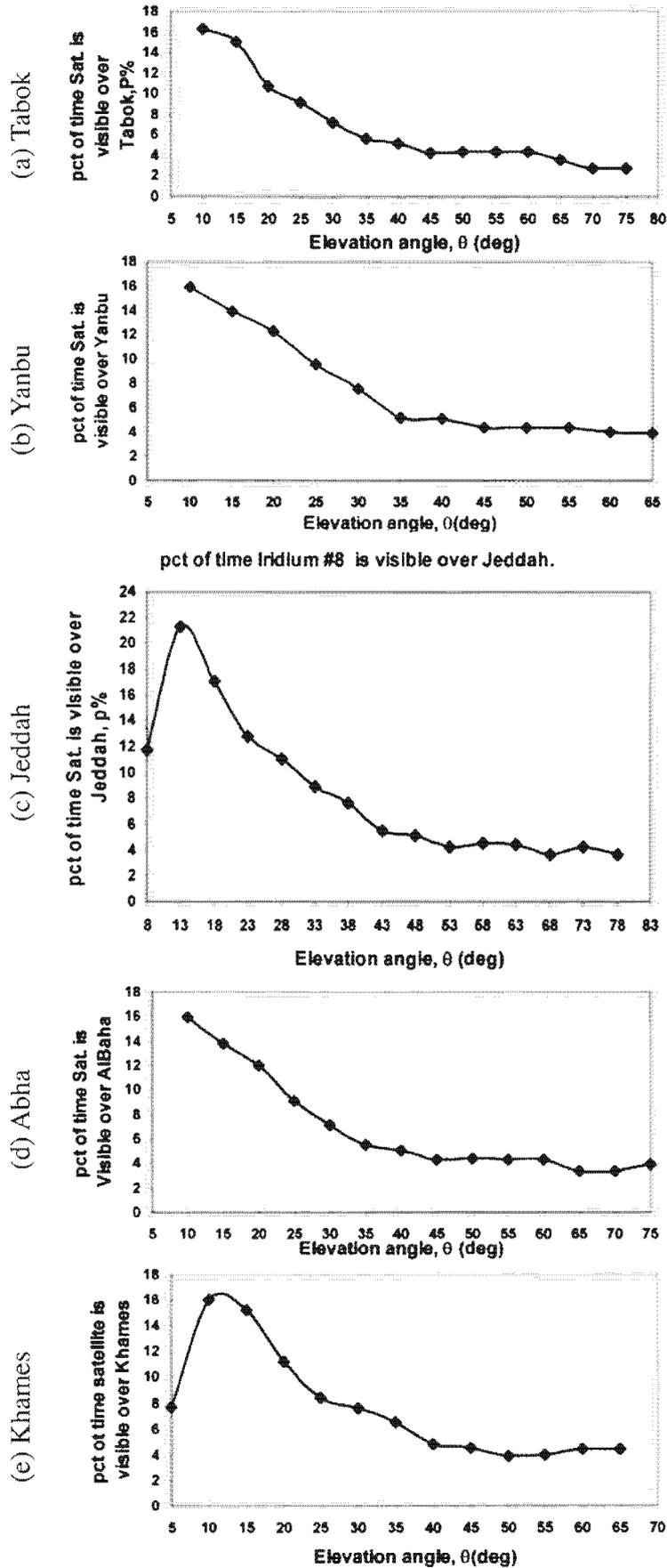


Fig. 4. Percent of Time Iridium #8 is Visible over Jeddah, Abaha, Yanbo, Tabok and Khames Meshait Regions.

The general decrease of P% with the increase of  $\theta$  may be attributed to the behaviour of the rise of the LEO satellite over the horizon with a very small elevation angle  $\theta$ , and the elevation angle increases as the LEO satellite continues to move until it becomes almost vertical over the earth station, then  $\theta$  starts to decrease until the LEO satellite sets beyond the horizon. Therefore the time at which the satellite is visible to the earth station is larger at small  $\theta$ . The limited increase of P% with the increase of  $\theta$  within small range in some regions of KSA maybe attributed to the unsymmetrical variation of change of the elevation angle with time due to the latitude and longitude of the earth station relative to the orbit plane of the LEO satellite.

Each region is represented by a city located within that region as shown in Table (1). Table (1) also summarizes the maximum elevation angle and the best fit of time-variation of Iridium LEO Satellite, where  $R^2$  is the square of the correlation coefficient. The minimum elevation angles  $\theta_{\min}$  is more than zero and ranges from  $5^\circ$  to  $13^\circ$  according to the geographical region of the earth station, which is in agreement with cited literatures. Operating at elevation angle lower than  $\theta_{\min}$  increases atmospheric attenuation and will be practically unachievable due to the blocking of the line of sight by terrestrial objects such as buildings, trees, and higher white noise.

## CONCLUSIONS

Calculated rain attenuation for LEO links depends heavily on the accuracy of the rain rate, and the determination of the time variation of the elevation angle. A new method to calculate the time variation of the elevation angle for a LEO satellite is used to provide a realistic study of the rain attenuation on LEO satellite links working in different regions of KSA and operating in the Ka-band.

Generally, the percentage of time at which the LEO satellite is visible decreases as the elevation angle increases. The variation of the percentage of time - at which the LEO satellite is visible over KSA- with elevation angle, can be represented by a simple decreasing power law.

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**Table 1.** Parameters of the Selected Gateway Locations and Best Fit of the Variation of Percent of Time, Iridium is Visible, with Elevation Angle.

City	Latitude	Longitude	Hr	Angle $\theta^\circ$ of Maximum	Relation of Best Fit of	Correlation Coefficient
	(Degree)	(Degree)	(km)	P% (Degree)	P% (viz $\theta$ )	( $R^2$ )
Tabok	28.5	36.5	1.1	10	$171.43 \theta^{-0.9452}$	0.9639
Riyadh	24.50	36.5	0.6	18	$122.10 \theta^{-0.81}$	0.8230
Yanbo	23.75	37.5	0.4	10	$139.5 \theta^{-0.8755}$	0.949
Jeddah	21.00	39.0	0.1	10	$336.980 \theta^{-1.0541}$	0.9718
Albaha	19.50	41.0	1.1	10	$123.18 \theta^{-0.8395}$	0.94
Khames	17.75	42.0	1.7	13	$132.09 \theta^{-0.8571}$	0.9421

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