

## Evidence for the "Rotational Phase Change" in $^{158}\text{Er}$ and $^{156}\text{Dy}$

M.I. El-Zaiki and H.O. Nafie

Physics Department, Faculty of Science (Banha), Banha, Egypt

ABSTRACT. A drastic change in the moment of inertia as a function of the rotational frequency is observed in the band crossing region for the deformed nuclei  $^{158}\text{Er}$  and  $^{156}\text{Dy}$  ( $N=90$ ). These changes may be interpreted as a phase transition from a super fluid state to a non  $\mathcal{J} / \hbar^2$  super fluid state.

In the present work the  $(2\mathcal{J} / \hbar^2)$  values for the yrast sequences in  $^{158}\text{Er}$  and  $^{156}\text{Dy}$  were calculated from lifetime measurements which were recently reported. The two nuclei investigated here have neutron number  $N = 90$  which lies close to the onset of prolate deformation taking place around  $N=88$  in this mass region.

From previous reported data, we noticed that the collectively drastic effect are due to the alignment of individual nucleon pairs. The calculated values of  $(2\mathcal{J} / \hbar^2)$  in the present work results in a quite systematic trend of the collective behaviour of these nuclei at high frequencies and give strong evidence for phase changes.

### Method of Calculation

For an axially symmetric rotor the rotational frequency and the moment of inertia can be defined according to (Harris 1965) as:

$$\hbar\omega = dE / d\sqrt{I(I+1)} \approx \frac{1}{2} \Delta E_{\gamma} (I \rightarrow I-2) \quad (1)$$

and

$$2\mathcal{J} / \hbar^2 = (dE / dI(I+1))^{-1}$$

The derivative is calculated approximately from the observed energies as:

$$\frac{dE}{dI(I+1)} = \frac{E(I) - E(I-2)}{4I - 2} \quad (2)$$

### Results and Discussions

#### (a) $^{158}_{68}\text{Er}$

Recently, Simpson *et al.* (1984) reported the observation of the rotational side bands in the N=90 and 91 nuclei  $^{158}\text{Er}$  and  $^{159}\text{Er}$  and compared the experimental features with the prediction of the cranked shell model. In  $^{158}\text{Er}$ , the yrast band has been observed up to  $I^\pi = 26^+$  and the ground state band has been observed to continue after the first backbend in the yrast sequence to  $I^\pi=20^+$ . Excitation energies and  $\gamma$  -ray transition energies are given in Table 1. Ward *et al.* (1973) reported the mean lifetimes for the  $14^+$ ,  $16^+$  and  $18^+$  levels, which are  $3.0+0.7$ ,  $2.5+0.9$  and  $< 2.2$  psec. respectively. These results are consistent with the

Table 1.  $E_x$ ,  $E_\gamma$  and  $(2I/h^2)$  values for the g- and s-bands in  $^{158}\text{Er}$ .

I	$E_x$ (MeV)	$E_\gamma$ (MeV)	$\hbar\omega$ (MeV)	$(\hbar\omega)^2$ (MeV) <sup>2</sup>	$(2I/h^2)$ (MeV) <sup>-1</sup>
<b>g-band</b>					
2 <sup>+</sup>	0.19218	0.19218	0.09609	0.0092	31.20
4 <sup>+</sup>	0.52718	0.335	0.1675	0.0280	41.79
6 <sup>+</sup>	0.97027	0.4431	0.2215	0.0491	49.65
8 <sup>+</sup>	0.49317	0.5229	0.2615	0.0683	57.37
10 <sup>+</sup>	2.07257	0.5794	0.2897	0.0839	65.58
12 <sup>+</sup>	2.68077	0.6082	0.3041	0.0924	75.63
14 <sup>+</sup>	3.37427	0.6935	0.34675	0.1202	77.86
16 <sup>+</sup>	4.02607	0.6518	0.3259	0.1062	95.04
18 <sup>+</sup>	4.67337	0.6479	0.3240	0.1049	108.04
20 <sup>+</sup>	5.37337	0.6994	0.3497	0.1223	111.52
<b>s-band</b>					
12 <sup>+</sup>	2.899				
14 <sup>+</sup>	3.1886	0.3087	0.1544	0.0238	174.90
16 <sup>+</sup>	3.6612	0.4726	0.2363	0.0558	131.18
18 <sup>+</sup>	4.227	0.566	0.283	0.0800	123.67
20 <sup>+</sup>	4.885	0.6584	0.3292	0.1084	118.47
22 <sup>+</sup>	5.625	0.7401	0.3700	0.1369	116.20
24 <sup>+</sup>	6.431	0.8057	0.4029	0.1623	116.67
26 <sup>+</sup>	7.275	0.8445	0.4223	0.1783	120.78

rotational model in spite of the severe backbending of the level spacings. The energies reported (Ward *et al.* 1973) for the transitions  $14^+ \rightarrow 12^+$ ,  $16^+ \rightarrow 14^+$  and  $18^+ \rightarrow 16^+$  are 0.510, 0.4732 and 0.567 MeV, respectively. However, these energy values as has been shown lately (Simpson *et al.* 1984) belong to the transitions  $14_g^+ \rightarrow 12_g^+$ ,  $16_s^+ \rightarrow 14_s^+$ , and  $18_s^+ \rightarrow 16_s^+$ , respectively. In the present work, we investigate the behaviour of the rotational bands (g- and s-) in  $^{158}\text{Er}$  at higher spin values.

Figure (1-a) shows the crossing of the g- and s-bands plotting the experimental lab energy  $E$  versus  $I$  for  $^{158}\text{Er}$ . As indicated by the broken line, the yrast sequence switches at  $I^\pi \approx 12^+ \hbar \sim 14^+ \hbar$  from the g- to the s-band. It is seen from the figure that the regularity of the points assigned to the g-band is broken when the  $14^+ \rightarrow 12^+$ ,  $16^+ \rightarrow 14^+$  and  $18^+ \rightarrow 16^+$  transitions are reached. This effect is also exhibited in Fig. (1-c). Moreover, the interband transitions between members of the s- and g-bands  $12_s^+ \rightarrow 10_g^+$ ,  $14_g^+ \rightarrow 12_s^+$ , and  $14_s^+ \rightarrow 12_g^+$  are shown -dashed lines-in Fig. (1-a).

Figure (1-b) displays the angular momentum  $I$  as a function of the rotational frequency  $\omega$ . Compared to the g-band, the s-states additionally carry an amount of aligned angular momentum which causes the backbending of the yrast sequence. The sharp discontinuity shown in the moment of inertia plot (1-c), reflects that the interaction between both bands is assumed to be very weak. This figure shows that,

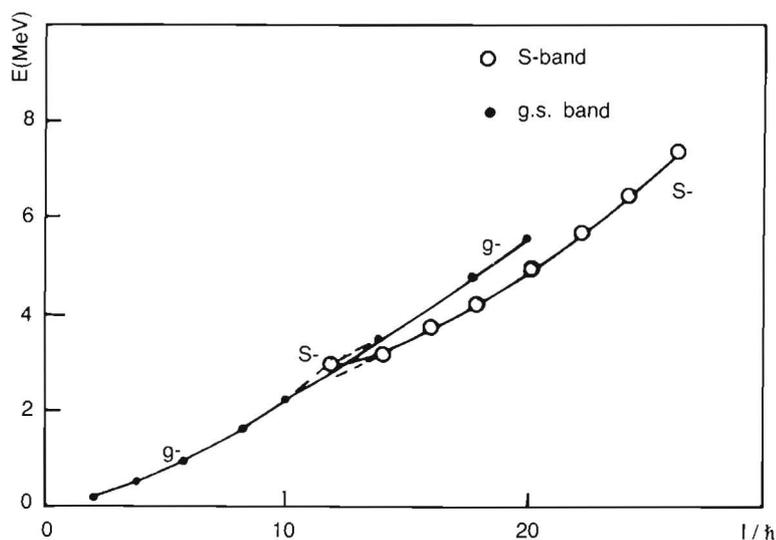


Fig. (1-a). Plot of excitation energy  $E_x$  as function of the angular momentum  $I$  in  $^{168}\text{Er}$

for the g-band, the moment of inertia increases smoothly up to  $I^\pi = 12^+\hbar$ . It shows irregularities in the crossing region,  $I \approx 12^+\hbar \sim 14^+\hbar$  and for sufficiently large spins it approaches the rigid rotor value. In the s-band, the decrease of the moment of inertia is more pronounced in the crossing region, then it changes very slowly over the considered states.

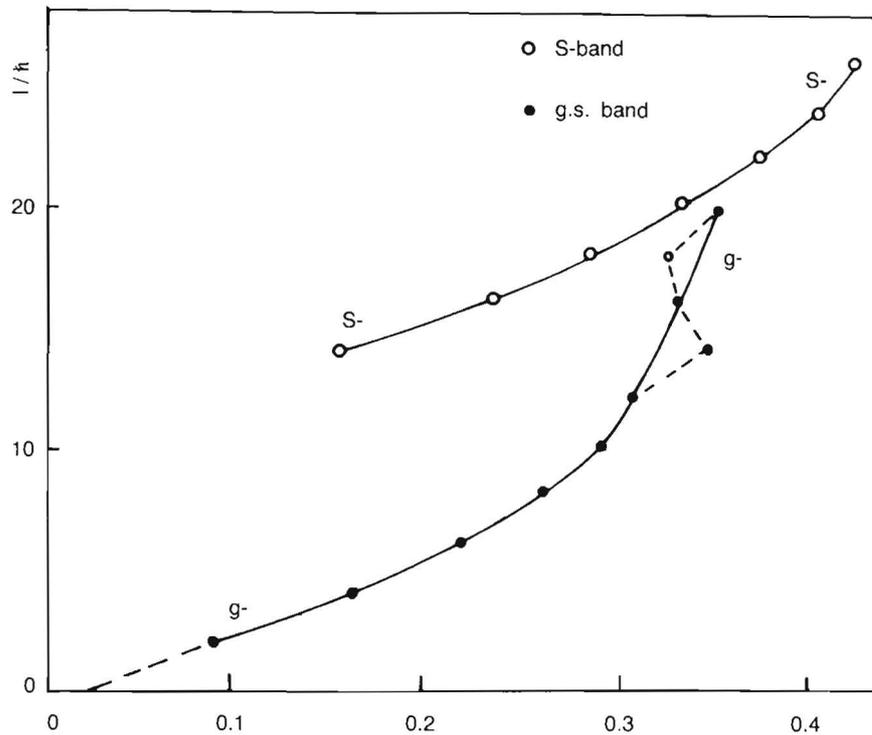


Fig. (1-b). The angular momentum  $I$  as function of the rotational frequency for  $^{168}\text{Er}$

The drastic change in the moment of inertia can be interpreted as due to a phase transition between a super fluid state and a non-super fluid state (Deleplanque *et al.* 1983), and the value of the moment of inertia is likely to be somewhat below the rigid rotor value after the first transition. The increase of the moment of inertia values as shown in Fig. (1-c) occurring at  $(\hbar\omega)^2 = 0.17829 \text{ MeV}^2$ ,  $\hbar\omega = 0.4223 \text{ MeV}$ , corresponding to spin value  $I^\pi = 26^+$  supports the idea of a

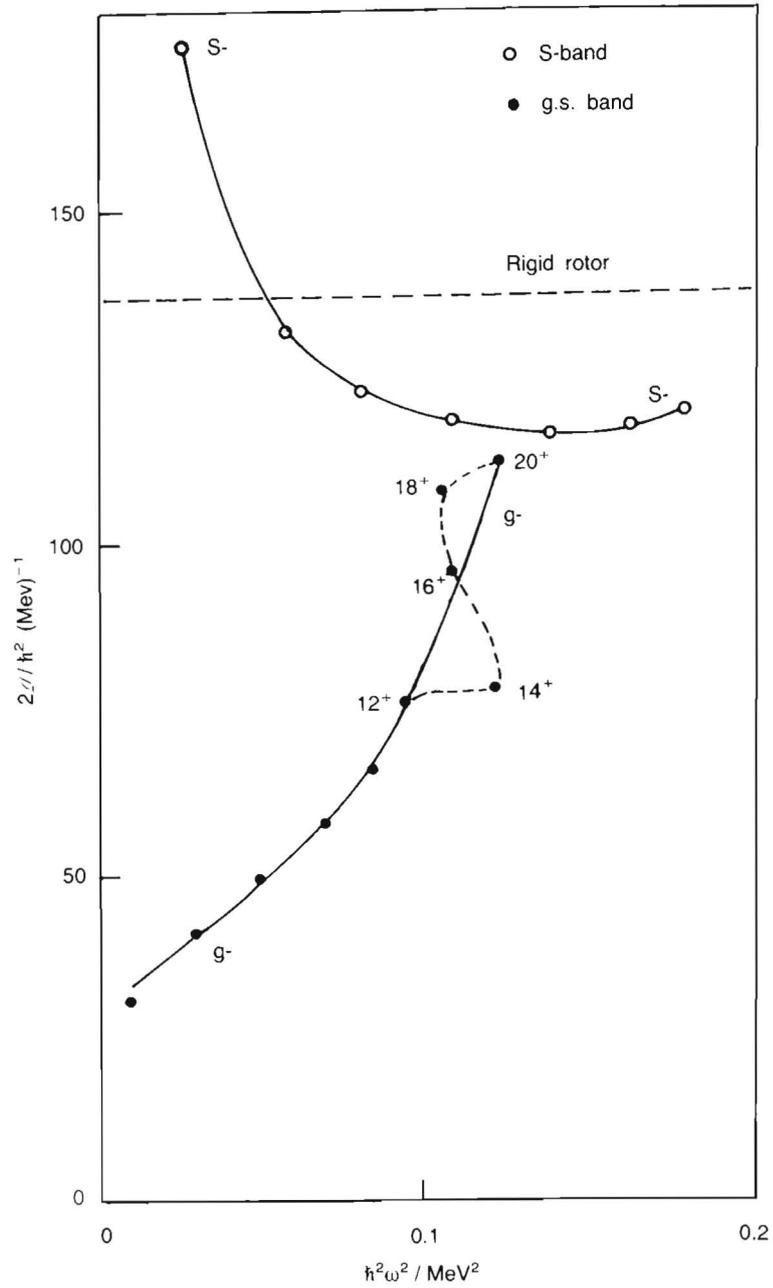


Fig. (1-c). The momentum of inertia for  $^{168}\text{Er}$  as a function of the rotational frequency. The dashed curve represents the calculated value of the moment of inertia for a rigid rotor

second band crossing between s- and g-bands (Brude *et al.* 1982) and (Holtzmann *et al.* 1985).

(b)  $^{156}_{66}\text{Dy}$

The energy level structure of  $^{156}\text{Dy}$  has been investigated in many experiments (Lieder *et al.* 1974, Andrews *et al.* 1974, El-Masri *et al.* 1976, Ward *et al.* 1979 and Emling *et al.* 1984) up to spin  $30^+$  and  $26^+$  in the yrast sequence and ground state band, respectively. In the present work, we investigate the behaviour of the g- and s-bands and deduce qualitative information concerning the band crossing. Excitation energies and  $\gamma$ -ray transition energies are given in Table 2.

Table 2.  $E_x$ ,  $E_\gamma$  and  $(2\mathcal{I}/\hbar^2)$  values for the g- and s-bands in  $^{156}\text{Dy}$ .

I	$E_x$ (MeV)	$E_\gamma$ (MeV)	$\hbar\omega$ (MeV)	$(\hbar\omega)^2$ (MeV) <sup>2</sup>	$(2\mathcal{I}/\hbar^2)$ (MeV) <sup>-1</sup>
<b>g-band</b>					
2 <sup>+</sup>	0.1378	0.1378	0.0689	0.0047	43.54
4 <sup>+</sup>	0.4042	0.2664	0.1332	0.0177	52.55
6 <sup>+</sup>	0.7704	0.3662	0.1831	0.0335	60.07
8 <sup>+</sup>	1.2167	0.4453	0.2226	0.0496	67.37
10 <sup>+</sup>	1.7257	0.5090	0.2545	0.0647	74.65
12 <sup>+</sup>	2.2866	0.5609	0.2804	0.0786	82.01
14 <sup>+</sup>	2.8886	0.6020	0.3010	0.0906	89.70
16 <sup>+</sup>	3.5223	0.6357	0.3179	0.1010	97.53
18 <sup>+</sup>	4.1777	0.6557	0.3277	0.1073	106.80
20 <sup>+</sup>	4.8581	0.6804	0.3402	0.1157	114.64
22 <sup>+</sup>	5.5731	0.7150	0.3575	0.1278	120.28
24 <sup>+</sup>	6.3293	0.7562	0.3781	0.1429	124.31
<b>s-band</b>					
14 <sup>+</sup>	3.066	0.3588	0.1794	0.0321	150.50
16 <sup>+</sup>	3.4985	0.4325	0.2163	0.0467	143.35
18 <sup>+</sup>	4.0255	0.5270	0.2635	0.0694	132.83
20 <sup>+</sup>	4.6353	0.6098	0.3049	0.0929	127.91
22 <sup>+</sup>	5.3195	0.6842	0.3421	0.1170	125.69
24 <sup>+</sup>	6.0695	0.7500	0.3750	0.1406	125.33
26 <sup>+</sup>	6.8773	0.8078	0.4039	0.1631	126.27
28 <sup>+</sup>	7.7387	0.8614	0.4307	0.1855	127.70
30 <sup>+</sup>	8.6510	0.9123	0.4562	0.2081	129.34

Figure (2-a) shows the experimental lab energy  $E$  versus  $I$  for  $^{156}\text{Dy}$ . As seen, the yrast sequence switches at  $I^\pi \approx 14^+\hbar - 16^+\hbar$  from the g- to s-band. The interband transitions between members of the two bands, *i.e.*  $16^+_g \rightarrow 14^+_g$ ,  $14^+_g \rightarrow$

$12_g^+$ , and  $16_g^+ \rightarrow 14_s^+$  are not shown in Fig. (2-a). These transitions were assumed to be absent because the corresponding transition energies were not observed. Figure (2-b) displays the smooth dependence of the angular momentum of both bands on the rotational frequency. Only one distinct discontinuity occurs around  $\hbar\omega \approx 0.3$  MeV which is interpreted as being due to the intersection of the g-band with the weakly interacting s-band built on a rotational aligned  $i_{13/2}$  neutron pair (Diebel *et al.* 1980).

To confirm this behaviour in a more quantitative way, the  $(2I/\hbar^2)$  values corresponding to the yrast states of  $^{156}\text{Dy}$  are calculated and given in Table 2. Figure (2-c) shows the discontinuity, which is sharp and reflects that the interaction between the g-band and the s-band is assumed to be very weak. In the g-band the moment of inertia increases smoothly up to  $I^\pi \approx 14^+\hbar \sim 16^+\hbar$ , then continues slowly to approach the rigid rotor value. For the s-band the decrease of the moment of inertia is more pronounced in the crossing region  $14^+\hbar - 16^+\hbar$ , then changes very slowly over the considered states. The increase of the moment of inertia again around  $(\hbar\omega)^2 = 0.1855 \text{ MeV}^2$ ,  $\hbar\omega = 0.4307 \text{ MeV}$  corresponding to  $I^\pi = 28^+\hbar$  is in agreement with the cranked Hartree-Bogoliubov model as well as cranked shell model calculations (Faessler *et al.* 1978). Moreover, these calcula-

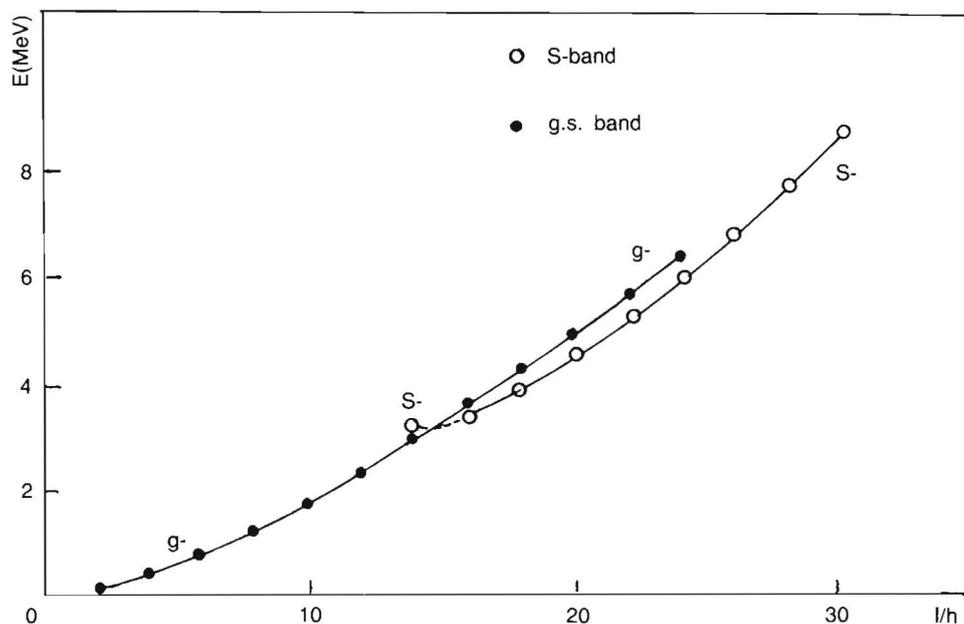


Fig. (2-a). Plot of excitation energy  $E_x$  as function of the angular momentum  $I$  in  $^{156}\text{Dy}$

tions predict that the second discontinuity should be smeared out due to a very strong interaction between the crossing two and four quasi-particle bands. If one interprets the curve in Fig. (2-c) as an indication of the nucleus undergoing a phase change, then the states  $I^\pi = 16^+\hbar$  (and up) should belong to the normal phase whereas the states  $I^\pi = 14^+\hbar$  (and down) should belong to the super fluid phase.

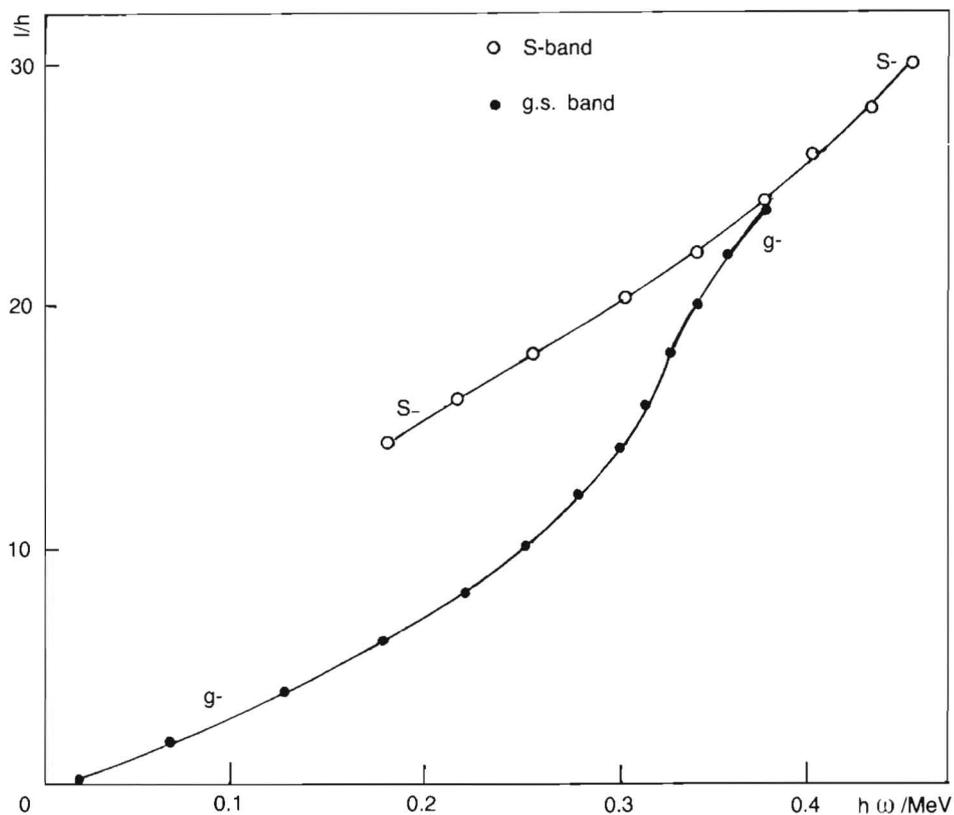


Fig. (2-b). The angular momentum  $I$  as function of the rotational frequency for  $^{166}\text{Dy}$

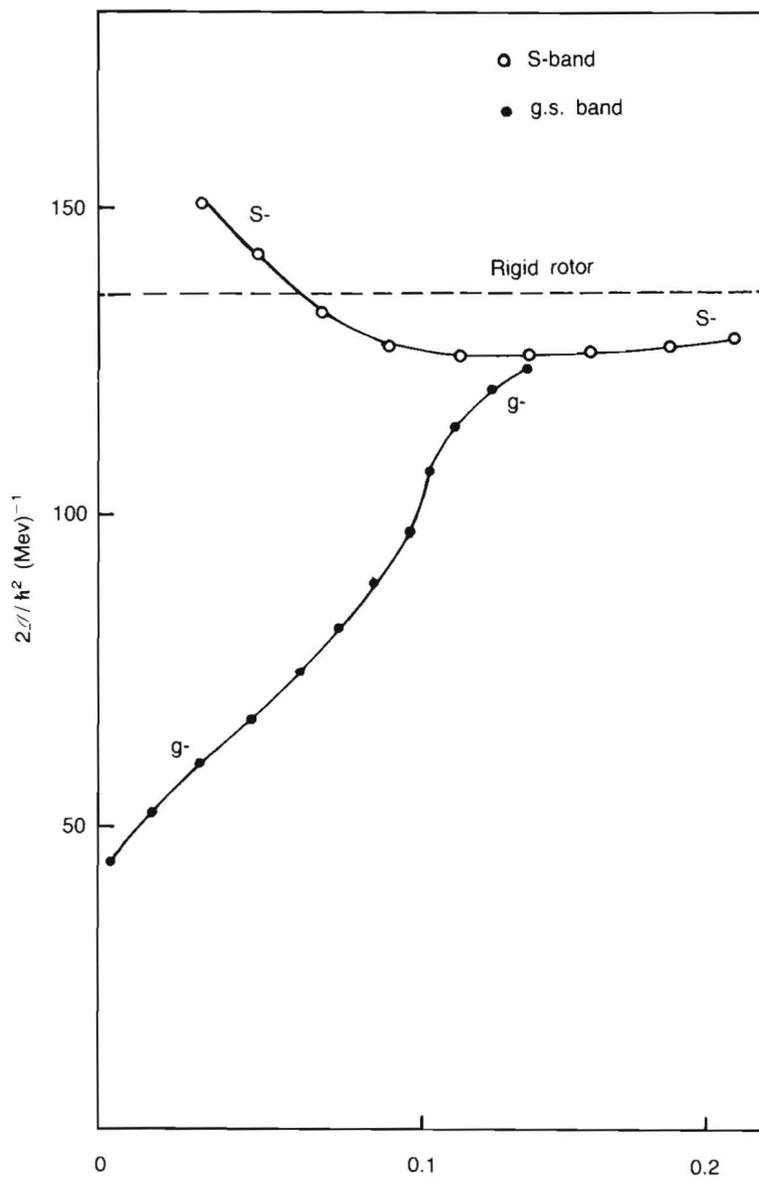


Fig. (2-c). The moment of inertia for  $^{166}\text{Dy}$  as a function of the rotational frequency. The dashed curve represents the calculated value of the moment of inertia for a rigid rotor

## References

- Andrews, H.R., Ward, D., Graham, R.L. and Gieger, J.S. (1974) High spin states in the ground and  $\beta$ -bands of  $^{156}\text{Dy}$  and the band crossing interpretation of backbending, *Nucl. Phys.* **A219**: 141-56.
- Burde, J., Dines, E.L., Shih, S., Diamond, R.M., Draper, J.E., Lindenberg, K.H., Schück, C., and Stephens, F.S. (1982) Third discontinuity in the yrast levels of  $^{158}\text{Er}$ , *Phys. Rev. Lett.* **48**: 530-33.
- Deleplanque, M.A., Macchiavelli, A.O., Diamond, R.M., Stephens, F.S., Dines, E.L. and Draper, J.E. (1983) Shell effect at high angular momentum, *Phys. Rev. Lett.* **51**: 1854-57.
- Dieble, M., Mantri, A.N. and Mosel, U. (1980) Energies and electromagnetic moments of deformed nuclei in the rare - earth region, *Nucl. Phys.* **A345**: 72-92.
- El-Masri, Y., Ferte, J.M., Janssens, R., Michel, C., Steyaert, J. and Vervier, J. (1976) Energy levels in the nucleus  $^{156}\text{Dy}$  through the  $^{159}\text{Tb}$  (p,4n  $\gamma$ )  $^{156}\text{Dy}$  reaction, *Nucl. Phys.* **A271**: 135-49.
- Emling, H., Grosse, E., Kulesa, R., Schwalm, D. and Wollersheim, H.J. (1984) Rotation-induced shape transitions in Dy nuclei, *Nucl. Phys.* **A419**: 187-220.
- Faessler, A. and Ploszajczak, M. (1978) Anomaly of the moment of inertia in  $^{158}\text{Er}$  above the backbending region, *Phys. Lett.* **76B**: 1-4.
- Harris, S.M. (1965) High order corrections to the cranking model, *Phys. Rev.* **138B**: 509-513.
- Holzmann, R., Loiselet, M., Van Hove, M.A., and Vervier, J. (1985) High spin states in  $^{157,158}\text{Er}$  and  $^{158,159}\text{Tm}$ , *Phys. Rev.* **C31**: 421-39.
- Lieder, R.M., Beuscher, H., Davidson, W.F., Neskakis, A., Mayer-Boricke, C., El-Masri, Y., Monson, P., Steyaert, J. and Vervier, J. (1974) Twin backbending in  $^{156}\text{Dy}$ , *Phys. Lett.* **49B**: 161-64.
- Simpson, J., Butler, P.A., Forsyth, P.D., Sharpyschafer, J.F., Garrett, J.D., Hagemann, G.B., Herskind, B. and Ekstrom, L.P. (1984) Rotational alignment in  $^{158}\text{Er}$  and  $^{159}\text{Er}$ , *J. Phys. G.; Nucl. Phys.* **10**: 383-416.
- Ward, D., Andrews, H.R., Gieger, J.S., Graham, R.L. and Sharpyschafer, J.F. (1973) Life time for levels above the "Rotational Phase Change" in  $^{158}\text{Er}$ , *Phys. Rev. Lett.* **30**: 493-96.
- Ward, D., Andrews, H.R., Hausser, O., El-Masri, Y., Aleonar, M.M., Lee, I.Y., Diamond, R.M. and Stephens, F.S. (1979) Measurement of lifetimes and feeding times for high spin states in  $^{156}\text{Dy}$ , *Nucl. Phys.* **A332**: 433-447.

(Received 16/12/1985;  
in revised form 29/06/1986)

## دلالة تغير الطور الدوراني في نواتي الأوربيوم<sup>(١٥٨)</sup> والديسبروسيوم<sup>(١٥٦)</sup>

مصطفى إبراهيم الزعقي و حسن عمر نافع

قسم الطبيعة - كلية العلوم - بنها - مصر

درست التغيرات المفاجئة والحدية لعزم القصور الذاتي كدالة للتردد الدوراني عند منطقة عبور الشرائط الدورانية في النواتين المشوهتين أوربيوم<sup>(١٥٨)</sup> وديسبروسيوم<sup>(١٥٦)</sup>. ان هذه التغيرات يمكن تفسيرها كإنتقال طوري من حالة فائقة الميوعة إلى حالة غير فائقة الميوعة.