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Depth Estimation of Three-dimensional Salt Structures from Second Derivative Gravity Anomalies

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ABSTRACT. This paper emphasizes the basic ideas behind the usefulness of the least-squares method in determining the optimum depth to a buried spherical body from second vertical derivative maps by finding a solution of a non-linear equation in the form of f(z) = O. The derived formulae can be directly computerized.

The depth to the center of the Humble Salt Dome, estimated by our present approach, is found to be comparable with other literatures previously published.

Salt domes are of special interest in gravity prospecting and a reliable estimation of the depth to such buried salt structures from gravity anomaly maps is always considered as one of the primary concerns of explorationists. However, computing such depths from second vertical derivative maps by transforming the problem into a problem of finding a solution of a non-linear equation of the form f(z) = O, is considered more objective than any other estimating technique (Abdelrahman *et al.* 1985), particularly when we consider the equation:

$$g_{zz}(x,y,z) = C\,H(x,y,z) \tag{1}$$

as being convenient definition for the second vertical derivative of gravity due to a spherical body (3-D), where

$$H(x,y,z) = z(2z^2 - 3x^2 - 3y^2)/(x^2 + y^2 + z^2)^{7/2},\ C = 4\,\pi G\,\sigma R^3 \tag{2}$$

and where G denotes the universal gravitational constant, σ is the density contrast, R is the radius (Fig. 1). This simple geometrical form gives useful approximation to some geological structures of interest in petroleum exploration (Hammer 1974) and particularly to salt domes (Nettleton 1976).

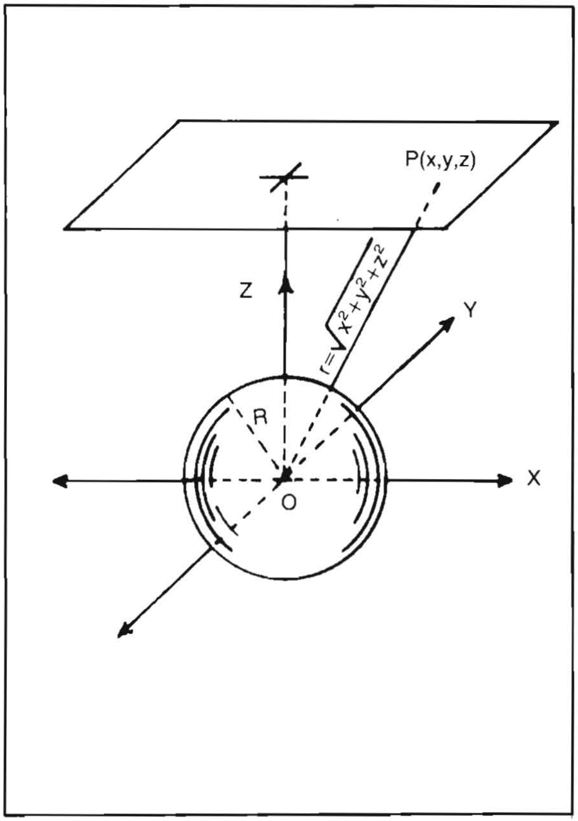

Fig. 1. A diagram for a three-dimensional spherical structure.

Formulation of the Problem

In such case, it is obvious that $g_{zz}(x,y,z)$ attains its maximum at x=y=O, with maximum value being defined as:

$$g_{zz}max = 2C/z^4 \qquad , x=y=O \tag{3}$$

for a fixed value of z.

In view of equation (3), equation (1) can be rewritten as:

$$g_{zz}(x,y,z) = g_{zz}max\, H_1(x,y,z) \tag{4}$$

where

$$H_1(x,y,z) = z^4H(x,y,z)/2 \tag{5}$$

Also, from equation (4), the following equation can be obtained at $x=x_i$, $y=y_j$, $i=1,2,3,..,N$ and $j=1,2,3,...,M$

$$g_{zz}(x_i,y_j,z) = g_{zz}max\, H_1(x_i,y_j,z) \tag{6}$$

where (x_i,y_j) are the discrete points on the ground surface at which the $g_{zz}(x_i,y_j,z)$ is numerically obtained using second derivative methods such as those given by Elkins (1951), Rosenbach (1953), Paul (1961) and Agarwal and Lal (1971), from the Bouguer anomaly values.

The unknown z in equation (6) can be obtained by minimizing

$$\varphi(z) = \sum_{i=1}^{N} \sum_{i=1}^{M} (g_{zz}(x_i,y_j,z) - g_{zz}max\, H_1(x_i,y_j,z))^2 \tag{7}$$

with respect to z in a least-squares sense (see Gupta 1983). Minimization of $\varphi(z)$ in the least-squares sense, *i.e.*, $(d/dz\ \varphi(z) = O$, leads to the following equation

$$f(z) = \sum_{i=1}^{N} \sum_{i=1}^{M} (g_{zz}(x_i,y_j,z) - g_{zz}max\, H_1(x_i,y_j,z))(H^*_1(x_i,y_j,z) = O \tag{8}$$

where

$H^*_1(x_i,y_j,z) = (d/dz)\ H_1(x_i,y_j,z)$ and is given as:

$$H^*_1(x_i,y_j,z) = \frac{5z^4\,(4x_i^2z^2 + 4y_j^2z^2 - 3x_i^4 - 6x_i^2y_j^2 - 3y_j^4)}{2\,(x_i^2 + y_j^2 + z^2)^{9/2}} \tag{9}$$

In fact, there are many standard approaches to solve equation (8) having the form $f(z) = O$. Here, it is first transformed into an equation of the form $z = f(z)$, and then solved by a simple iterative method (Demidovich 1973). It was found that equation (8) gives the exact value of z when using synthetic second derivative maps. It is also found that only a few points around $g_{zz}max$ on the map can be considered sufficient to get the exact value of z. However, the data with random errors require more points around $g_{zz}max$. It is evident that the advantages of the present method over the profile graphical techniques (Elkins 1951 and Pick *et al.* 1973) lie with the facts that: (1) all the observed values can be used, and hence the optimum depth, and (2) the interpretation procedure can be computerized.

Field Example

The present technique has been applied to the gravity anomaly area over the Humble Salt Dome Houston, USA, which is shown in Fig. 2 (Nettleton 1976, Fig. 8.16) after being digitized (400 km^2, forms an 21×21 square grid pattern with grid separation of 1 km) and used for the construction of the second vertical derivative map shown in Fig. 3 by Agarwal and Lal method (1971) using equation (23).

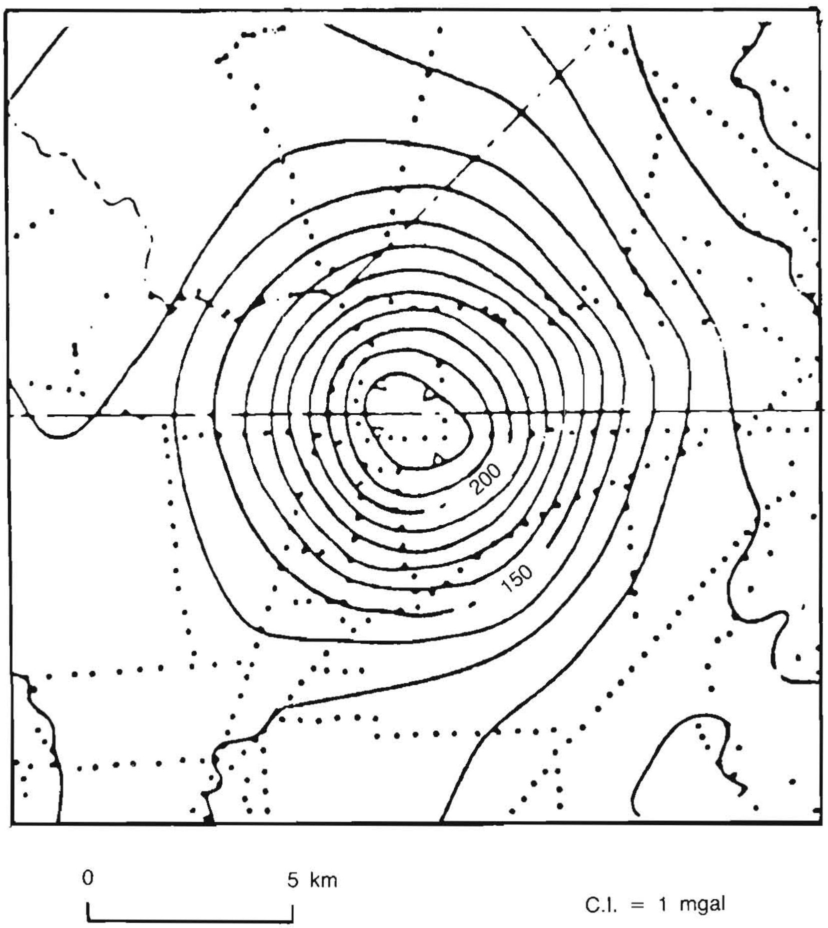

Fig. 2. Observed Bouguer gravity anomaly map of Humble Salt Dome, near Houston, USA, (Nettleton, 1976).

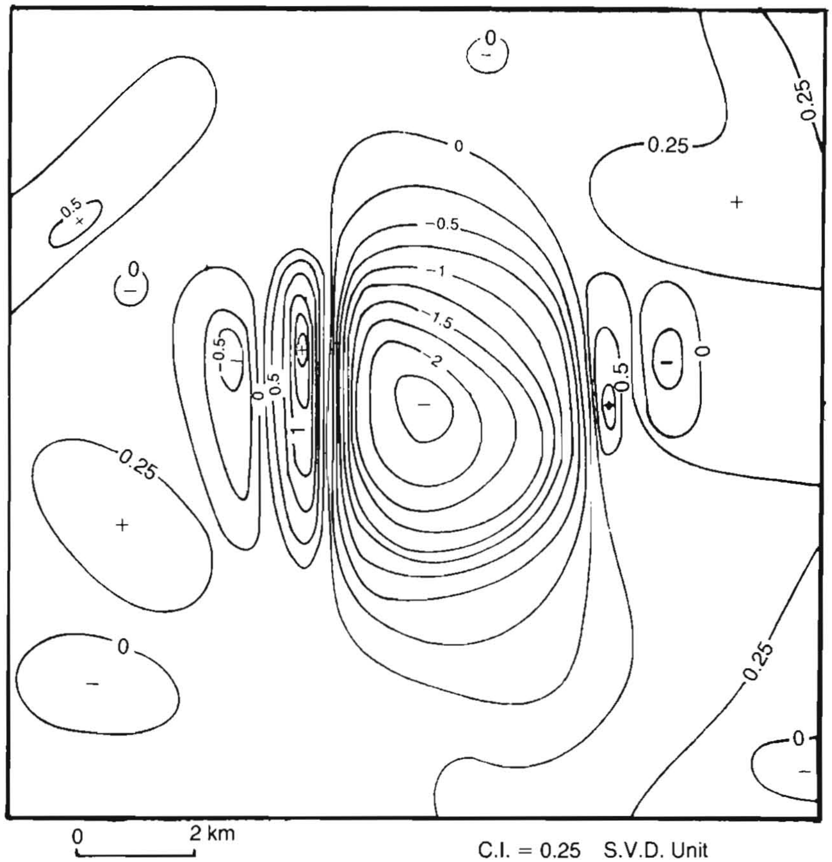

Fig. 3. Second vertical derivative gravity anomaly of Humble Salt Dome, near Houston, USA.

Using the computer programme given by El-Araby (1986) and by making use of −2.36 S.V.D. units for g_{zz}max, it was found that the depth to the center of the salt mass being spherical in shape is 4.63 km which is comparable with the depth obtained by Nettleton (1976) and by Mohan *et al.* (1986) using Mellin transform (about 4.97 km). However, our estimated value is less than that computed by them but is more reliable since it is computed from all data points on the second vertical derivative map and not form a single profile.

Conclusion

The problem of depth determination of a buried structure approximated by a sphere from the second vertical derivative anomaly map has been transformed into the problem of finding a solution of a non-linear equation of the form f(z) = O. The present three-dimensional approach is capable of determining the optimum depth of a buried structure from gravity data in a small area over the buried structure, *i.e.*, from the small segment of the second derivative area around g_{zz}max.

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تقدير أعماق تراكيب القباب الملحية من واقع المشتقة الثانية للجاذبية الأرضية

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يؤكد البحث الحالي جدوى استخدام طريقة أقل المربعات في تقدير العمق الأمثل لجسم كروي مدفون من واقع قيم خرائط المشتقة الرأسية الثانية للجاذبية الأرضية وذلك بإيجاد حل لمعادلة غير خطية في صورة د (Z) = صفر. ويمكن استخدام العلاقات المشتقة مباشرة للبرمجة. حيث تم بنجاح تطبيق الطريقة على قبة ملح هامبل.