Arab Gulf Journal of Scientific Research, 23 (1), pp 37-41 (2005)

Original Paper Ref. No 2302

Steady (Couette) Flow with Temperature Dependent Viscosity and the (Hall) Effect

سريان (كوتي) المستقر مع لزوجة معتمدة على الحرارة وتيار (هول)

حازم عطية

Hazem Attia

Dept. of Mathematics, College of Science, Al-Qasseem University
P.O. Box 237, Buraidah 81999, KSA.
E.mail: ah1113@yahoo.com

Abstract: The steady Couette flow with heat transfer of a conducting fluid is studied, taking into consideration the Hall effect. The viscosity of the fluid is assumed to vary with temperature. The fluid is subjected to a constant pressure gradient and an external uniform magnetic field perpendicular to the plates, which are kept at different but constant temperatures. The effect of Hall's current on the velocity components, as well as on the temperature, is more pronounced for higher values of viscosity exponent. It was also found that the Hall term has a marked effect on the axial and transverse components of the skin friction and the Nusselt number at both walls of the channel.

Key words: Couette, Temperature, Hall, magnetic field, conducting fluid, electrically insulating plates.

المستخلص: تم في هذا البحث دراسة سريان كوتي المستقر مع انتقال الحرارة لمائع موصل للكهربية مع أخذ تيار هول في الاعتبار. وقد تغير أفترض اللزوجة مع الحرارة وتعرض المائع لمعدل ضغط ثابت، ومجال مغناطيسي خارجي منتظم، وعمودي، على المستويين اللذين حفظا عند درجتي حرارة ثابتتين. وجد أن تأثير تيار هول على مركبات السرعة، ودرجة الحرارة، يكون أكبر للقيم الكبيرة لمتغير اللزوجة. وظهر أيضا تأثير واضح لحد هول على المركبات المحورية والعرضية لمعامل السطح، وعدد نسلت عند كل من المستويين.

الكلمات الدخلية: كوتي، حرارة، تيارهول، مائع موصل كهربية، مجال مغنطيسي.

Introduction

The flow with heat transfer of a viscous incompressible electrically conducting fluid between two parallel plates, which has important applications in magnetohydrodynamic (MHD) power generators and pumps, accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, and in the petroleum industry in the purification of crude oil and fluid droplet sprays, has attracted the attention of many authors (Cramer and Pai, 1973); (Tani, 1962); (Soundal-gekar, *et al.* 1979); (Attia, 1998). Most of these studies are based on constant physical properties. However, some physical properties vary with temperature (Herwig and Wicken, 1986), and assuming constant properties is a good approxima-tion as long as small differences in temperature are involved. However, more accurate prediction for the flow and heat transfer can be achieved by considering the variation of these physical properties with temperature. (Klemp, *et al.* 1990) studied the effect of temperature dependent viscosity on the entrance flow in a channel in the hydrodynamic case. The (MHD) fully developed flow and heat transfer of an electrically conducting fluid between two parallel plates with temperature dependent viscosity is studied (Attia and Kotb, 1996); (Attia, 1999) without taking into consideration the Hall effect.

In this paper, the steady Couette flow of a viscous incompressible electrically conducting fluid with heat transfer between two electrically insulating plates is studied, considering the Hall effect. The upper plate is moving with a constant speed, and the lower plate is kept stationary, while the fluid is acted upon by a constant pressure gradient and an external uniform magnetic field is applied perpendicular to the plates. The magnetic Reynolds number is assumed to be small so that the induced magnetic field is neglected (Cramer and

Pai, 1973); (Attia, 1998). The two plates are kept at two constant, but different temperatures, while the viscosity of the fluid is assumed to vary with temperature. Thus, the coupled set of the equations of motion and the energy equation including the viscous and Joule dissipation terms becomes non-linear and is solved numerically using the finite difference approximations to obtain the velocity and temperature distributions.

Formulation of the Problem

The fluid is assumed to be flowing between two infinite horizontal plates located at the ($y=\pm h$) planes. The upper plate moves with a uniform velocity, (U_o), while the lower plate is stationary. The two plates are assumed to be electrically insulating and kept at two constant temperatures (T_1) for the lower plate and (T_2) for the upper plate with $T_2>T_1$). A constant pressure gradient, (dP/dx), is applied in the (x-direction). A uniform magnetic field, (B_o), is applied in the positive (y-direction) which is the only magnetic field in the problem as the induced magnetic field is neglected by assuming a very small magnetic Reynolds number (Cramer and Pai, 1973); (Attia, 1998). The Hall effect is taken into consideration, and consequently a (z-component) for the velocity is expected to arise. The viscosity of the fluid is assumed to vary exponentially with temperature and the viscous and Joule dissipations are taken into consideration. The flow of the fluid is governed by the Navier-Stokes equation which has the two components (Cramer and Pai, 1973); (Attia, 1998).

$$\left[-\frac{dP}{dx}+\mu\frac{d^2u}{dy^2}+\frac{d\mu}{dy}\frac{du}{dy}-\frac{\sigma B_o^2}{1+\beta_e^2}(u+\beta_e w)=0.\right]$$

Eq. (1)

$$\left[\mu\frac{d^2w}{dy^2}+\frac{d\mu}{dy}\frac{dw}{dy}-\frac{\sigma B_o^2}{1+\beta_e^2}(w-\beta_e u)=0.\right]$$

Eq. (2)

Where (μ) is the viscosity of the fluid, ($u=u(y)$) is the electric conductivity of the fluid, ($u=u(y)$) is the velocity component of the fluid in the (x-direction), ($w=w(y)$) is the velocity component of the fluid in the (z-direction), and (β_e) is the Hall parameter (Cramer and Pai, 1973). The no-slip condition at the plates implies that the energy equation describing the temperature distribution for the fluid is given by (Cramer and Pai, 1973); (White, 1991)

$$\left[k\frac{d^2T}{dy^2}+\mu[(\frac{du}{dy})^2+(\frac{dw}{dy})^2]+\frac{\sigma B_o^2}{1+\beta_e^2}(u^2+w^2)=0,\right]$$

Eq. (3)

Where (T) is the temperature of the fluid and (k) is the thermal conductivity of the fluid. The last two terms in the left-hand side of Eq. (4) represent, respectively, the viscous and Joule dissipations. The temperature of the fluid must satisfy the boundary conditions,

[$y=-h$: $T=T_1$, $y=h$: $T=T_2$,] **Eq. (4)**

The viscosity of the fluid is assumed to vary with temperature and is defined as, ($\mu=\mu_o f_1(T)$). By assuming the viscosity to vary exponentially with temperature, the function ($f_1(T)$) takes the form,

($f_1(T)=\exp(-a_1)(T-T_1)$) (Klemp, *et al.* 1990)

Where (a_1) is a constant which in some cases may be negative, i.e. the coefficient of viscosity increases with temperature (Attia and Kotb, 1996); (Attia, 1999).

The problem is simplified by writing the equations in the non-dimensional form. To achieve this, we define the following non-dimensional quantities (Table 1).

Table (1): Definition of Non-dimentional quantities.

Non-diamentional quantities	**Definitions**
$\hat{f}_1(\theta)$ = $\exp(-a_1(T_2-T_1)T)$ = $\exp(-aT)$,(a)	the viscosity exponent.
$\hat{f}_2(\theta)$ = $1+b_1(T_2-T_1)\theta=T)$, = $1+b\theta$,(b)	The thermal conductivity parameter.
R = $\rho U_o h/\mu_o$,	The Reynolds number.
Ha² = $\sigma B_o^2 h^2/\mu_o$,Ha	The Hartmann number.
Pr = $\mu_o c_p/k_o$	The Prandtl number.
Ec = $U_o^2/c_p(T_2-T_1)$	The Eckert number.
τ_{xL} = $(d\hat{u}/d\hat{y})\hat{y}=-_1/R$	The axial skin friction coefficient at the lower plate.
τ_{zL} = $(d\hat{w}/d\hat{y})\hat{y}=-_1/R$	The transverse skin friction coefficient at the lower plate.
τ_{zU} = $(d\hat{u}/d\hat{y})\hat{y}=-_1/R$	The axial skin friction coefficient at the upper plate.
τ_{zU} = $(d\hat{w}/d\hat{y})\hat{y}=-_1/R$	The transverse skin friction coefficient at the upper plate.
Nu_L = $(d\theta/d\hat{y})\hat{y}=)\ y_1/=_1$	The Nusselt number at the lower plate.
Nu_U = $(d\theta/d\hat{y})\hat{y}=)\ y_1/=_1$	The Nusselt number at the upper plate.

Where (ρ) and (c_p) are, respectively, the density and the specific heat at a constant pressure of the fluid. In terms of the above non-dimensional quantities Eqs. (1) to (4) read (the hats are dropped for convenience).

$$\left[(\hat{x},\hat{y},\hat{z})=\frac{(x,y,z)}{h},\hat{P}=\frac{P}{\rho U_o^2}.(\hat{u},\hat{v},\hat{w})=\frac{(u,v,w)}{U_o}.\theta=\frac{T-T_1}{T_2-T_1},G=-\frac{d\hat{P}}{d\hat{x}},\right]$$

Eq. (5)

$$G + f_1(\theta)\frac{d^2u}{dy^2} + \frac{df_1(\theta)}{dy}\frac{du}{dy} - \frac{Ha^2}{1+\beta_e^2}(u+\beta_e w) = 0 \quad \textbf{Eq. (6)}$$

$$f_1(\theta)\frac{d^2w}{dy^2} + \frac{df_1(\theta)}{dy}\frac{dw}{dy} - \frac{Ha^2}{1+\beta_e^2}(w-\beta_e u) = 0 \quad \textbf{Eq. (7)}$$

[y=-1: u=w=0, y=1: u=1, w=0] **Eq. (8)**

$$\frac{1}{R\,\Pr}\frac{d^2\theta}{dy^2} + \frac{Ec}{R}f_1(\theta)\left[\left(\frac{du}{dy}\right)^2 + \left(\frac{dw}{dy}\right)^2\right] + \frac{EcHa^2}{R(1+\beta_e^2)}(u^2+w^2) = 0$$

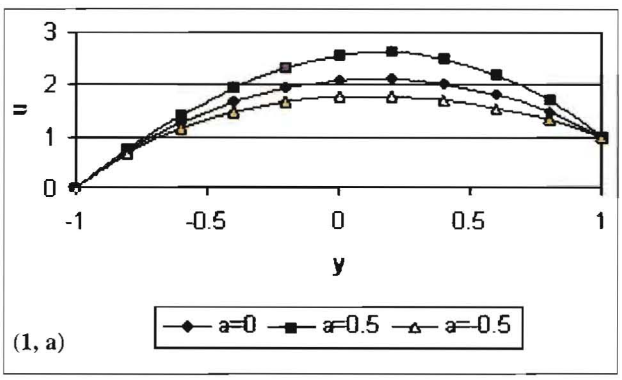
[y=-1: θ=0, y=1: θ=1] **Eq. (9)**

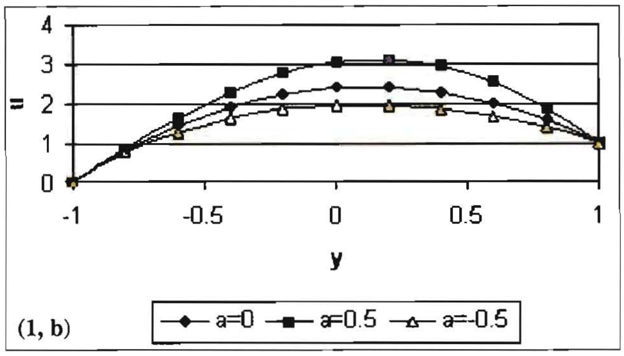
Equations (5), (6), and (8) represent a system of coupled non-linear partial differential equations which can be solved numerically under the initial and boundary conditions (7) and (9) using the finite difference approximations. The Crank-Nicolson implicit method is used (Ames, 1977). Finite difference equations relating the variables are obtained by writing the equations at the mid point of the computational cell and then replacing the different terms by their second order central difference approximations in the (y-direction). The diffusion terms are replaced by the average of the central differences at two successive time levels. The non-linear terms are first linearized and then an iterative scheme is used at every time step to solve the linearized system of difference equations. All calculations have been carried out for

(G=5),(R=1), (Pr=1), and (Ec=0.2).

Results and Discussion

(Figures (1) and (2)) present the profile of the velocity components (u) and (w), respectively, for various values of the parameter (β_e) and (a) and for Ha=1. (Figure 1) shows that increasing (β_e) increases (u) for all values of (a).





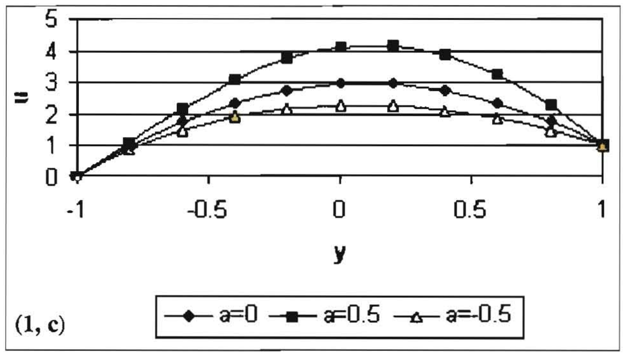
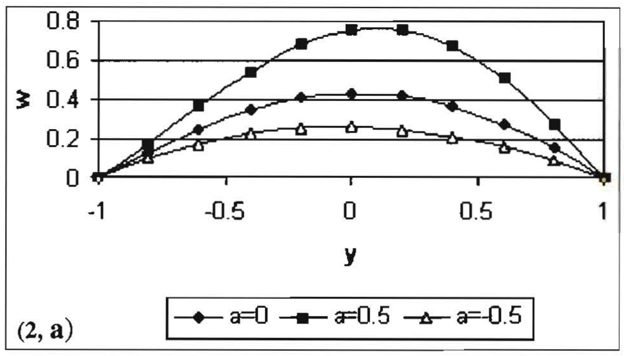
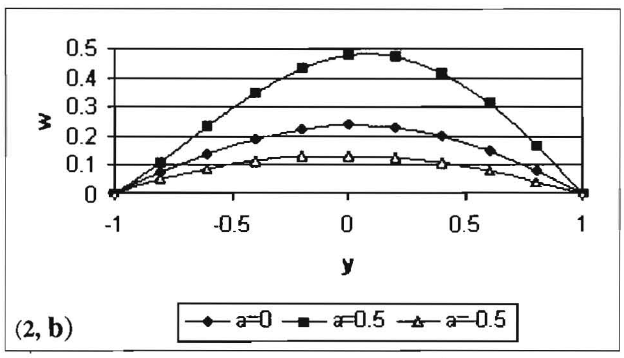


Fig.(1) (a), (b) & (c) Effect of (β_e) on the profile of (u) for various values of (a)





βe=0; (b) βe=1; (c) βe=5

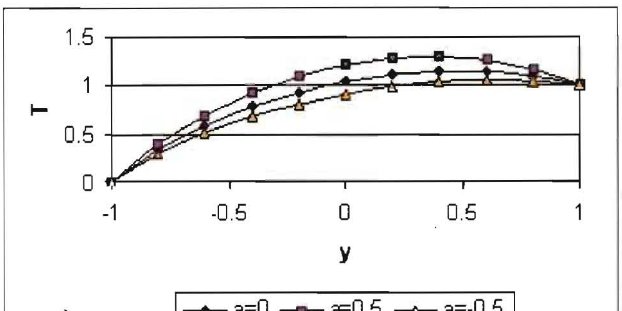
Fig. (2) (1) (a) and (b) Effect of (βe) on the profile of (w) for various values of (a)

(a) βe=1; (b) βe=5

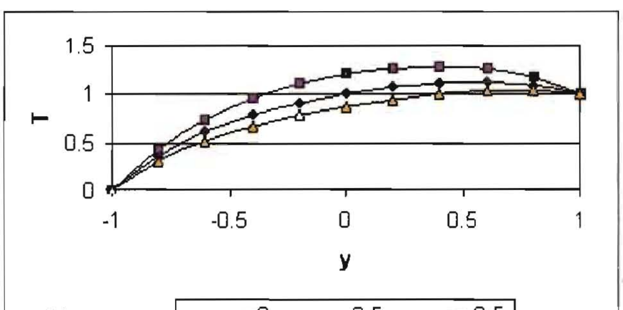
This is because the effective conductivity decreases with increasing (βe) which reduces the magnetic damping force on (u). In Fig. (2), for small values (β_e), of the velocity component, (w), increases with increasing (β_e), since (w) is a result of the Hall effect. For large values of (β_e), increasing (β_e) decreases (w) since it decreases the source term for (w ($\beta eu/(1+\beta e^2)$)).

(Figures (1) and (2)) show that increasing the viscosity exponent (a) increases both (u) and (w) as a result of decreasing the viscosity. It is also seen in Fig. (2) the influence of the viscosity exponent (a) on the symmetry of the profiles of (w) about the plane (y=0). It is clear from Figs. (1) and (2) that the effect of (β_e) on (u) or (w) is more pronounced for higher values of (a).

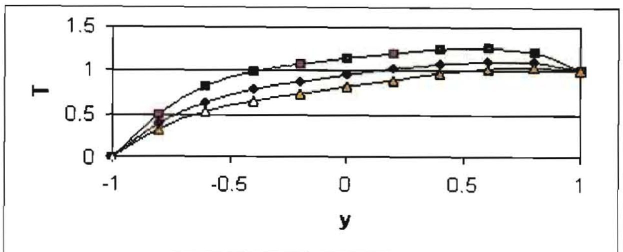
Figure (3) (a), (b) & (c) presents the profile of the temperature (T) for various values of the parameters (β_e) and (a) and for (Ha=1). It is of interest to find that the influence of (β_e) on (T) depends on (y). Near the center of the channel (y=0), increasing (β_e) decreases (T), while away from the center of the channel, increasing (β_e) increases (T). This can be attributed to the fact that large values of (β_e) decreases (w) significantly near the center of the channel which, in turn, decreases the Joule and viscous dissipations and then decreases (T) near the center. On the other hand, away from the center of the channel, the effect of large values of (β_e) on increasing (u) is more important than decreasing (w), which increases the dissipations and then increases (T). It is clear from the figure that the effect of β_e on (T) is more pronounced for higher values of (a). It is difficult to predict the effect of the parameter (a) on (T), because while increasing (a) increases the velocities and their gradients which increases the dissipation, it decreases the function (f_1) which decreases the dissipation. In all cases, Fig.(3), indicates that increasing (a) increases (T).



(3, a)



(3, b)



(3, c)

(a) β_e=0; (b) β_e=1; (c) β_e=5

Fig. **(3)** **(a)**, **(b)** **&** **(c)** Effect of β_e on the profile of (T) for various values of (a).

Table (2) presents the variation of the axial and the transverse skin friction coefficients and the Nusselt number, (Nu), at both walls of the channel with the Hall parameter (β_e) and the viscosity exponent (a). The results are estimated for (Ha=1).

It is clear from the table that increasing the Hall parameter (β_e) increases the magnitude of the axial skin friction coefficient and the (Nu), while decreases the magnitude of the transverse component of the skin friction. Increasing the viscosity exponent (a) decreases the magnitude of the axial skin friction coefficient at the lower plate while it increases its magnitude at the upper plate. On the other hand, increasing the viscosity exponent (a) increases the magnitude of the transverse skin friction coefficient and the (Nu) at both plates.

Table (2) Variations of the steady state skin friction coefficients and the Nusselt number at both walls of the channel with (β_e) for: (a) a=0, (b) a=0.5, and (c) a=0.5.

		β_e=0	β_e=1	β_e=5
(a)		4.0836	4.5572	5.3942
	τ_{zL}	0.0	0.6679	0.3688
	Nu_L	1.9289	2.1244	2.4656
		-2.7737	-3.3927	-4.3806
		0.0	-0.8225	-0.4319
		-0.6694	-0.7513	-0.9062
(b)		3.9307	4.4042	5.4362
		0.0	0.7927	0.4964
		2.1678	2.4636	3.0761
		-3.8778	-4.8302	-6.6939
		0.0+	-1.4145	-0.8481
		-1.0385	-1.2204	-1.5979
(c)		4.0744	4.4989	5.1419
		0.0	0.5436	0.2734
		1.7349	1.8594	2.0468
		-1.9916	-2.3778	-2.9082
		0.0	-0.4774	-0.2283
		-0.3961	-0.4196	-0.4659

Conclusion

The steady Couette flow of a conducting fluid under the influence of an applied uniform magnetic field has been studied with temperature dependent viscosity, considering the Hall effect. It is found that the effect of the Hall current on the velocity components (u) and (w) and the temperature (T) is more pronounced for higher values of the viscosity exponent (a).

It is also shown that increasing the Hall parameter increases the velocity component (u), however, its effect on the velocity component (w) depends on whether it is small or large.

It is of interest to find that the effect of the Hall parameter on the temperature (T) depends on the coordinate (y). The Hall term or the viscosity exponent has a marked effect on the axial and transverse components of the skin friction and the (Nu) at both walls of the channel.

References

Cramer, K.R. and Pai, S.-I. (1973) Magnetofluid dynamics for engineers and applied physicists. McGraw-Hill, New York.

Tani, I. (1962) Steady motion of conducting fluids in channels under transverse magnetic fields with consideration of Hall effect. J. of Aerospace Sci. **29**: 287-296.

Soundalgekar, V.M., Vighnesam, N.V. and Takhar, H.S. (1979) Hall and ion-slip effects in MHD Couette flow with heat transfer. IEEE Transactions on Plasma Sciences PS-**7**(3): 178-182.

Soundalgekar, V.M. and Uplekar, A.G. (1986)Hall effects in MHD Couette flow with heat transfer. IEEE Transactions on Plasma Science PS-**14**(5): 579-583.

Attia, H.A. (1998) Hall current effects on the velocity and temperature fields of an unsteady Hartmann flow. Can. J. Phys. **76**(9): 739-746.

Herwig, H. and Wicken, G. (1986) The effect of variable properties on laminar boundary layer flow. Warme-und Stoffubertragung **20**: 47-57.

Klemp, K., Herwig, H. and Selmann, M. (1990) Entrance flow in channel with temperature dependent viscosity including viscous dissipation effects. *(In)*: Proceedings of the Third International Congress of Fluid Mechanics, Cairo, Egypt **3**:1257-1266.

Attia, H.A. and Kotb, N.A. (1996) (MHD) flow between two parallel plates with heat transfer. Acta Mechanica **117**: 215-220.

Attia, H.A. (1999) Transient (MHD) flow and heat transfer between two parallel plates with temperature dependent viscosity. Mechanics Research Communications **26** (1): 115-121.

White, M.F. (1991) Viscous fluid flow. McGraw-Hill, New York.

Ames, W.F. (1977) Numerical solutions of partial differential equations 2nd ed. Academic Press, New York.

Ref: 2317
Rec. 29/05/2004
In revised form 15/01/2005