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Decentralized Self-Tuning Controller of Suboptimality β

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ABSTRACT. A simple state decentralized self-tuning is designed for suboptimal control of multivariable systems. The suboptimal decentralized self-tuner is calculated by using a cost function with degree β and the recursive least squares identification method. The suboptimal decentralized adaptive control law for the large-scale system is obtained from the adaptive control laws of the subsystems. The alogrithm is extremely easy to program on a computer.

Cost function methods are numerically efficient but are based on necessary condition to minimize the cost when applied to initial condition plant Horisberger and Belanger (1974) and Allwright and Mao (1982). Many other methods have been devised to avoid the above problem Levine and Athans (1974) and Levine *et al.* (1971) but are relatively complex. Man (1970), Dabke (1970), Kosut (1970) and Berger (1982) had studied the suboptimal in the sense that the cost function is minimal for all initial states and minimal design. However, the computational is still high for most practical application.

In this paper a decentralzied self-tuning method based on the cost function with degree β is presented. This method is suboptimal, but is very fast and easy to implement. The idea of cost function method with degree β was first proposed by Zheng (1989), and was developed and assessed by Skeliton and Xu (1990).

The cost function method with degree β for suboptimality, El-Shahat (1992), El-Shahat (1992a) and El-Shahat (1992b), has been successfully applied to the design of different control systems. The regulator which is given in this paper uses an input-output model to characterize the process dynamics. The algorithm will be applied to a suboptimal decentralized self-tuning control in a very simple manner.

I. Formulation of the Problem

In this note, we use the input/output model. This model is defined for a decentralzied system which can be written in multivariable input / output. The Auto Regressive Moving Average (ARMA) model given by Luenberger (1966) is assumed to have the same number of inputs and outputs q. The (ARMA) model is written in the following general form:

$$y_i(t) = A_i\,(Z^{-1})\,y_i\,(t) + B_i\,(Z^{-1})\,U_i\,(t-1) + e_i\,(t)\ ,\quad i = 1,2,\ \ldots\ldots r \tag{1}$$

where $A_i\,(Z^{-1})$ and $B_i\,(Z^{-1})$ are defined as follow:

$$A_i\,(Z^{-1}) = I + A_{1i}\,Z^{-1} + A_{2i}\,Z^{-2} + \ldots\ldots\ldots\ldots + A_{ni}\,Z^{-ni}$$

$$B_i\,(Z^{-1}) = B_{0i} + B_{1i}\,Z^{-1} + B_{2i}\,Z^{-2} + \ldots\ldots\ldots + B_{mi}\,Z^{-mi} \tag{2}$$

Where n and m are the orders of $A(Z^{-1})$ and $B(Z^{-1})$, respectively, for the subsystem i, I is the identity matrix (qxq), t is the discrete time index, Z^{-1} denotes the backward shift operator, U_i is the system input, y_i is the system output, and e_i is the random, zero-mean Gaussian white noise with the covariance R.

In a more compact form the plant of equation (1) can be written in state-space equation as follow:

$$X_i\,(t) = \begin{vmatrix} 0 & 0\ldots\ldots\ldots\ldots & & A_{ni} \\ & & & \cdot \\ I_{mi} & 0\ldots\ldots\ldots\ldots & & \cdot \\ & & & \cdot \\ 0 & I_{mi} & & A_{21} \\ 0\ldots\ldots & .0 & I_{mi} & A_{1i} \end{vmatrix} X_i(t-1) + \begin{vmatrix} B_{ni} \\ \vdots \\ B_{2i} \\ \vdots \\ B_{1i} \\ B_0 \end{vmatrix} U_i\,(t-1) \tag{3}$$

$$= A_iX_i\,(t-1) + B_i\,U_i\,(t-1), \tag{4}$$

$$y_i = C_iX_i$$

The problem is to devise a suboptimal decentralized adaptive controller for the plant (4). This controller uses the well known cost function with degree β combined with the recursive least squares identification method.

II. Recursive Least Squares Identification

Recursive least-squares identification method (Bohon and Debeer (1977) and Dugard and Landan (1980)) is used to obtain the parameters of the system. In decentralzied self-tuning control, we introduced a vector of parameters for every subsystem:

$$\theta_i \, (a_{1i} \, ... \, a_{ni}, \quad b_{0i} \, ... \, b_{mi}) \tag{5}$$

and a vector of regressors:

$$\phi_i = [y_i(t) \, ... \, y_i \, (t-n_i) \, U_i(t) \, ... \, U_i \, (t-n_i)] \tag{6}$$

The recursive least squares estimates are then given by:

$$\theta_i(t+1) = \theta_i(t) + P_i(t+1) \, \phi_i(t+1) \in_i(t+1) \quad , i = 1,2,..........r \tag{7}$$

where

$$\in_i(t+1) = y_i(t+1) - \phi_i^T(t+1) \, \theta_i(t+1) \tag{8}$$

and

$$P_i(t+1) = [P_i(t) + P_i(t)\phi_i(t)\{1+ \phi_i^T(t)P_i(t)+\phi_i(t)\}^{-1}\phi_i(t)P_i(t)] \, / \, \mu_i(t+1) \tag{9}$$

where P_i is the covariance matrix and μ_i is the exponential forgetting factor which is given by the following recursive equation Borison (1979):

$$\mu_i(t+1) = \mu_{0i} \, \mu_i(t+1) + (1-\mu_{0i}) \tag{10}$$

III. Suboptimal Decentralized Controller

The control objective is to find a suboptimal decentralzied output feedback gain F_i from the decentralized linear control law of the form:

$$\overset{*}{U}_i = -F_i y_i = -R_i^{-1} \, B_i^T \, \tilde{P}_i \, X_i \tag{11}$$

which minimizes the following cost function:

$$J_i = X_i^T \, \tilde{P}_i \, X_i \tag{12}$$

where

$\tilde{P}_i = \beta P_i$

P_i is the solution of the matrix riccati equation

$$P_i(A_i + \alpha I) + (A_i + \alpha I)^T P_i + Q_i - P_i B_i R_i^{-1} B_i^T P_i = 0 \tag{13}$$

If there exists a gain F_i which satisfies the following equation for a given $\beta \geq 1$ and $\alpha \geq 0$.

$$\tilde{P}_i(\overset{n}{A}_i + \alpha I) + (\overset{n}{A}_i + \alpha I)^T \tilde{P}_i + Q_i - \tilde{P}_i B_i R_i^{-1} B_i^T \tilde{P}_i = 0 \tag{14}$$

and

$$C_i^T F_i^T R_i F_i C_i \leq \tilde{P}_i B_i R_i^{-1} B_i^T \tilde{P}_i = 0 \tag{15}$$

where

$$\overset{n}{A}_i = A_i + B_i R_i^{-1} B_i^T \tilde{P}_i - B_i F_i C_i \tag{16}$$

then there exists a suboptimal decentralized output feedback control $\overset{*}{U}_i$ which minimizes its cost function $\overset{*}{J}_i$.

IV. Suboptimal Decentralized Self-Tuning Controller

The suboptimal decentralzied controller in section III is combined with the decentralzied least squares identification in section II. The algorithm of the suboptimal decentralized self-tuning controller is given as follows:

Step 1. Choose initial stabilized decentralized feedback gain $F_i \in \theta$ where $\theta = F_i \in R^{m.r}$:

$A_i + B_i F_i C_i$ is asymptotically stable and a real scalars $\alpha \geq 0$ and $\beta \geq 1$.

Step 2. Find the polynomial matrices $\hat{A}_i$, $\hat{B}_i$ from the model of equation 1 by recursive least squares identification.

Step 3. Determine A and B from equation 2.

Step 4. Find the suboptimal decentralized feedback gain, F_i from section III.

Step 5. Use F_i and time-varying reference signal to estimate the control signal U_i from the equation

$$U_i = L\,r(t) - F_i y_i(t) \tag{17}$$

where
r(t) is an mxl reference vector,
L is an mxm input gain matrix,
F is an mxm feedback gain matrix, and
y(t) is an mxl vector.

V. Example

The present example was used by Yu and Siggers (1971) and El-Shahat (1998), where the optimal decentralized controller was obtained by the optimal cost function. However, the computation required by the present algorithm is negligible compared with that used by Yu and Siggers (1971) and El-Shahat (1998). Here we consider a six-order discrete system consisting of noninteracting subsystems defined by the matrices as follows:

$$A_{11} = \begin{vmatrix} -2.66 & -0.099 \\ -1.36 & -0.037 \end{vmatrix}, A_{12} = \begin{vmatrix} -0.087 & 0.002 \\ 1.110 & -0.001 \end{vmatrix}, A_{13} = \begin{vmatrix} -0.25 & 0.003 \\ 2.80 & -0.020 \end{vmatrix}$$

$$B_1 = \begin{vmatrix} 0.002 & 0.600 \\ -0.001 & 0.450 \end{vmatrix}, C_1 = I$$

$$A_{21} = \begin{vmatrix} 0.121 & 0.003 \\ -0.620 & -0.015 \end{vmatrix}, A_{22} = \begin{vmatrix} 1.6 & -0.005 \\ 9.3 & -0.120 \end{vmatrix}, A_{23} = \begin{vmatrix} -0.46 & 0.002 \\ 1.40 & -0.040 \end{vmatrix}$$

$$B_2 = \begin{vmatrix} 0.900 & 0.900 \\ 0.021 & 0.015 \end{vmatrix}, C_2 = I$$

$$A_{31} = \begin{vmatrix} 0.46 & 0.005 \\ -1.10 & -0.090 \end{vmatrix}, A_{32} = \begin{vmatrix} 0.22 & 0.053 \\ 1.70 & 0.123 \end{vmatrix}, A_{33} = \begin{vmatrix} -1.2 & -0.003 \\ 7.0 & -2.370 \end{vmatrix}$$

$$B_3 = \begin{vmatrix} 0.8 & 0.4 \\ 0.0 & 0.9 \end{vmatrix}, C_3 = I$$

and let $Q = 0.1\ I_{2x2}$, $R = 10\ I_{2x2}$ and $P = I_{2x2}$, where I is the unity matrix.

The plant is described by the one subsystem model as follows:

$$y(t) = \begin{vmatrix} -0.087 & 0.020 \\ 1.110 & -0.001 \end{vmatrix} y(t-1) + \begin{vmatrix} 0.002 & 0.60 \\ -0.001 & 0.45 \end{vmatrix} U(t-1) + \begin{vmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{vmatrix} e(t-1) + e(t)$$

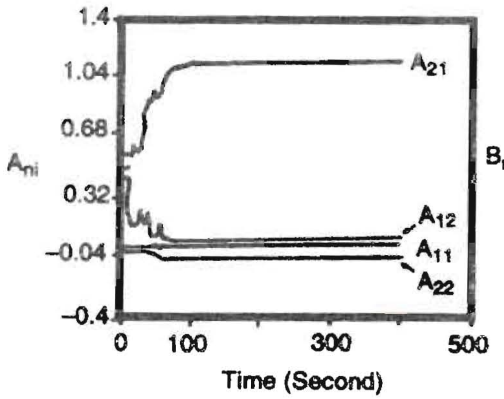

Fig. 1. Variation of the estimated parameters A_{ni} with time.

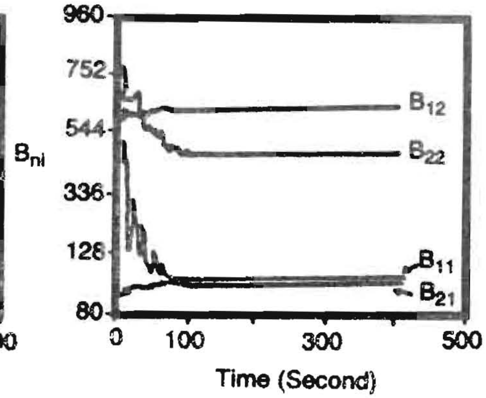

Fig. 2. Variation of the estimated parameters B_{ni} with time.

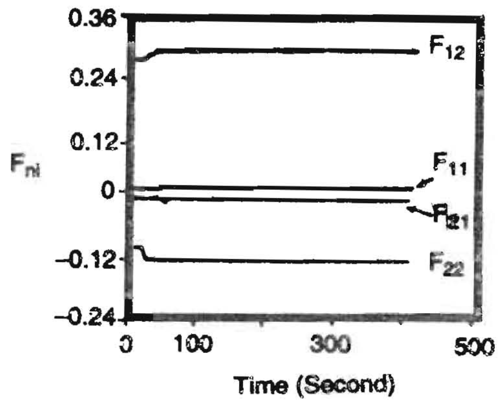

Fig. 3. Variation of the controller outputs F_{ni} with time.

The simulation results for the example with the decentralized adaptive controller are presented in Figs. (1-3). From these Figures, it is clear that the subsystem parameters and feedback gains converge to their real values in 100 iteration, which is about 25%

of the number of iteration required for the convergence of the self-tuner to the optimization feedback gains. Figs. 4-7 show that the closed loop-outputs Y_1 and Y_2 closely follow the reference inputs $U_1(t)$ and $U_2(t)$. From this study, it becomes clear that the application of cost function method with degree β on the class of decentralzied adaptive control yields the same results as those reported by El-Shahat (1998) for the applciation of cost function algorithm on the decentralzied adaptive control.

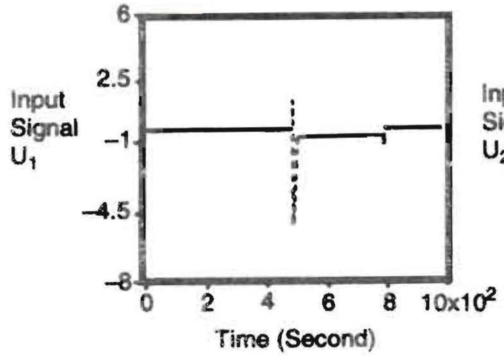

Fig. 4. Variation of input signal U_1 with time.

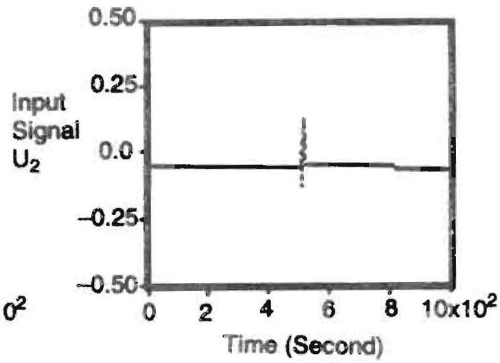

Fig. 5. Variation of input signal U_2 with time.

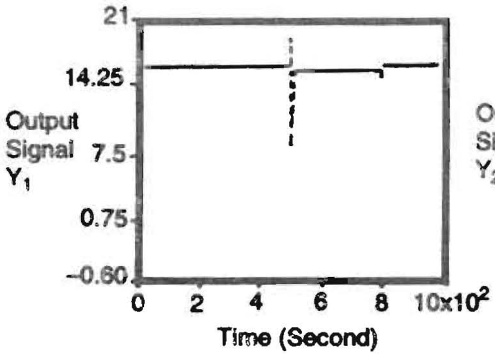

Fig. 6. Variation of output signal Y_1 with time.

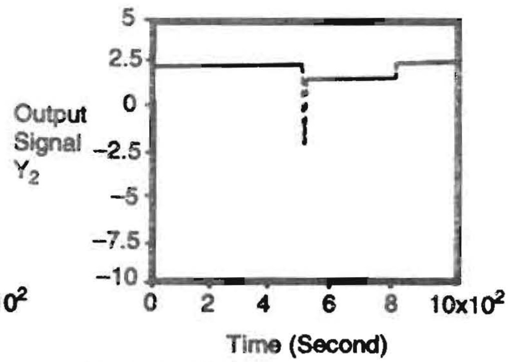

Fig. 7. Variation of cutput signal Y_2 with time.

Conclusions

The suboptimal self-tuning controller has beene extended to include decentralzied large-scale systems. The objective devise is to use the cost function with degree β to determine the suboptimal controller. The controller is easy to implement and the fast acting simulation results illustrate the good behavior of the proposed design as a suboptimal decentralzied adaptive control technique.

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تصميم حاكم لامركزي ذاتي الضبط شبه مثالي من الدرجة β

إسماعيل أبو زيد الشحات

هيئة المواد النووية - ص .ب (٥٣٠) - المعادي - القاهرة - مصر

الحاكم المقترح في هذا البحث مبني على أساس أن تكون المتغيرات المستخدمة في حساب إشارة التحكم ليست متغيرات الخرج وحدها فقط بل جميع المتغيرات المأخوذة في الاعتبار للنظام والمسماة بمتغيرات الأحوال .

تعرض هذا البحث لتصميم الحواكم شبه المثالية واللامركزية ذاتية الضبط ومتعددة المتغيرات ، وقد بنى على أساس الجمع بين أسلوب دالة التكلفة باستخدام مضروب متعدد القيمة العددي يسمى β وطريقة المربعات الدنيا .

لقد تم تطبيق الأسلوب المقترح بعد عمل برنامج للمعادلات الرياضية المستخدمة في صورة محاكاة على مثال عددي من أمثلة التحكم الآلي ومنه تتضح بجلاء سرعة وسهولة الحسابات والتصميم مقارنة بالأساليب المستخدمة سابقا .