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Modelisation of Water Balance by Simulation of Potential Evapotranspiration and the Rainfall for the Region of Annaba

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ABSTRACT. The water balance modelling of the Annaba region is achieved by integrating the chronological series (20 years) of rainfall and potential evapotranspiration (ETP). Rain constitutes the main contribution in bringing water to the soil; it is included in the water balance in terms of the effective rain.

The ETP allows us to quantify the crop's need for water. It determines the water volume lost by soil evaporation and plant transpiration and depends on atmospheric demand, on the stage of development of the vegetative cover as well as on the water supply.

The water balance modelling, rainfall and potential evapotranspiration simulation are established on a daily basis.

The rainfall following an aleatory phenomenon is simulated by the discreet Markov's chains. In fact, two states are taken into consideration:

state 0: if the rainfall is inferior or equal to a previously determined threshold.

state 1: if the rainfall is superior to that threshold.

The potential evapotranspiration whose variations are cyclical is simulated by the decomposition of the historical series into seasonal and fluctuating components. These parameters (rainfall and potential evapotranspiration) enable the modelling of the water balance which allows to quantify the supply water volumes obtained by irrigation.

The synthetic series of water deficits allow us to anticipate and to predict certain problems due to climate hazards thereby enabling water resource managers to avoid them.

1. Bibliographical Review

1.1. Introduction

The soil-plant-atmosphere system share water as a common element. Water plays a fundamental role in the functionning of the system as a whole. The stated problem is among others, the study of water availability for the plants during their development cycle, since the rarefaction of water resources and the exponential increasing of demand require the most favorable management of resources.

In Algeria, the climatic conditions do not allow the obtaining of regular crops without resorting to irrigation, water resources not being abandant and the potential users being more and more numerous (adjustment policy, extension of irrigated perimeters, *etc*...).

All this forces the managers to better valorize the resource which gets more and more scarce, and gets them to find out methodologies allowing to express the agro-climatic relations whose understanding is necessary for planning and organizing the introduction of projects that aim at the improvement of new crops and ecotypes.

The statistical study of the existing links in this context allows us to anticipate and predict certain problems due to climate hazards thereby enabling us to avoid them. In this field, modelling and simulation play a great role providing means for managers to take action.

Being subject to the Mediterranean climate where the Bounamoussa irrigation perimeter lays, and supplied by the Cheffia dam, the Annaba region (Algeria) has been chosen as the study area.

1.2. Background

Thirriot and Dechemi (1988), after a comparative study of five formulas pertaining to the calculation of the ETP (Penman - Blaney Criddle - Thornthwaite - Truc and Riou), demonstrated that Penman's formula, based on the taking into account of the ETP physical phenomenons, led to better results for the North Algerian climate.

Choisnel (1984) developed a model, coupled with water and surface energy balances of the soil, for a vegetable cover using weather data (temperature, relative humidity, wind speed, flow of radiation and precipitations). The output data act as a synthetic agroclimatic predictor.

Eldin and Lhomme (1984) developed a model which simulated the soil water balance from the daily rainfall, the monthly averages of the potential evapotranspiration and some soil and crops characteristics. This model is recurrent and gives an idea about the water reserve as well as the water deficit which allows for the characterization of the risks of aridity or exceeding water.

Several models were proposed for the simulation of the chronological series in hydrometeorology: Auto Regressive (AR) self regressive models (Thomas and Fiering (1962)), gaussian noise models (Matalas and Wallis (1971)), Auto Regressive Moving Average (ARMA) self-regressive and mobile average models (Carlson *et al.* (1970), O'connel (1971)), broken line models (Megia (1971)), ARMA Markov models (Lettenmaier and Burges (1977)), general mixed models (Boes and Salas (1978)), and models based on the main components analysis (Dechemi and Smith. (1994)).

The choice of use of one of these models depends on the following factors (Salas and Smith (1981)):

- Judgment, experience, personal preference of modellizer.
- Physical process of the model to be studied.
- Statistical characteristics of the chronological series.

2. Rainfall Simulation

The daily rainfall simulation of the Annaba region which is characteristically random (Fig. 1) is composed of Markov chains. The rainfall is subdivided into two classes constituting the discreet Markovian process, with:

$X_t \geq 0$ if the rainfall is inferior to S.

$X_t = 1$ if the rainfall is superior or equal to S,

S, being comprised between 0 and 5 mm, characterizes the occurence of dry and rainy days.

A Markov chain is a system which is subject, in course of time, to aleatory transition state changes, and which, without being deprived of memory, keeps just its most recent past.

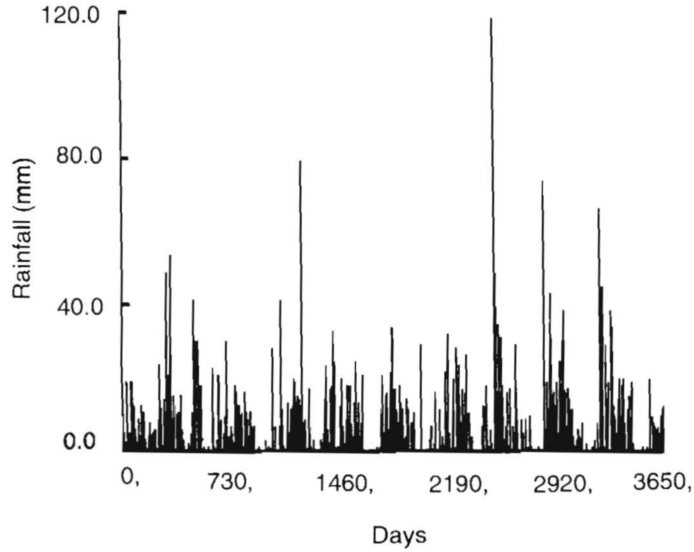

Fig. 1. Evolution of the daily rainfall.

A k order Markov chain is so called if the equation p $\{X_t, X_{t-1}, ..., X_{t-k}, X_{t-k-1}\}$ = p $\{X_t, X_{t-1}, ..., X_{t-k}\}$, where X is an aleatory variable.

As for the k order, the realization of a given state depends only on the preceding k realizations.

If independent properties exist, the Markov chains are to be given the order 0 (unconditional probabilities).

2.1. Analytic deduction of parameters

The daily rainfalls are described by means of the following stochastic process:

$$Z_t = X_t \cdot Y_t$$

where: X_t characterizes the state on t day $\begin{cases} X_t = 0 \text{ on dry day} \\ X_t = 1 \text{ on rainy day} \end{cases}$

and Y_t the rain height (if $X_t = 1$)

To determine the passage matrix in a general case, we just take into consideration the X_t variable which describes the passage from a state E_i to E_j.

2.1.1. *Order 1 process*

Let X be the aleatory variable characterizing the state; X may be assigned two values (binary state system) either 0 on a dry state or 1 on a rainy state.

This process is going to be characterized by the conditional probabilities of the passage, from the state of the day prior to today's state.

Also, the k day state depends only on the (k–1) day state; it is to be noticed that: prob $(X_k = j / X_{k-1} = i)\ \alpha_{ij}$

α_{ij} : represents the probability to obtain on k day the state j, knowing that on (k–1) day we had the state i.

By considering two possible states, we obtain the passage matrix P described below :

Table 1. Passage matrix coefficients (order 1)

(k-1) day state	**(k) day state**	
	0	**1**
0	α_{00}	α_{01}
1	α_{10}	α_{11}

The matrix P having the stochastic matrix characteristics, leads to the following relationship :

$\alpha_{i0} + \alpha_{i1} = 1 \qquad i = 0 \text{ or } 1$

The P matrix coefficients are obtained by the reckoning of the days characterized by the states (0,1).

The marginal probability (or unconditional) P_0 can be represented as follows:

$$p_0 = \frac{\alpha_{10}}{1 - \alpha_{00} + \alpha_{10}}$$

Given the bolean character of the aleatory variable, the self-correlation coefficient can be represented by:

$$\phi = \frac{\alpha_{00} - p_0}{1 - p_0} = \frac{\alpha_{11} - (1 - p_0)}{p_0} = \alpha_{00} - \alpha_{10} = \alpha_{11} - \alpha_{01}$$

By inserting the two grades of liberty ø and P_0, the passage matrix becomes:

$$p = \begin{vmatrix} p_0 + \varnothing\ (1-p_0) & (1-p_0) - \varnothing\ (1-p_0) \\ p_0 + \varnothing\ p_0 & (1-p_0) + \varnothing\ p_0 \end{vmatrix}$$

2.1.2. *Order 2 process*

Let's calculate today's state probability (class 0 or 1) in terms of the states of the two preceding days (already known).

The transition matrix is represented in the following table:

Table 2. Passage matrix coefficients (order 2)

States on (k-1) and (k-2) days	**States on (k-1) and (k) days**			
	00	**01**	**10**	**11**
00	β_{000}	β_{001}	–	–
01	–	–	β_{010}	β_{011}
10	β_{100}	β_{101}	–	–
11	–	–	β_{110}	β_{111}

β_{ijk} represents the conditional probability to obtain a (j, k) class doublet succeeding another (i, j) class one.

Because there is an overlap of two couples on the day before, it is necessary to have equal transition classes in the two couples.

To meet this requirement, the state on k day depends on the states on (k–1) and (k–2) days.

In the passage matrix definition, expressing the probabilities of successive couples, some combinations cannot be accomplished considering the succession of certain doublets.

The comparison between the first order α_{ij} and second order β_{ijk} conditional probabilities is, in short, the verity proof for the first order Markovian model if the margin is not considerable; the first order model is to be used to define a given climate.

When the first order transition matrix, raised to an exponent n, converges to infinity taking the form of an asymptotic matrix, the system is then stable.

In probabilities, this kind of system is said to possess an ergotic characteristic which allows us to determine the maximum order we can work with.

2.2. Principle of simulation

The simulation leads to the obtention of synthetic series of daily rainfalls from a more or less long historical series.

The stochastic simulation of a variable x aims at reproducing the probability structure of this variable. These probabilities are defined by the density F, or the distribution function F.

As we have seen before the daily rainfalls are described as follows:

$$Z_t = X_t \cdot Y_t$$

For the first order, from the conditional probabilities α_{00} and α_{10}, we generate the synthetic series daily state after drawing an aleatory number comprised between 0 and 1 (and constituting a probability) to be compared to α_{io}, and this after having previously fixed an initial state which doesn't reflect on the synthetic series given the fact that the memory of the phenomenon is too short (7 days for Annaba station).

If the aleatory number is inferior to α_{io} (i = 0 or 1), the simulated day is dry, if not, rainy.

The principle of simulation is similar for the superior orders.

In this case the obtained probability by aleatory drawing will be compared to the conditional probabilities of the passage matrix corresponding to simulation process order.

If the simulated day appears to be rainy, it will be assigned a rainfall height drawn from conditional functions of distribution.

Considering the humid day position in a given episode, four functions of distribution have been chosen (F010: rainy day comprised between two dry days, F011, F110 and F111). To assign to each simulated rainy day a rainfall height, a second aleatory number drawing is accomplished, and which will be compared to the considered distribution function.

2.3. Application to the rainfall serial of Annaba

In the following, a threshold of 0.1 is worked with, and by right of information the passage matrix will be presented from order 0, 1, and 2, as shown in Tables 3, 4, and 5.

Table 3. Order 0 passage matrix

state of (k) day	
0	**1**
0.787	0.213

Table 4. Order 1 passage matrix

State of (k-1) day	**State of (k) day**	
	0	**1**
0	0.869	0.131
1	0.486	0.514

Table 5. Order 2 passage matrix

States on (k-1) and (k-2) days	**States on (k-1) and (k) days**			
	00	**01**	**10**	**11**
00	0.884	0.116	–	–
01	–	–	0.480	0.520
10	0.769	0.231	–	–
11	–	–	0.491	0.509

Considering the order 0 passage matrix (in this case, the states of successive days are taken as independant), we notice that unconditional probability of dry state is greater than 78%. The order 1 passage matrix shows that the probability of having two successive dry days is 86.9%. However, this probability is only about 51.4% to obtain two successive humid days. Taking into account the order 2 passage matrix we have the confirmation of the semi-arid character of Annaba region climate.

In order to study the reliability of the rainfall simulation model, we stressed on three fundamental parameters: the threshold, the number of seasons to be taken into account and the order of Markov chains. As for the latter, its influence is not preponderant, the tests accomplished having shown good results using just the order 1.

In order to study the influence of the number of seasons, the year has been divided to two (October - April, May - September) and four seasons (September - November, December - February, March - May and June - August). This resulted in large variations of the results. The four season model reflects the evolution of the Annaba region rainfall in a sharper way. In fact, it's the nearest model to the physical reality of the studied phenomenon, that is, rainfall.

The threshold influence is not preponderant as shown where, three have been taken into consideration (0.1mm, 1mm and 5mm) but with no considerable variations in the results.

4. Simulation of the potential evapotranspiration (ETP)

4.1. Characteristics of the historical series

The examination of the evapotranspiration series graphic representation (Fig. 2) allows us to establish the verisimilitude of some fundamental components:

* a cyclical movement (Fig. 3) denoting oscillations where periods and amplitudes are more or less irregular.

* an aleatory element (Fig. 4), lacking precision and leading to more or less sporadic and unexpected variations sometimes called residual variations. So, rough date is to be looked at as the superposition of two independent series, the former taking into account the seasonal variations and the latter the residual variations or fluctuations (fluctuating component).

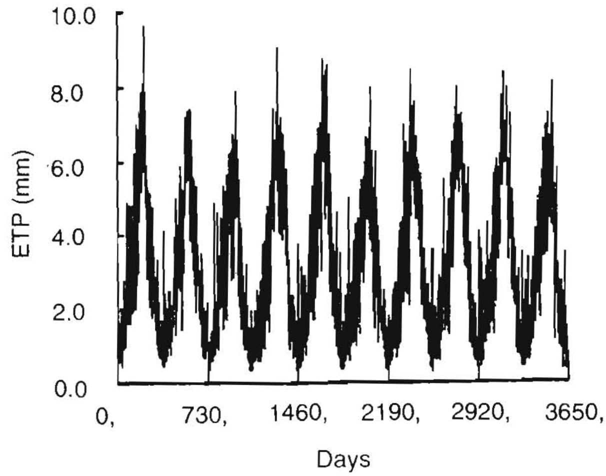

Fig. 2. Evolution of the daily evapotranspiration.

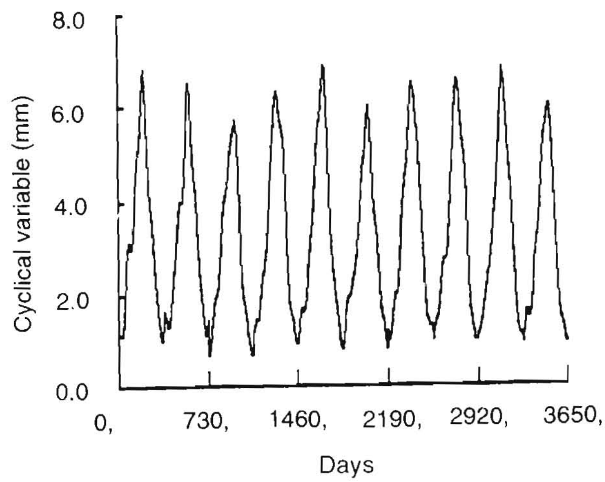

Fig. 3. Evolution of cyclical variable with time.

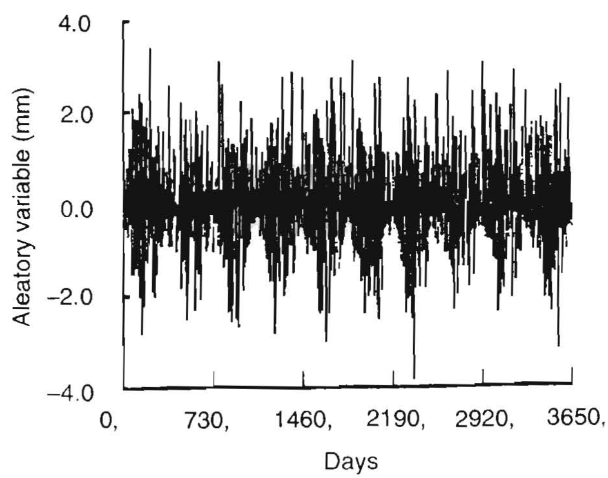

Fig. 4. Evolution of aleatory variable with time.

To separate these two components, we resort to the smoothing techniques where the series will be decomposed as follows (Arlery 1973):

$X_{it} = V_{it} + W_{it}$.

where: X_{it} = Rough data.
V_{it} = Seasonal component.
W_{it} = Fluctuating component.
t = 1 to n, where n represents the number of years of the historical series.

The decomposition into Fourier series of the initial series allows as to deduce the seasonal component whreas the fluctuating component will be represented by:

$$W_{it} = X_{it} - V_{it}$$

The simulation of both the seasonal and fluctuating components enable us to obtain the synthetic series through their superposition.

4.2 Estimate of the smoothing parameters

Knowing that the seasonal component may be represented by $V_t = A_0 + a_1 \sin t + b_1 \cos t + a_2 \sin t + b_2 \cos t + ...$,

Given that m is the number of harmonics, the smoothin will be written as follows:

$$V_t = A_0 + \sum_{j=1}^{m} \sum_{j=1}^{T} a_j \cos \frac{2\pi jt}{T} + b_j \sin \frac{2\pi jt}{T}$$

Where: A_0 = Rough series annual average.
a_j, b_j = Fourier series coefficients.
m = Number of harmonics.
T = Number of days per year.

The seasonal component is defined by an annual average to which periodical variations are to be added. These variations are characterized by their coefficients (a_j, b_j), the amplitude ($\sqrt{a_j^2 + b_j^2}$), the period (T/j) and the phase angle cotg (a_j/b_j), in such a way as:

$$X_t = V_t + W_t \quad \text{and}$$

$$\sum_{t=1}^{T} W_t = \emptyset \qquad A_0 = X_0 = \frac{1}{T} \sum_{t=1}^{T} X_t$$

The terms a_j, b_j are estimated using the least squares method consisting in minimizing the margin between X and V, which is calculated as follows:

$$\Delta = \sum_{t=1}^{T} \varepsilon_t^2 = \sum_{t=1}^{T} (X_0 - V_t)^2$$

$$\Delta = \sum_{t=1}^{T} \left[X_t - X_0 - \sum_{t=1}^{T} \left[a_j . \cos \frac{2\pi jt}{T} + b_j \sin \frac{2\pi jt}{T} \right] \right]^2$$

Whence we can find out the a_j and b_j values which nullify the derivatives in respect to these variables.

The fluctuations are obtained by substracting from the potential evapotranspiration rough data series the seasonal component obtained by the decomposition of the historical series in Fourier series.

They have a null average and are simulated by way of Markov chains (rainfall simulation). Two cases are then taken into consideration:

State ø if fluctuation is negative or null;
State 1 if fluctuation is positive.

In order to get a better simulation of these fluctuations the year has been subdivided into 1, 2, 4 seasons, and different orders of Markov chains have been tested.

a_0 term :

We have :

$$\sum_{t=1}^{\tau} x_t = \sum_{t=1}^{\tau} v_t = \tau * a_0$$

whence :

$$a_0 = x. = \frac{1}{\tau} * \sum_{t=1}^{\tau} x_t$$

a_j and b_j terms: Estimates by the least squares method.

The best estimates of these parameters by the least squares are those which minimize the margin between x and v which is calculated by :

$$\Delta = \sum_{t=1}^{\tau} \varepsilon_t^2 = \sum_{t=1}^{\tau} (x_t - v_t)^2$$

$$\Delta = \sum_{t=1}^{\tau} [x_t - x. - \sum_{t=1}^{m} (a_j * \cos\frac{2\pi jt}{\tau} + b_j * \sin\frac{2\pi jt}{\tau})]^2$$

$$a_j = \frac{2}{\tau} * \sum_{t=1}^{\tau} x_t * \cos\frac{2\pi jt}{\tau}$$

$$b_j = \frac{2}{\tau} * \sum_{t=1}^{\tau} x_t * \sin\frac{2\pi jt}{\tau}$$

It was observed that, the mean annual values (Fig. 5) and Fourier coefficients (Fig. 6 and 7) follow Gaussian distribution.

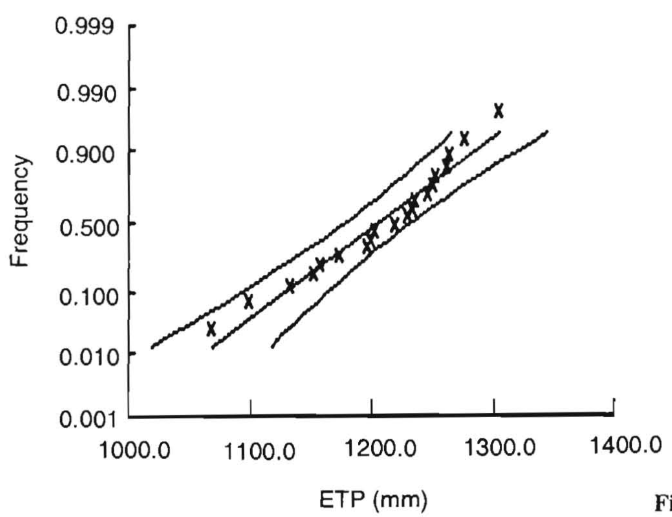

Fig. 5. Annual evapotranspiration.

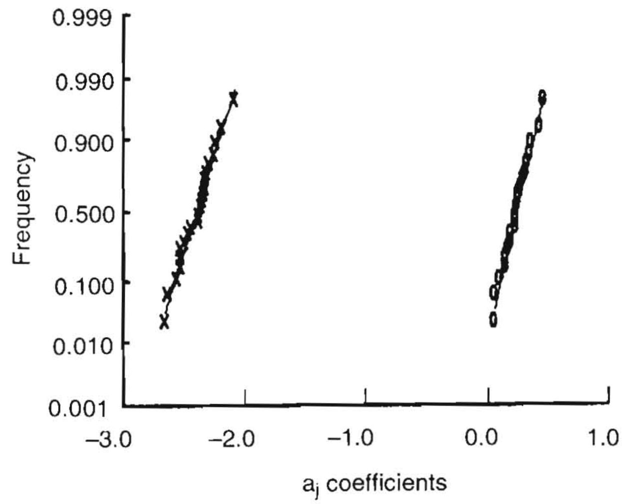

Fig. 6. Fourier coefficient (a_j).

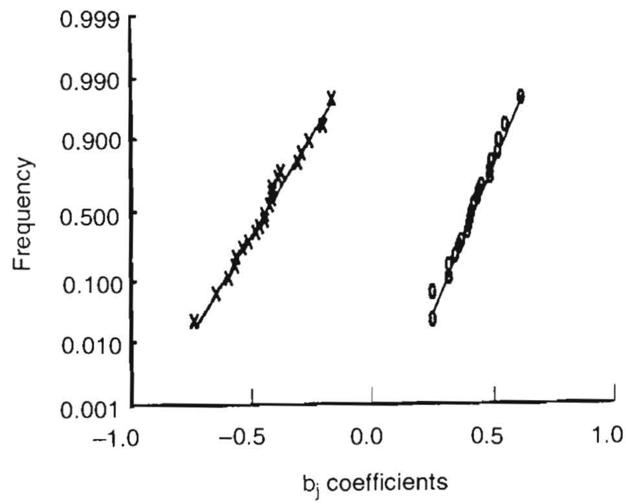

Fig. 7. Fourier coefficient (b_j).

4.3. Application of the simulation method

We consider the ETP serial of Annaba, three elements are to be taken into consideration to study the reliability of the potential evapotranspiration simulation model:

- The number of seasons.
- The order of Markov chains.
- The number of harmonics.

The fact of increasing the number of seasons (we used 1,2 and 4 simulations in our study) doesn't permit the simulations improvement on the basis of comparison criterions taken into account. The same applies to the Markov chains where the order 1 is largely sufficient for the obtainment of good simulations.

These two parameters (number of seasons and Markov chains) reflect on the aleatory component simulation as shown by the obtain results. In fact, the aleatory component is not preponderant in respect to the cyclical one. However, the number of harmonics has a great influence over the simulation. The four and six harmonics models giving the best results.

The fact of taking into consideration only two harmonics doesn't enable us to suitably explain the total variance, whence the rejection of this option is obvious.

5. Water Balance Model

5.1. Introduction

In a semi-arid climate, water availability for crops constitutes a major concern, since water is the limiting factor of vegetable production.

For this reason the study of the water balance is of great importance because it indicates the soil water reserve evolution.

The water balance is based on the principle of the soil water mass conservation. It includes precipitation, evapotranspiration, streaming, and drainage.

5.2. The determination of the effective reserve

The Util Reserve (R.U.) evolution depends on the crop development. That's why we take into account two parameters:

* The $R.U_x$ value just before the harvesting (in general at the max. rooting).

* A C1 coefficient which defines a minimum R.U. representing the case of a nude soil.

Between the "Semis" (S) and the harvesting (H) dates, the reserve varies from RU_d to RU_x, and will be equal to:

RU_d = RU_x . (C1 + (1–C1) . DSJ/DSR).
RU_d = Effective reserve on j-Day.
RU_x = max. R.U.
C1 = Coefficient comprised between 0 and 1.
DSJ = Time or number of days between the semi's date and j-day.
DSR = Lasting of the crop vegetative cycle between harvesting date and the following Semi's date. we'll have:

$$UR_d = Cl \ . \ RU_x$$

Therefore, the yearly reserve will be equal to:

C1 . RU_x if d . [H,S]

(C1 + (1–C1) . DSJ/DSR). RU_x if d . [S,H]

As for the perennial crops, the effective reserve is supposed to remain at its maximum value and not to vary all year long.

5.3 Effective rain

The weak rain waters falling down on a very arid soil (water reserve inferior to a one which is hardly usable) don't contribute to the increasing of the soil water reserve, since they vaporize very quickly.

This can be expressed as follows:

$PE_d = P_d$ if $(P_d > P_m)$ on $(P_d < P_s$ and $RH_{d-1} > RDU)$.

$PE_d = 0$ if $(P_d \leq P_s)$ and $(RH_{d-1} \leq RDU)$.

Where: P_d : Falling rain on (d) day.
P_s : Threshold.
RH_{d-1} : Water reserve on the (d–1) day.
RDU : Hardly usable reserve.

5.4. Maximum evaporatranspiration (ETM)

The ETM depends on the crop development during its vegetative cycle. It is bound to the potential evapotranspiration by the cultural coefficient (K_c), which is specific to this development.

Between the Semi's and the harvesting the ETM will be equal to the evaporation of a nude soil:

ETM = C_0. ETP [R,S], where C_0 is a reducing coefficient comprised between 0 and 1.

For a perennial crop, the ETM will be equal to the ETP all year long.

5.5. Real Evapotranspiration (ETR)

The ETR varies according to the soil water reserve. When the soil reserve is easily usable for the plant, the ETR will be equal to the ETM. Below this value, that is, when we are at the hardly usable reserve level at which the plant roots are required to get a certain force of suction, the ratio ETR/ETM is supposed to increase linearly from 0 to 1 according to the soil water reserve until reach the hardly usable reserve (RDU):

$ETR_d = ETM_d \quad \text{if } RH_{d-1} \geq RDO$

$ETR_d = ETM_d \,.\, RH_{d-1} / RDU \quad \text{if } RH_{d-1} < RDO$

The crop daily deficit is defined by the following relationship.

$$DH_d = ETM_d - ETR_d$$

5.6 The Drainage

Streaming hasn't been taken into account widely but we can consider that as soon as there is a soil saturation, there is a loss in exceeding water either by way of drainage or by streaming involving the terrain slope, the rain density, the soil and the kind of vegetal cover.

At this stage, we take into account the retention capability (CR) of the soil, which is defined as the difference between the usable reserve (RU) on (d) day and the water reserve on (d–1) day (RH_{d-1}):

$$CR_d = RU_d - RH_{d-1}$$

If the rainfall on d-day is greater than the retention capability of the soil, there will be a loss by way of drainage, otherwise it will contribute to fill up the soil reserve:

$$DR_d = P_d - CR_d \qquad \text{if } P_d \geq CR_d$$

$$DR_d = 0 \qquad \text{if } P_d < CR_d$$

5.7. The water balance equation

The equation representing the daily water balance is as follows:

$$RH(d) = RH(d-1) + PE(d) - ETR(d) - DR(d)$$

This equation will be used to calculate the successive daily water balances. Therefore, the elaboration of the water balance is an interative process. It is also necessary to start with a hypothesis on the value of the reserve at a suitable date, chosen on the basis of the following hypotheses:

– If the start-up of the iterative process takes place in the dry season, RH_0 will be supposed to have a O value.

– If in the middle of the rainy season, RH_0 = RO, then a mean value will be assigned.

Results and Discussions

The developed model is applied to the Annaba region; five paires of synthetic series of rainfall and potential evapotranspiration have been introduced in the model over a period of twenty (20) years on a daily basis.

On the basis of statistical criteria, taken into consideration for the comparison of the historic and simulated series, that the total water deficits mostly fell in the interval of confidence (5% margin of error) of the interannual average and typical margin (Table 6).

Table 6. Historical and simulated water deficit (Annaba)

	$\bar{X}$	σ	Cv
Historical	603.0	58.4	0.097
Simulation 1	555.7	74.8	0.135
Simulation 2	558.9	73.0	0.131
Simulation 3	561.2	76.0	0.135
Simulation 4	588.3	52.2	0.089
Simulation 5	558.4	73.5	0.132

The inter-annual variation coefficients (Cv) of the simulated water deficits range from 0.089 to 0.135.

The graphics (Fig. 8) showing the evolutions of the historic and simulated deficits demonstrate that:

– The maximum annual observed historic water deficit approaches 711 mm while it is about 735 mm for the simulated deficits.

– The evolution of the historic of water deficits is characterized by a series of two to three successive dry years, followed by one relatively humid year. The different simulations provide informations on tendancies of probable water deficits. Their analysis allows to have a better understanding of their evolution mainly during dryness period, and enables to anticipate on the way of managing the water resource.

We used these results as input parameters for the management of the Cheffia dam, which feeds in water not only the irrigation perimeter of Bounamoussa (Annaba), but fulfills domestic and industrial water needs of the Annaba region as well.

This allowed us to contribute to a scientific management of the water resources of the Annaba region, and to provide the necessary aids for decision making to the dam managers.

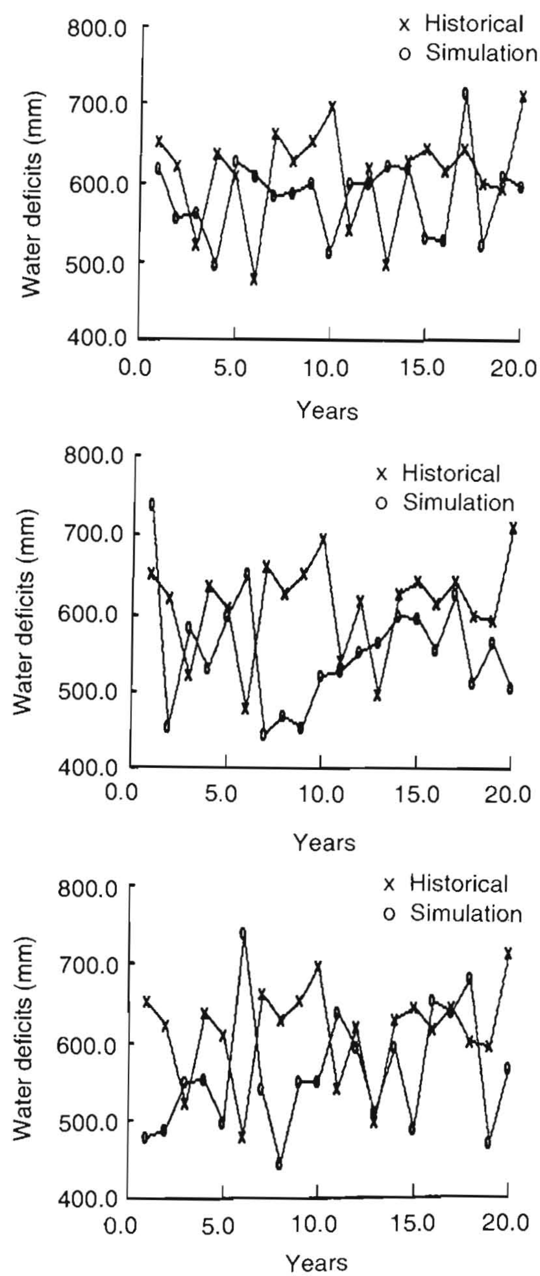

Fig. 8. Comparison between the historical and simulated series of the water deficits.

Conclusion

The crop's need in water depend on several factors; the ones pertaining to the climatic demand (environment evaporation demand); the ones pertaining to the soil factors (dynamics and availability of water in the soil), and the ones linked to the plant physiology.

The knowledge of these factors contributes to the amelioration of water management, to its valorization, thereby coming to the optimization of the outputs.

The two input parameters of the water balance model, namely, rainfall and potential evaporation, were simulated by means of statistical models, taking into account the specific characteristics of these two phenomena.

The rainfall simulation by a hybrid model, which takes into consideration the aleatory feature of this phenomenon, enables to stand out the diverse factors having an influence over this model, thus deducing the one giving the closest results to the physical reality. The number of seasons taken into consideration deeply refines the model to be developed.

The potential evapotranspiration has been decomposed into aleatory and cyclical parts which have been simulated by Markov chains and Fourier series respectively. The simulated series of rainfall and potential evapotranspiration have been used and input to the water balance model, which allowed to generate the simulated series of water deficits.

The generated water deficits analysis brought out not only the dry years but their successions in the course of time as well. That allows the water resource managers to be provided with guidance in irrigation, in determining the quantities of water to be allocated to the different irrigators.

The water needs of the irrigation perimeter, determined by means of the water balance model, can be used in a large context, as a partial introduction of a dam management model, assuring a number of functions, as fulfilling the irrigation needs, as well as the domestic and the industrial ones.

Appendix

a_j and b_j terms: Estimates by the least squares method.

The best estimates of these parameters by the least squares are those which minimize the margin between x and v which is calculated by:

$$\Delta = \sum_{t=1}^{\tau} \varepsilon_t^2 = \sum_{t=1}^{\tau} (x_t - v_t)^2$$

$$\Delta = \sum_{t=1}^{\tau} [x_t - x_. - \sum_{t=1}^{m} (a_j * \cos\frac{2\pi jt}{\tau} + b_j * \sin\frac{2\pi jt}{\tau})]^2$$

This comes to the same as finding out the a_j and b_j values which nullify the Δ derivatives in respect to these variables.

We can write that:

(In the following, $\frac{d}{da_i}$ and $\frac{d}{db_i}$ represent partial derivatives)

$$\frac{d\Delta}{da_i} = \sum_{t=1}^{\tau} 2 * \varepsilon_t * \frac{d\varepsilon_t}{da_i}$$

$$\frac{d\varepsilon_t}{da_i} = \sum_{j=1}^{m} \frac{d}{da_i} (a_j * \cos\frac{2\pi jt}{\tau} + b_j * \sin\frac{2\pi jt}{\tau})$$

$$\frac{d\varepsilon_t}{da_i} = \sum_{j=1}^{m} \cos\frac{2\pi jt}{\tau} * \frac{da_j}{da_i} + \sum_{j=1}^{m} \sin\frac{2\pi jt}{\tau} * \frac{db_j}{da_i} = \cos\frac{2\pi it}{\tau}$$

Whence:

$$\frac{d\Delta}{da_i} = \sum_{t=1}^{\tau} \left\{2 * [x_t - x - \sum_{j=1}^{m} (a_j * \cos\frac{2\pi jt}{\tau} + b_j * \sin\frac{2\pi jt}{\tau})] * \cos\frac{2\pi it}{\tau} \right\}$$

$$= 2 * \sum_{t=1}^{\tau} (x_t - x_\bullet) * \cos\frac{2\pi it}{\tau}$$

$$= -2 * \sum_{t=1}^{\tau} \sum_{j=1}^{m} a_j * \cos\frac{2\pi jt}{\tau} * \cos\frac{2\pi it}{\tau}$$

$$= -2 * \sum_{t=1}^{\tau} \sum_{j=1}^{m} b_j * \cos\frac{2\pi jt}{\tau} * \cos\frac{2\pi it}{\tau}$$

The same:

$$\frac{d\Delta}{db_i} = 2 * \sum_{t=1}^{\tau} (x_t - x_\bullet) * \sin\frac{2\pi it}{\tau}$$

$$= -2 * \sum_{t=1}^{\tau} \sum_{j=1}^{m} a_j * \cos\frac{2\pi jt}{\tau} * \sin\frac{2\pi it}{\tau}$$

$$= -2 * \sum_{t=1}^{\tau} \sum_{j=1}^{m} b_j * \sin\frac{2\pi jt}{\tau} * \sin\frac{2\pi it}{\tau}$$

We obtain:

$$\frac{d\Delta}{da_i} = 0 ===>$$

$$\sum_{j=1}^{m} a_j \sum_{t=1}^{\tau} \cos\frac{2\pi it}{\tau} * \cos\frac{2\pi it}{\tau}$$

$$+ \sum_{j=1}^{m} b_j \sum_{t=1}^{\tau} \sin\frac{2\pi jt}{\tau} * \cos\frac{2\pi it}{\tau}$$

$$= \sum_{t=1}^{\tau} (x_t - x_\bullet) * \cos\frac{2\pi it}{\tau}$$

And

$$\frac{d\Delta}{db_i} = 0 ===>$$

$$\sum_{j=1}^{m} a_j \sum_{t=1}^{\tau} \cos\frac{2\pi jt}{\tau} * \sin\frac{2\pi it}{\tau}$$

$$+ \sum_{j=1}^{m} b_j \sum_{t=1}^{\tau} \sin\frac{2\pi jt}{\tau} * \sin\frac{2\pi it}{\tau}$$

$$= \sum_{t=1}^{\tau} (x_t - x_\bullet) * \sin\frac{2\pi it}{\tau}$$

i and j varying from 1 to m, we get 2 * m equations with 2 * m unknown quantities that we've got to solve to estimate a_j and b_j.

Knowing that:

$$\sum_{t=1}^{\tau} \cos\frac{2\pi tj}{\tau} * \cos\frac{2\pi ti}{\tau} = \frac{\tau}{2} * \delta_{ij}$$

$$\sum_{t=1}^{\tau} \sin\frac{2\pi tj}{\tau} * \sin\frac{2\pi ti}{\tau} = \frac{\tau}{2} * \delta_{ij}$$

$$\sum_{t=1}^{\tau} \cos\frac{2\pi tj}{\tau} * \sin\frac{2\pi ti}{\tau} = 0$$

$$\text{with } \delta_{ij} \begin{cases} = 1 & \text{if } i = j \\ = 0 & \text{if } i \neq j \end{cases}$$

These equations then become:

$$\frac{\tau}{2} * a_i = \sum_{t=1}^{\tau} x_t * \cos\frac{2\pi it}{\tau}$$

$$\frac{\tau}{2} * b_i = \sum_{t=1}^{\tau} x_t * \sin\frac{2\pi it}{\tau}$$

Then we get for $i = 1$ to m:

$$a_i = \frac{2}{\tau} * \sum_{t=1}^{\tau} x_t * \cos\frac{2\pi it}{\tau}$$

$$b_i = \frac{2}{\tau} * \sum_{t=1}^{\tau} x_t * \sin\frac{2\pi it}{\tau}$$

We verify that:

$$\text{Var}(v) = \frac{1}{\tau} * \sum_{t=1}^{\tau} \left[\sum_{j=1}^{m} (a_j * \cos\frac{2\pi jt}{\tau} + b_j * \sin\frac{2\pi jt}{\tau}) \right]^2$$

$$= \sum_{j=1}^{m} \frac{a_j^2 + b_j^2}{2}$$

The harmonics are two by two orthogonal and each participates in the explained variance of the quantity:

$$\frac{a_j^2 + b_j^2}{2}$$

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نموذج التوازن المائي بواسطة محاكاة التبخر النتحي الكامن والمطر في منطقة عنابة

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صمم نموذج التوازن المائي لمنطقة عنابة (الجزائر) بإدراج تسلسل زمني (٢٠) سنة للمطر والتبخر النتحي الكامن (ETP) .

يشكل المطر المورد الرئيسي للماء على الأرض وهو يؤثر في التوازن المائي بواسطة المطر الفعال ، أي كمية الماء الساقطة التي تساهم في سد احتياجات الزراعة من الماء .

التبخر النتحي الكامن يتيح لنا معرفة ما تحتاجه المزروعات من الماء . كما يحدد حجم الماء المفقود من التبخر من التربة ونتح النباتات ويعتمد على مقتضيات الطقس ، وعلى مراحل النمو النباتي ، وأيضاً على الامدادات المائية .

يقيم التوازن المائي – محاكاة المطر والتبخر النتحي الكامن – على أساس يومي . إن المطر الناتج عن ظاهرة معتمدة على المصادفة يحاكي بسلاسل ماركوف . في الواقع ، هناك حالتان أخذت بعين الاعتبار :-

الحالة 0 : عندما يكون المطر الساقط أقل أو يساوي حداً سبق تحديده .

الحالة 1 : عندما يكون المطر الساقط أكبر من الحد الذي سبق تحديده .

التبخر النتحي الكامن ذو التغيرات الدورية يحاكي بتحليل السلسلة التاريخية إلى مركبات فعلية ومتغيرة والتي تحاكى طبقاً لسلاسل فوريه وسلاسل ماركوف . المتغيرات (المطر والتبخر النتحي الكامن) تمكن من إعداد نموذج التوازن المائي (لمحيط ري عنابة) والذي بدوره يمكن من معرفة حجم الإمدادات المائية اللازمة من سد الشافيه والذي يستخدم أيضاً لتزويد الصناعة بالماء وكذلك تزويد منطقة عنابة بماء الشرب .

السلاسل المركبة للعجز المائي تمكننا من التوقع والتنبؤ لبعض المشاكل الناجمة عن تقلبات الطقس ، وبالتالي تمكن المسؤولين عن مصادر المياه من تجنبها .