Arab Gulf J. Scient. Res., **9** (3), pp. 29-40 (1991)

Influence of the Electron-Phonon Interaction on the De Haas-van Alphen effect on β-Orbit in Mercury for First Ten Harmonics

Maarib A. Khalid

Department of Physics, College of Science, University of Basrah, Basrah, Iraq

ABSTRACT. The influence of the electron-phonon interaction on the De Haas-van Alphen effect amplitudes is calculated using a theory of Engelsberg and Simpson (1970). Calculations are presented on Hg using self-energies calculated from the $\alpha^2(v)$ F(v) spectrum found from superconducting tunnelling data. The largest changes are found for the first harmonic: 30% in amplitude for the β_{100} orbit at 15 tesla. The 10th harmonic shows large deviation also 2%.

The influence of the electron-phonon interaction on the de Haas-van Alphen (dHvA) effect has an interesting history. Theoretically, it was first demonstrated by Wilkins and Woo (1965), at temperature T such that $KT << v_D$ (Debye energy), that the electron-phonon interaction should increase the value of the electron mass over its band structure value. The many-body claculation of the electron-phonon interaction in the presence of a magnetic field was contained also in a paper of Fowler and Prange (1965). The more rigorous treatment of the same problem by Englesberg and Simpson (1970, to be referred to as ES) provides a convenient means of evaluating the effect of the electron-phonon interaction upon the dHvA effect. The results of ES indicate, under laboratory conditions, departures from the semiclassical theory of Lifshitz and Kosevich (1956 to be referred to as LK).

Palin (1972) measured electron life times as determined via the Dingle temperature (Dingle 1952). Suprisingly, even for mercury with its low Debye temperature (about 63K) and strong electron-phonon interaction, measurements by Palin failed to show such an effect.

Accurate measurement of dHvA effect amplitude in mercury up to 5 tesla was studied by Elliott (1978) who reported departures from quasiparticle behaviour of LK theory. The departures arise from the influence of the electron-phonon interaction.

A recent publication (Khalid *et al.* 1988) presents the results of an experimental investigation of the influence of the electron-phonon intraction on the dHvA effect on β_{100} in mercury up to 10 tesla. The many-body theory of the dHvA effect of ES is found to be in good agreement with experiment.

In this paper we present theoretical work on the same orbit to the tenth harmonic, in fields up to 15 tesla, to see the influence of the electron-phonon interaction on harmonics.

Theoretical Background

In this section, I briefly outline the electron-phonon and other interactions in the dHvA effect, and indicate the important parameters involved which lead to departures from LK theory.

Because the dHvA effect is a purely thermodynamic property (the magnetisation) of a metal, a natural starting point is the expression for the thermodynamic potential

$$\Omega = -\frac{1}{\beta}\sum_{i} \mathrm{Ln}\,(1 + \exp[\beta(\mu - E_i)]) \quad (1)$$

where $\beta = 1/KT$, μ is the chemical potential and E_i are the single-particle energy levels of the system. This expression was used by Lifshitz and Kosevich (1956) for their calculation in which there were no interactions. Expression (1) is also equivalent to that used by Fowler and Prange (1965). However, Engelsberg and Simpson (1970) showed that the effect of electron-phonon interactions could be incorporated in the oscillatory part of the thermodynamic potential by replacing, at a certain state, the non-interacting single-particle electron energies by the Non-interaction energy plus the full electron-phonon self energy Σ,

$$\Sigma(\varepsilon_i, K) = \Delta(\varepsilon_i, K) - i\Gamma(\varepsilon_i, K) \quad (2)$$

Normally Σ is taken to be averaged over the portion of Fermi surface (Grimvall 1967). For more information the reader is referred to Engelsberg and Simpson (1970) or to the discussion by Elliott (1978). The final result for the oscillatory magnetisation is

$$\widetilde{M} = \frac{-ek}{\pi\hbar}\left(\frac{2\pi e}{\hbar}\right)^{1/2} \cdot \frac{FTB^{-1/2}}{[A'']^{1/2}} \sum_{r} r^{-1/2} \cdot 2a_r \cos\left(\frac{\pi rgm}{2m_0}\right).$$

$$\sin\left[2\pi r\left(\frac{F}{B} - \gamma\right) \pm \frac{\pi}{4}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

where m is the cyclotron effective mass, F is the dHvA frequency, B is the magnetic field, A″ is the Fermi surface curvature and $\gamma = 1/2$ for free electron. Each quantity is to be evaluated at the extremal cross section (A_f^{ext}) of the Fermi surface perpendicular to the field direction.

According to ES, the entire effect of the electron-phonon interaction is contained in a term, a_r, for the amplitude of the r^{th} harmonic of the dHvA effect is that

$$a_r = \sum_{n=0} \exp\left[\left(\frac{-2\pi r}{\hbar w_c}\right)\left\{w_n + \zeta(w_n)\right\}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

where $w_c = eB/m$.

In equation (4) $\zeta(w_n)$ is related to the full self energy evaluated at the poles, iw_n, of the Fermi function, where

$$w_n = (2n + 1)\,\pi KT.$$

Thus

$$\zeta(w_n) = \pi KT \int_0^{\infty} \frac{2\,\alpha^2(v)\,F(v)}{v}\left\{1 + 2\sum_{l=1}^{n}\left[1 + \left(\frac{2\pi lKT}{v}\right)^2\right]^{-1}\right\} dv \quad \ldots\ldots \quad (5)$$

In this formula $F(v)$ is the phonon density of states as a function of phonon energy v, and $\alpha^2(v)$ is the effective electron - phonon coupling strength.

Returning to equation (4) if there is no electron-phonon interaction, $\zeta(w_n) = 0$, and

$$a_r = \sum_{n=0} \exp \left[\left(\frac{-2\pi r}{\hbar w_c}\right) w_n\right] = \left[2 \sinh \left(\frac{2\pi^2 r KT}{\hbar w_c^*}\right)\right]^{-1} \quad \text{......................} \quad (6)$$

so that equation (4) exactly produces the LK formula.

The general behaviour of equation (4) in various limits of magnetic field and temperature has been discussed by Engelsberg and Simpson (1970), Engelsberg (1978) and Mueller and Myron (1986). Then we may summarize the results as

1. At both high temperature and low magnetic field limit, $w_n \rightarrow 0$, only the first pole, w_0, contributes effectively to the summation. From equation (4),

   $$\zeta(w_0, T) = \lambda_{ep} \pi KT \quad \text{..} \quad (7)$$

   where λ_{ep}, the electron-phonon renormalization constant, is given by

   $$\lambda_{ep} = \frac{m_c^*}{m_c} - 1 = \int_0^\infty \frac{\alpha^2(v)\ F(v)}{v}\, dv \quad \text{..................................} \quad (8)$$

   so that in this limit equation (4) reduces to the LK formula with m_c renormalized to m_c^*.

2. In the low temperature, high magnetic field limit, an increasing number of terms is effective in the summation of equation (4). If in equation (2), $\frac{2\pi l KT}{v} \sim 1$ at the highest values of l ($l = n$) which contribute effectively to the summation of equation (4), then departures from quasiparticle behaviour and hence from LK theory will ensue.

In order to calculate the precise deviation from LK behaviour, one needs to perform the sum in equation (4) numerically, for both limits, using values of $\zeta(w_n)$ evaluated from equation (5). Thus it is convenient to write the deviation in terms of the logarithmic factor

$$A_r = \ln\left\{a_r \cdot 2\sinh\left(\frac{2\pi^2 rKT}{\hbar w_c^*}\right)\right\} \quad \text{.......... (9)}$$

so that the true dHvA amplitude will be given by the multiple of LK amplitude in equation (6) by e^{A_r}.

Results and Discussion

As mentioned earlier, at low temperature and high magnetic field that the evaluation of equation (5) leads to significant departures from LK. Inspection shows that this is favoured by low phonon frequencies and low effective masses. β_{100} orbit in mercury has been chosen for such study for its low lying phonon frequency and low effective mass (see Poulsen *et al.* 1971).

The influence of the electron-phonon interaction has been numerically calculated using the $\alpha^2(v)$ F(v) spectrum depicted in Fig. (1), taken from the work of Hubin and Ginsberg (1969).

Fig. (2) shows the variation of logarithmic amplitude, with harmonic, for both LK and ES at 1.2K, for three different field values. It is clear that the influence of electron-phonon interaction on dHvA amplitude depends on magnetic field B and decreases with increasing harmonic.

It is worth mentioning that the departures from LK behaviour depend on B rather than T (see Khalid 1987).

It is clear from the foregoing that all the effects of phonon are contained in a_r. Fig. (3) shows the variation of A_r with harmonic for the same orbit at different field values. The information in Fig. (1) was used in the calculation of $\zeta(w_n)$ using equation (5).

In Fig. (3), the first harmonic shows 30% deviations from LK formula at 15 tesla. Approximately 2% deviation occurs for 10th harmonic.

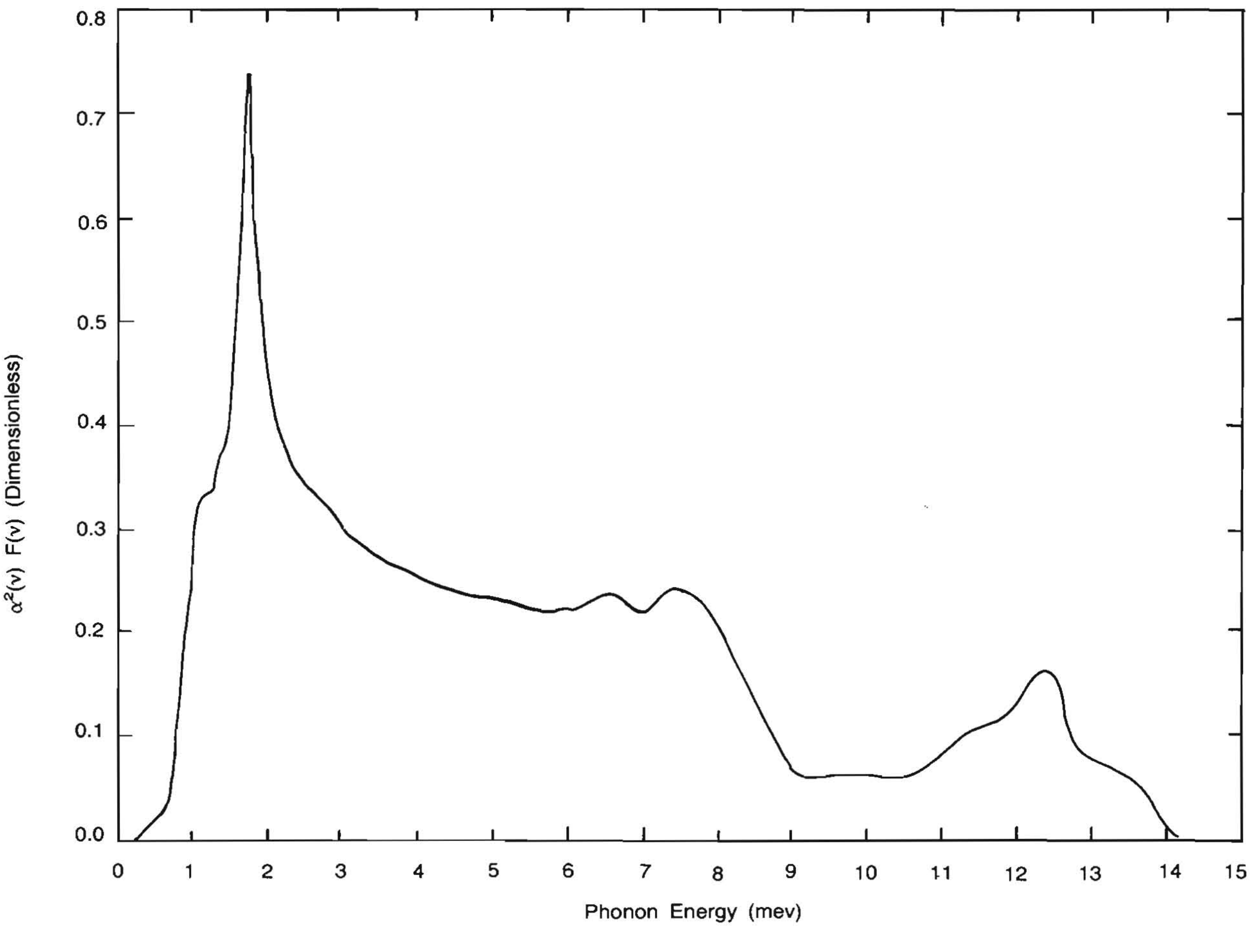

Fig. 1. The $\alpha^2(v)$ F(v) spectrum for mercury, after Hubin and Ginsberg (1969).

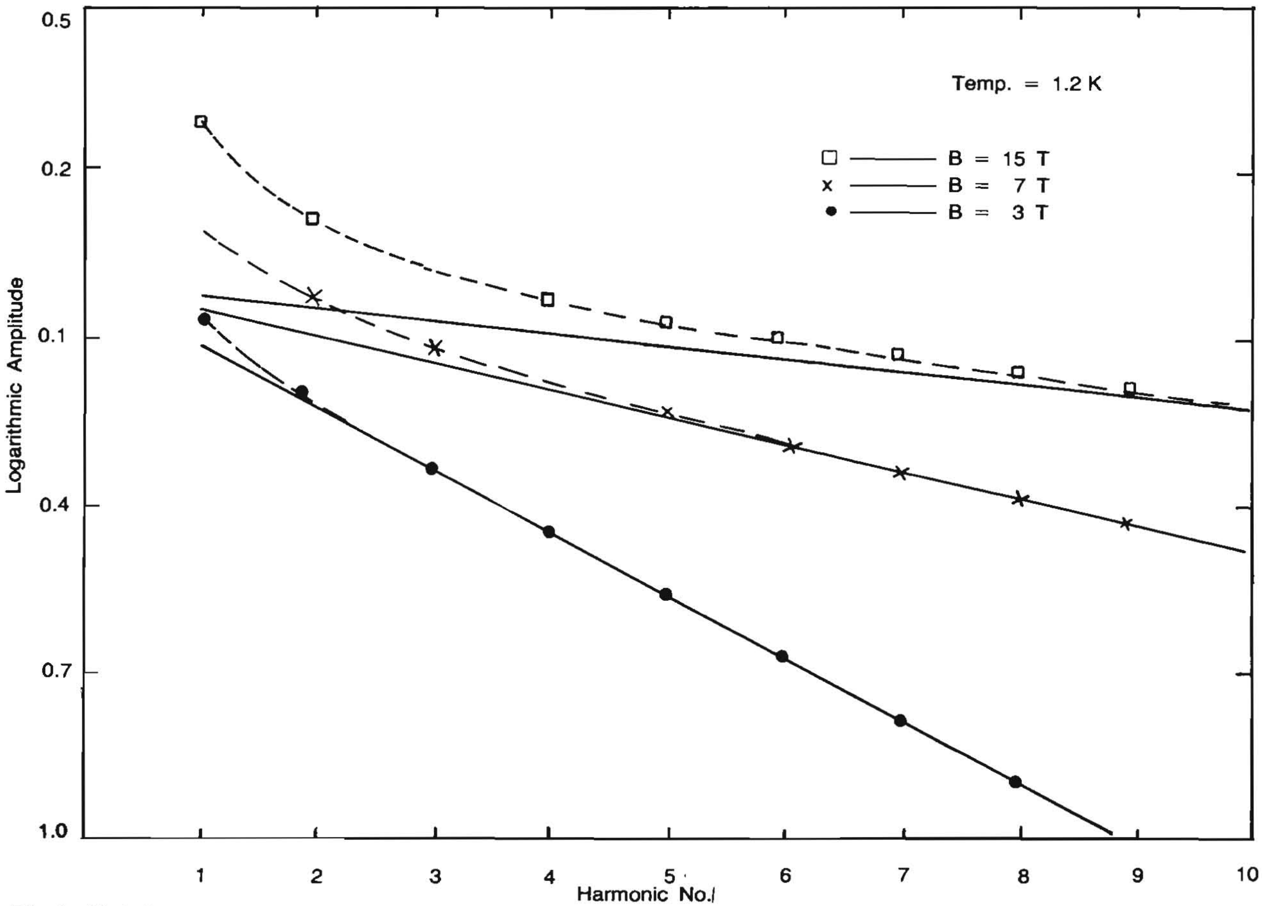

Fig. 2. Variation of Logarithmic amplitude (equation (4)) with harmonic number at three values of magnetic field and at T = 1.2K. The full line represents the LK behaviour, whilest the doted line incorporates the deviations according to ES theory.

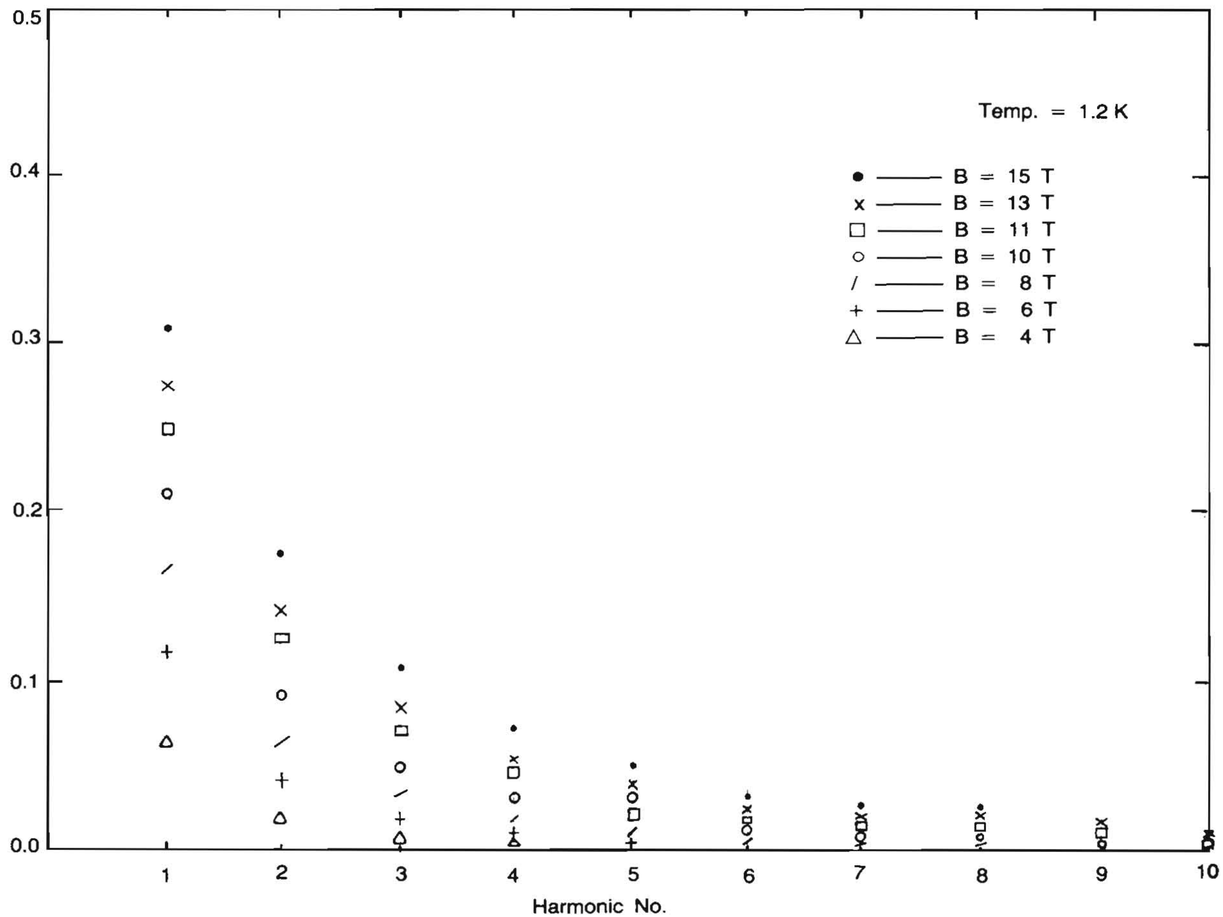

Fig. 3. Variation of A_r (equation (9)) with harmonic number at 1.2K. The seven curves correspond to field of 15, 13, 11, 10, 8, 6 and 4 T from the uppermost to the lowest curve respectively.

Conclusion

In this paper, we have seen that significant departures in amplitude occur from the semiclassical theory of Lifshitz and Kosevich (1956). These departures are attributed to the influence of the electron-phonon interaction. We have demonstrated from dHvA amplitudes Fig. (2), that the departures from LK theory increase with increasing magnetic field.

It is clear from the variation of A_r with harmonic Fig. (3) that the influence of the electron-phonon interaction can be detected even for 10th harmonic. The deviations found for the first harmonic are 30% in amplitude at 15 tesla while for the 10th harmonic they are 2%.

References

Dingle, R.B. (1952) Some magnetic properties of metals. II-The influence of collisions of the magnetic behaviour of large systems. *Proc. Roy. Soc.* **211**(A): 517-525.

Elliott, M. (1978) *Influence of electron-phonon interaction in the de Haas-van Alphen effect.* D. Phil thesis University of Sussex.

Engelsberg, S. (1978) Quasiparticle picture for the de Haas-van Alphen effect. *Phys. Rev.* **18**(B): 966-969.

Engelsberg, S. and **Simpson, G.** (1970) Influence of Electron-Phonon Interaction on the de Haas-van Alphen Effect. *Phys. Rev.* **2**(B): 1657-1665.

Fowler, M. and **Prange, R.E.** (1965) Electron-phonon renormalization effects in high magnetic fields. The de Haas-van Alphen effect *Physics* **1**: 315-328.

Grimvall, G. (1967) Numerical calculations on the electron-phonon systems in Sodium. *Phys. Kond. Matt.* **6**: 15-22.

Hubin, W.N. and **Ginsberg, D.M.** (1969) Electron tunneling into superconducting mercury films at temperatures below .4K. *Phys. Rev.* **188**; 716-722.

Khalid, M.A. (1987) *A study of the electron-phonon interaction on the de Haas-van Alphen effect* D. Phil thesis, University of Sussex.

Khalid, M.A., Reinders, P.H.P. and **Springford, M.** (1988) A study of the electron-phonon interaction in the de Haas-van Alphen effect *J. Phys. F: Met. Phys.* **18**: 1949-1964.

Lifshitz, I.M. and **Kosevich, A.M.** (1956) Theory of Magnetic Susceptibility in metals at low temperatures. *Sov. Phys. JETP* **2**: 636-645.

Mueller, F.M. and **Myron, H.W.** (1976) Field dependence of the amplitude of the de Haas-van Alphen effect in simple metals. *Comm. on Phys.* **1**: 99-104.

Palin, C.J. (1972) Amplitude of the de Haas-van Alphen effect in mercury. *Proc. Roy. Soc.* **329**(A): 17-34.

Poulsen, R.G., Moss, J.S. and **Datars, W.R.** (1971) de Haas-van Alphen Effect in Mercury. *Phys. Rev.* **3**(B): 3107-3115.

Wilkins, J.W. and **Woo, J.W.F.** (1965) Effect of many body interactions on the temperature dependent amplitude of de Haas-van Alphen oscillation. *Phys. Lett.* **17**: 89-90.

*(Received 25/03/1990;
in revised form 15/10/1991)*

دراسة تأثير تصادم الإلكترون مع الفونون على سعة موجة دي هاز ـ ڤان ألفِن للعشر توافقيات الأولى على المدار β لعنصر الزئبق

مآرب عبدالله خالد

قسم الفيزياء ـ كلية العلوم ـ جامعة البصرة ـ البصرة ـ العراق

الهدف من هذا البحث هو إجراء دراسة نظرية حول تأثير تصادم الإلكترون مع الفونون على سعة موجة دي هاز ـ ڤان ألفِن للعشر توافقيات الأولى باستخدام نظرية أنجلسبرغ وسامپسون (Engelsberg and Simpson) لعام ١٩٧٠ م .

وللوصول إلى هذا الهدف تمَّ اختيار مدار عنصر الزئبق لأسباب عديدة أهمها أنه يمكن ـ بسهولة ووضوح ـ إيجاد تأثير تصادم الإلكترون مع الفونون على سعة موجة دي هاز ـ ڤان .

كما أن الدراسة النظرية تمَّت عند درجة حرارة ٢ , ١ كلفن ، وقد أمكن الحصول ـ عملياً ـ على درجة الحرارة هذه باستخدام سائل الهيلوم H_e^4 للوصول إلى درجة حرارة ٢ , ٤ كلفن ومن ثمَّ تبريده بأجهزة معيّنة للوصول إلى درجة حرارة ٢ , ١ كلفن .

وقد تبينَّ بنتيجة الدراسة أنه بسبب تصادم الإلكترون مع الفونون فإن إنحرافاً يحصل مقداره ٣٠ ٪ من سعة الموجة التوافقية الأولى وذلك بوجود مجال مغناطيسي مقداره ١٥ تسلاً . وتبين بوضوح أنه يمكن أن نجد تأثير تصادم الإلكترون مع الفونون على سعة الموجة بالنسبة للعشر توافقيات الأولى ، إذ أنه

ظهر إنحراف مقداره ٢ ٪ من سعة الموجة للعشر توافقيات الأولى وذلك طبعاً بوجود مجال مغناطيسي مقداره ١٥ تسلاّ .

تبيَّنَّ من الدراسة أيضاً أنه بزيادة المجال المغناطيسي ، فإن تأثير تصادم الإلكترون مع الفونون على سعة موجة دي هاز - ڤان إلفن يكون أوضح .