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An Iterative Approach to Depth Determination of a Buried Sphere from Vertical Magnetic Anomalies

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ABSTRACT. In the present paper, a simple numerical method is developed to estimate the depth of a buried spherical ore body from the zero points of the vertical magnetic anomaly. The depth determination problem from the characteristic distances has been transformed into the problem of finding a solution of a nonlinear equation of the form z = f(z). Procedures are also formulated to estimate the magnetization inclination and the magnetic moment of the sphere. The method is tested on theoretical data with and without random errors and also on a field example from Bankura area in West Bengal, India.

The sphere model is frequently used in magnetic interpretation to find the depth, magnetic inclination, and the magnetic moment of a class of ore bodies. Several methods have been developed for interpreting the magnetic anomalies due to spheres such as those given by Smellie (1956), Paul (1964), Murthy (1967 and 1974), Rao *et al.* (1973 and 1977), Rao and Murthy (1979), Prakasa and Subrahmanyam (1988). Most of these methods use characteristic points and distances for interpreting the anomalous magnetic field caused by spherical ore bodies. For examples, Smellie (1956) derives magnetic anomaly expressions and factors are calculated which may be multiplied by the half-maximum distances on the anomaly profile to yield depth estimates, whereas Murthy (1974) describes a method which involves obtaining and plotting the differences of the two anomalies on either side of the origin and measuring twice the distance of the turning point of the difference curve to give the depth of the sphere. Also, an empirical method has been outlined by Rao *et al.* (1977) for computing the magnetization inclination using the measured distance between the principal maximum, principal minimum, and zero anomaly positions on the vertical magnetic anomaly profiles; and more recently, Prakasa and Subrahmanyam (1988) have presented nomograms for interpreting vertical and total components of the magnetic field. Their method uses several characteristic points and distances of the anomaly profile. Moreovor, the half-width and the maximum slope methods can be also used to determine the depth of the buried structure from magnetic anomalies. An interesting review is given in Nettleton (1976) and Telford *et al.* (1976).

In the present paper, a simple and rapid numerical approach is developed to estimate the depth of a buried spherical ore body from the sum and ratio of the positive and the negative zero anomaly distances of the vertical magnetic anomaly profile.

The depth determination problem from these two characteristic distances has been transformed into the problem of finding a solution of a nonlinear equation of the form $z = f(z)$. The method is tested on four theoretical examples with and without random errors and also on a field example from West Bengal, India.

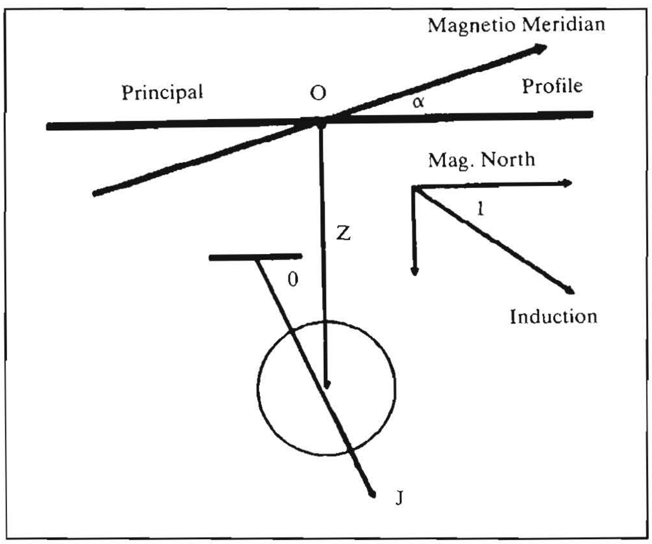

Fig. 1. Geometry of a magnetized sphere.

The Iterative Method

Let us consider a magnetized sphere located below the origin of a cartesian coordinate system (Fig. 1). Let θ be the magnetization inclination in a plane that makes an angle α with the magnetic meridian. The magnetization, J, of the body may be resolved into three components along the axes of the coordinate system. If x, y, z are taken along the magnetic north, magnetic east, and vertically downward directions, then the three-dimensional vertical magnetic anomaly expression at a point (x, y) on the z=0 plane is given as

$$V(x, y, z) = M \frac{(2z^2 - x^2 - y^2)\sin(\theta) - 3xz\cos(\theta)\cos(\alpha) - 3yz\cos(\theta)\sin(\alpha)}{(x^2 + y^2 + z^2)^{5/2}}. \quad (1)$$

In equation (1), M is the magnetic moment of the sphere and z is depth to its center. If θ is the effective angle of magnetization in the plane of the principal profile coinciding with the x direction, and only induced magnetization is considered, then equation (1) reduces to

$$V(x, z) = M \frac{(2z^2 - x^2)\sin(\theta) - 3xz\cos(\theta)}{(x^2 + z^2)^{5/2}}. \tag{2}$$

Setting equation (2) to zero, we obtain the positive and the negative distances at which it attains its zero value as

$$XN = A/2 + \sqrt{A^2/4 + 2Z^2}\ , \tag{3}$$

$$XS = A/2 - \sqrt{A^2/4 + 2Z^2}\ , \tag{4}$$

where $A = XN + XS = -3z\cot(\theta)$.

The positive distance XN is taken from the origin to the magnetic north and the negative distance XS is taken from the origin to the south (Figure 2). In all cases, the positive and the negative zero value distances (XN and XS) can be determined from the observed magnetic data using a simple linear interpolation technique (*e.g.* Davis 1973).

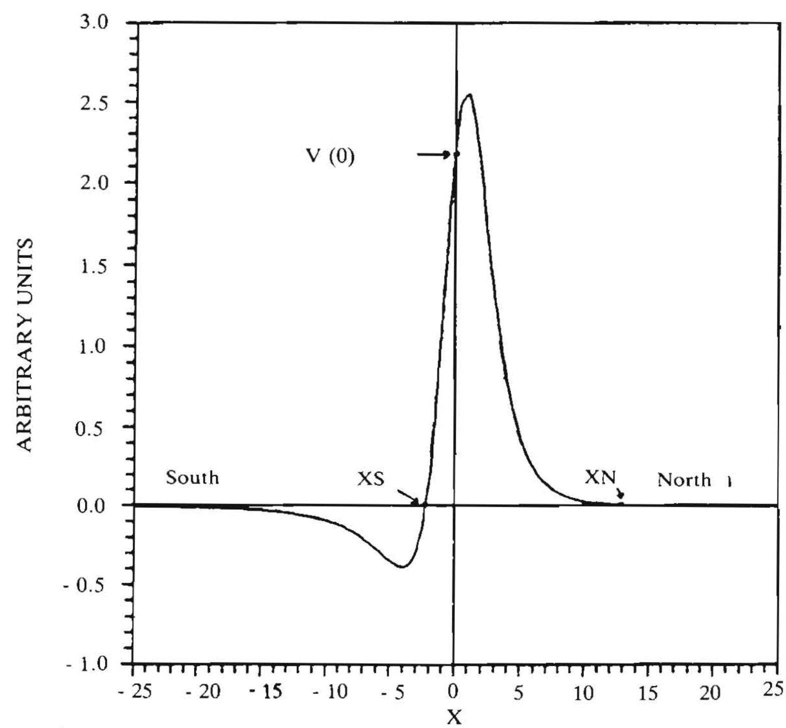

Fig. 2. A typical vertical magnetic anomaly over a sphere showing the definition of XN, XS, and V(0).

The ratio between XN and XS is obtained from equations (3) and (4) as

$$K = XN/XS = \frac{A+\sqrt{A^2+8z^2}}{A-\sqrt{A^2+8z^2}} = \frac{A^2+4z^2+A\sqrt{A^2+8z^2}}{-4z^2} \quad (5)$$

Finally from equation (5), we obtain the following nonlinear equation in z

$$z = \sqrt{\frac{A^2+A\sqrt{A^2+8z^2}}{-4(K+1)}} \quad (6)$$

Equation (6) can be solved using a simple iterative method (Demidovich and Maron, 1973). The iterative form of equation (6) is given as

$$z_f = F(z_i)\,, \quad (7)$$

where z_i is the initial depth and z_f is the final depth. Starting from the initial depth z_i, we calculate the final depth z_f from equation (7). If $| z_f - z_i | \leq 10^{-5}$ then z_f is the solution for the depth. On the other hand if $| z_f - z_i | > 10^{-5}$, then we set $z_i = z_f$, and recalculate z_f from equation (7) using the new value of z_i. We repeat these steps until $| z_f - z_i | \leq 10^{-5}$.

Because z and A are known, the magnetic inclination θ can be determined from the following relationship

$$\theta = \cot^{-1}(-A/3z). \quad (8)$$

Knowing z and θ, the magnetic moment can be determined from the following relationship

$$M = V(0)\, z^3 / 2 \sin(\theta), \quad (9)$$

where V(0) is the anomaly value at x = 0, the origin.

For most values of the magnetization inclination, both the positive and the negative zero anomaly distances will be located in close proximity of the sphere, thus making these distances relatively immune from the neighboring interference. However, it should be noted that for very low inclination angles (θ approaching 0 or 180 degrees), either the negative or the positive distance (respectively) will depart significantly from the origin and hence it will be difficult to pick A accurately.

The value of θ, as obtained from the present method, lies in the range of -90 to 90 degrees, although, in reality, they can take any value between 0 and 360 degrees. The actual value can be determined by a careful examination of the field profile. Position and sign of the dominant maximum and dominant minimum, with respect to the magnetic north or northeast or northwest, determine the actual value following the criteria given in Table 1. In all cases shown in Table 1, the minus sign of θ is not disregarded.

Table 1. Criteria for determining the actual value of the magnetization inclination

Position and sign of dominant anomaly	Actual value of θ for V profile	
	Northern hemisphere	Southern hemisphere
Dominant positive to the south	θ	180 + θ
Dominant positive to the north	180 + θ	360 + θ
Dominant negative to the south	180 + θ	θ
Dominant negative to the north	360 + θ	180 + θ

Application to Theoretical Data

The parameters of four theoretical models are given in Table 2. The vertical magnetic anomalies are generated from equation (1) with a station separation of 1 depth unit. In each case, a search algorithm is used to determine XN and XS values using a simple linear interpolation technique between the observed values (Davis 1973). When XN and XS are obtained, A and K can be calculated. Using A and K values thus obtained, z can be estimated from (6), and θ and M values are then computed using equations (8) and (9), respectively. In this way the parameters are evaluated and given also in Table 2 for comparison.

Table 2 reveals that equations (6), (8), and (9) give accurate values of z, θ, and M when using synthetic data with and without random error. The depth and the inclination angle obtained are within ±4 percent whereas the percentage of error in the magnetic moment is within ±9 percent. Even when using XN and XS values approximated to the first significant decimal figure, good results are obtained by using the present method (Table 2), particularly, for depth estimation, which is a prime concern in magnetic prospecting and other geophysical work.

Table 2. Theoretical examples (in arbitrary units)

	Model I		Model II		Model III		Model IV	
Parameters assumed								
z	3.0000		4.0000		5.0000		6.0000	
θ	30°		135°		240°		300°	
M	100		100		100		100	
Parameters evaluated using synthetic data (XN and XS are measured to 5 significant figures)								
		% of error		% of error		% of error		% of error
z	3.0824	2.75	4.0984	2.46	5.0206	0.41	6.0432	0.72
θ	30.69°	2.29	134.18°	-0.61	239.95°	-0.02	299.76°	-0.08
M	106.27	6.27	106.06	6.06	101.29	1.29	101.93	1.93
Parameters evaluated using synthetic data with 10% random error (XN and XS are measured to 5 significant figures)								
		% of error		% of error		% of error		% of error
z	3.0859	2.86	4.0998	2.50	5.0194	0.39	6.0405	0.67
θ	30.72°	2.39	134.15°	-0.63	239.97°	-0.01	299.79°	-0.07
M	109.10	9.10	108.67	8.67	103.63	3.63	104.28	4.28
Parameters evaluated using synthetic data (XN and XS are measured to 1 significant figure)								
		% of error		% of error		% of error		% of error
z	3.0305	1.02	4.1425	3.56	5.0398	0.80	6.0399	0.66
θ	30.23°	0.78	133.76°	-0.92	240.08°	0.03	299.85°	-0.05
M	102.36	2.36	108.74	8.74	102.32	2.32	101.86	1.86

Field Example

Fig. 3 shows a vertical magnetic profile from Bankura area in West Bengal, India (Verma and Bandopadhyay 1975). It represents a vertical anomaly due to a spherical mass of gabbroic composition. The principal profile shown in Fig. 3 has the dominant positive to the south. The positive zero anomaly value distance (XN) is 0.75 km to the northwest and the negative zero anomaly value distance is -5.30 km to the southeast. The anomaly value at the origin is about 1100 gammas. Hence, A = -4.55 km and k = -15/106. Results of interpretation by present iterative method are $\theta = 42.91°$, z = 1.4098 km, and M = 2263.77 gammas/km^3. The geomagnetic dip in the Bankura area is about 27°. The principal profile in Fig. 3 trends 43° west of north. The effective inclination of magnetization in the plane is therefore 35° because $\tan(\theta) = \tan(i) / \cos(\alpha)$ (Prakasa Rao and Subrahmanyam 1988). The estimated value is found to be generally in good agreement with the actual value. Our results are also in good agreement with the results obtained by Verma and Banopadhyay (1975), Rao *et al.* (1977) and Prakasa and Subrahmanyam (1988) (Table 3). A theoretical anomaly profile for a sphere with the parameters evaluated by the present method gives good agreement with the observed profile shown in Fig. 3.

Table 3. Computed parameters

Parameters	Using Verma and Bandopadhyay method (1975)	Using Rao *et al.* method (1975)	Using Prakasa and Subrahmanyam method (1988)	Using Abdelrahman and El-Araby method (present method)
z	1.32 km	1.32 km	1.52 km	1.41 km
θ	induction	40°	39°	43°
M				2264 γ/km^3

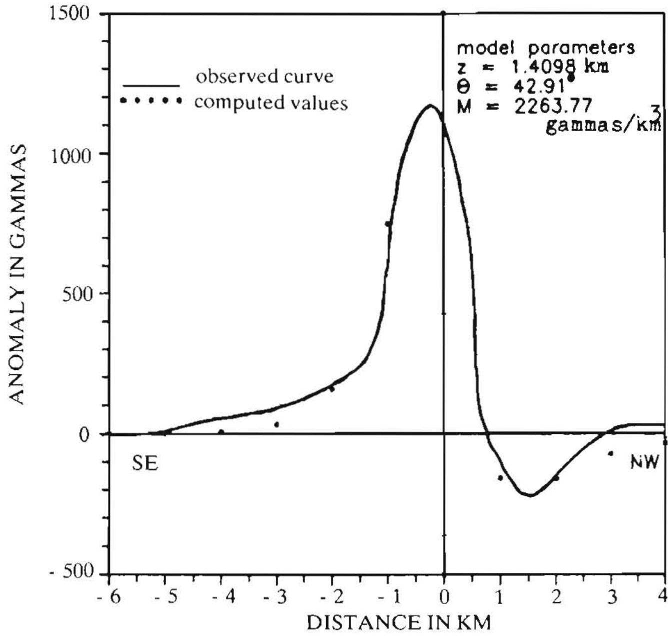

Fig. 3. Vertical magnetic anomaly profile over a spherical feature in the Bankura area in West Bengal, India (Verma and Bandopadhyay 1975).

Discussions and Conclusion

The problem of depth determination of a buried sphere magnetized vertically, has been transformed into the problem of finding a solution of a nonlinear equation of the form z = f(z). The method has the advantages that: (1) the interpretation procedure is vrery simple to execute in the field using a simple programmable calculator, (2) the anomaly curve or the actual field data can be used directly, and (3) the method can be extended to vertical anomalies but also horizontal anomalies due to spheres. In higher latitudes, the measured total field anomaly map approximately resembles the vertical anomaly map. Hence, the present method can be applied to the total magnetic anomaly profiles measured in higher latitude areas. Moreover, reduced to the pole data can be inverted effectively using present method.

It is also emphasized that the effect of noise, due to surrounding bodies is kept at minimum, as the characteristic distances are close to the origin. Results of Interpretation by the iterative method, are in good agreement with those of others, and the theoretically computed profile matched well with the observed profile (Fig. 3).

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طريقة عددية لتعين العمق
لكرة من الشاذات المغناطيسية الرأسية

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في هذا البحث تمّ اعطاء طريقة عددية لتعين العمق الى بعض التراكيب الجيولوجية المدفونة تحت سطح الأرض والتي يمكن تمثيلها بكرة وذلك بواسطة استخدام مجموع ونسبة المسافات الأفقية الموجبة والسالبة على بروفيل والتي تصل فيها قيمة الشاذات المغناطيسية الرأسية للصفر.

وقد تمّ تحويل مشكلة تعين العمق من مجموع ونسبة قيم هذه المسافات إلى إيجاد حلّ لمعادلة غير خطيّة في صورة د $Z = f(Z)$ باستخدام طريقة ديموفتش ومارون (١٩٧٣) واستخدمت طريقة دافيز (١٩٧٣) لتعين قيم المسافات الموجبة والسالبة من بيانات الشاذات المغناطيسيّة الرأسيّة والمقاسة على بروفيل. ثم أُعطِيَت طرق أيضاً لتعين الميل المغناطيسي والعزم المغناطيسي للكرة وذلك بدلالة العمق.

- تمّ تطبيق الطريقة على أربعة نماذج نظريّة ذات أعماق وميل مغناطيسي مختلف وذات عزم مغناطيسي ثابت بها خطأ عشوائي بنسبة ١٠ بالمائة وبدون خطأ عشوائي. ووجد في جميع الحالات المدروسة أن نسبة الخطأ في تقدير الأعماق لا تزيد عن أربعة بالمائة وفي تقدير الميل المغناطيسي وجد أن نسبة الخطأ لا تزيد عن ٢,٥ بالمائة.

كما تمّ تطبيق الطريقة على مثال حقلي من منطقة بنكورا، غرب البنغال بالهند ووجد أن العمق والميل المغناطيسي لخام جيولوجي والمقدّرين بالطريقة الجديدة يتفق مع النتائج التي توصّل إليها العديد من العلماء السابقين.

- تمتاز هذه الطريقة بأنها سهلة التطبيق في الحقل يمكن أيضاً تطبيقها على شاذات المجال المغناطيسي الأفقي للكرة لتقدير العمق والميل المغناطيسي والعزم المغناطيسي.