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State Space Analysis of the Spatial Variability Field-Measured Infiltration: A Case Study from Saudi Arabia

A.M. Al-Omran and Omer El-Bassir

Soil Science Department, College of Agriculture,
King Saud University, P.O. Box 2460,
Riyadh 11451, Saudi Arabia

ABSTRACT. The variability of infiltration rate within a field is expected to be correlated with that of soil physical and chemical properties. Observations of infiltration rate along two transects made in a field comprised of calcareous loamy soil (Torripasamments) were measured together with some parameters affecting infiltration rate (clay, sand and silt content; bulk density; and $CaCO_3$). State space approach was used to identify which of these parameters most affecting final infiltration rate and to improve the interpolation of the infiltration rate along the transects. Variability of final infiltration rate along the transects could be explained by the variability of bulk density or clay content using state space model of infiltration-bulk density and clay content. The results also show that the model could be used to estimate final infiltration rate with values of final infiltration rate considered missing at every other location along the transect when all observations of bulk density and clay % included. Adding other properties such as silt, sand, and $CaCO_3$ to the model did not improve the estimate of final infiltration rate.

Infiltration is one of the most important processes governing the movement of water into the soil. Infiltration rate varies within fields (Warrick and Nielsen 1980, Russo and Bresler 1981, Vieira *et al.* 1981) and it is affected by many soil properties. Warrick (1983) listed some of those fundamental properties such as soil water content, hydraulic conductivity, soil water characteristic relationship, bulk density, soil texture, chemical properties of the soil and plant population. An earlier review of the parameters governing the infiltration process has been published by Philip (1969). Recently, Ben-Hur *et al.* (1985) studied the effect of soil texture and $CaCO_3$ content on infiltration rate with water of different qualities. They found that with an increase of clay content up to 19.2%, the final infiltration rate decreased due to increase in rate of crust formation. With more clay content of 39.9% the final infiltration rate remained large because of the stability of aggregates. They also found that a change in silt (5.6-34.3%) and $CaCO_3$ (5-16%) did not cause differences on the final infiltration rate for water of different quality.

The large spatial variabilty of infiltration rate of the soil has suggested to previous investigators that many observations are essential. However, applying geostatistical methods in analyzing data provides means of minimizing the number of observations necessary to characterize the infiltration rate in field (Vieira *et al.* 1981). The state space model introduced by Shumway (1985) provides another approach to analyze field observations. Morkoc *et al.* (1985a) showed that observations of soil temperature and incomplete observations of soil water content can be jointly used to estimate missing soil water content values. Shumway *et al.* (1989) reported on the data of Morkoc *et al.* (1985a,b) using mean transect values of crop yield, water content, temperature and salt concentration at interval of 1 m. They compared a multivariate model of state space to univariate spline model, and concluded that multivariate model of crop yield, temperature, water content and salt concentration does not significantly improve the crop yield estimate over that of the univariate (yield) model. However, a joint analysis of correlated variables can be used to estimate the variable that is more time consuming and difficult to measure.

Our objectives in this study are:

i) to identify the most variables affecting measured infiltration rate along the transect.
ii) to explore the utility of the state space approach in identifying those properties.
iii) to demonstrate the utility of the approach using the identified parameters to interpolate the infiltration rate data.

Materials and Methods

The data analyzed and reported here are part of field study carried out at the King Saud University Experimental Research Station, Saudi Arabia (El-Bassir 1989). The experimental site consists of about 19 hectares in the northwestern corner of the station. The soil, classified as calcareous loamy soil (Torripsamments), was not cultivated having a natural vegetation of scattered bushes and annual grasses. The gravel and stone percentage increases from north toward the south edge of the field. Two transects were laid out within the experimental site. Transect I was sampled from south to north every 5 m for 500 m. Transect II, east of the first transect, was sampled every 20 m for 900 m from south to north. Infiltration rates were measured using double-ring infiltrometers (Maller and Sharma 1981, Sisson and Wierenga 1981). The inner (23 cm diameter) and outer (75 cm diameter), rings having a height of 15.3 cm, were inserted into the soil to depth of 5 cm with minimum disturbance of the soil surface. The times for cumulative infiltration of successive depths of water were recorded. Observations were terminated when the infiltration rate remained constant in 2-3 successive readings. Under flooded conditions, infiltration flux (i) as a function of time (t) can be estimated using the Philip equation (1957):

$$i = S/2\, t^{-1/2} + i_c \qquad (1)$$

where S is sorptivity and i_c is a variable related to soil water potential and hydraulic conductivity which are both functions of initial water content. A simple linear regression equation computed from measured values of i versus $t^{1/2}$ at each location along the transect provided estimate of the intercept i_c. The intercept was considered the final infiltration rate. Measurements of chemical and physical properties performed on the less-than-2mm fraction included particle size distribution, bulk density and $CaCO_3$. Particle size distribution was estimated by the hydrometer method (Black 1965). $CaCO_3$ content was estimated using a calcimeter procedure described by Allison and Moodie (1965).

State space Models

To achieve our objectives in determining the properties that are correlated with space and influence the infiltration rate, state space models were used. The state space approach was introduced by Kalman (1960), Kalman and Bucy (1961) in their aerospace research. The method has been applied to data in different academic fields such as economics (Shumway and Stoffer 1982), in hydrology (Georgakakos *et al.* 1990) and soil science (Shumway 1985, Morkoc *et al.* 1985 (a,b), Alemi *et al.* 1988, Nielsen *et al.* 1989). The general model as described by Shumway *et al.* (1989) in details, assumes that some unobserved px1 vector of $X_i = (X_{i1}, ...X_{np})$ which can observed through the qx1 observation equation:

$$Y_i = A_i X_i + V_i, \quad i = 1, 2 ....n \qquad (2)$$

where Y_i denote the observed vector of soil properties at spatial point i, A_i is qxp measurement matrix and V_i is a qx1 zero-mean vector with qxq covariance matrix, $cov(v_i) = R$. The state vector X_i satisfies the state space equation, which describes the way X_i moves through space,

$$X_i = \Phi X_{i-1} + W_i, \quad i = 1,2....n \qquad (3)$$

where Φ is a coefficient of pxp transiton matrix and w_i is the state noise vector with mean zero and covariance matrix Q. The initial value of X_i is to be X_0 with mean vector u and covariance E. In state space modeling u, Φ, E, Q, and R are estimated from the observed series Y_i, i=1,2...n by an iterative procedure using Kalman filtering, smoothing and expectation maximization (EM) algorithm given in Shumway (1988). The procedure is repeated until we obtain stable value of -log likelihood function. The equations represent the observation and state space equation for the first order state space model which we used for our data. Higher orders model can be used as described by Shumway (1988).

Results and Discussion

Final infiltration rate, bulk density and particle size percentage for transect I and II are shown in Fig. 1 & 2. In as much as only one measurements of soil properties at each location, we assume that the local error is incorporated in each measurement along the transects. For transect I large values of the final infiltration rates were observed at distance of 0-100 m. Subsequently, the infiltration rate decreased toward the north along the transect. Fig. (1b) shows that the greatest variation of bulk density along the transect occurs between 0-100 m. Fig. (1c) shows that more of sand combined with less of silt and clay occurs at the south edge of the transect. Transect II shows a larger variation of all properties. The largest variation of infiltration rate observed at south edge of the transect as shown in Fig. (2a & b) shows high values of soil bulk density at southern end of the field. Fig. (3c) shows large sand percentage and small of clay and silt percentage in the southern section of the filed. Silt and clay increased toward the northern section (400-900 m) of the field.

Figs. 3 through 5 show the observed final infiltration rate as open circles and 95% confidence limits of final infiltration rate predicted by the first order state space models as solid lines. Fig. 3 (a & b) shows the observed and estimated final infiltration rates with 95% confidence limits using state space model for single parameter (version of spline function) to model the final infiltration rate at i position with final infiltration rate at i-1 position as an input for transect I and II, respectively. The models in the figures indicate that infiltration rate coefficients Φ were heavily weighted on the previous position of the final infiltration rate.

Fig. 4 (a & b) shows the results of bivariate state space models of final infiltration rate and bulk density for transect I and II, respectively. The models indicate that the infiltration coefficients Φ were less than that of single models and there is some weight contributed to the bulk density parameter. Summary of the results of using state space models with different variables are shown in Table 1. The Table shows the values of r^2 of observed vs. estimated final infiltration rate, mean standard error(MSE) of the models, and Akaike's Information Criteria (AIC), (Akaike 1974).

$$AIC^{*} = -2 \log \text{likelihood} + 2 \text{ (number of parameters)}.$$

The best suited model can be chosen with minimum value of AIC. The results show that minimum values of AIC were observed with the infiltration rate-bulk density model for the two transects. The AIC values for transect I and II were −246 and −54, respectively. The bivariate model of transect II improve the r^2 to 0.86 compared to 0.79 obtained with single model. For transect I, r^2 was 0.92 for both single and bivariate models. The values of AIC indicate that the models of infiltration rate and bulk density are the best model to estimate infiltration rate. The results also show that the model of infiltration rate

* Aic is a statistical estimation procedure for determining the best of alternative parametric models fitted to the data and calculated by −log likelihood +2 (number of parameters), Shumway *et al.* 1989; Kitagawa and Gersch, 1984.

and clay content gave the second smallest value of AIC for both transects. Although, the AIC value of combined infiltration rate and $CaCO_3$ model was lower than infiltration rate and sand model, however the r^2 value of a combined infiltration rate and $CaCO_3$ model was lower which may indicate that the model is better in estimating $CaCO_3$ and not infiltration rate. Summary of the reuslts indicate that $CaCO_3$, sand and silt content did not improve the estimation of infiltration rate. This may suggest that $CaCO_3$ has no effect on infiltration rate, which is in agreement with the finding reported by Ben-Hur *et al.* (1985). The results indicate that state space approach as suggested by Shumway *et al.* (1989) can be useful technique to estimate the final infiltration rate and also to determine the properties that explain the variability of the final infiltration rate along the transects. Although, the infiltration rate is affected by different soil properties as listed by Warrick (1983), the state space model used in our study clearly indicates that the variability of final infiltration rate along the transect could be expained by the variability of bulk density and with less degree on clay content. The results seem to indicate that the bivariate model of bulk density and infiltration rate improved the estimate of final infiltration rate over that of single version model.

Table 1. AIC, mean standard error, and R squared values of the regression for the relationship between final infiltration rate and estimated infiltration rate from different variables using state space approach

Models	Transects					
	I			II		
	AIC*	r^2	MSE	AIC	r^2	MSE
in	149	0.09	0.54	126	0.79	0.97
inbk	−246	0.92	0.54	−54	0.86	1.02
insd	617	0.90	0.53	366	0.78	0.87
incl	355	0.91	0.53	229	0.78	0.96
inst	509	0.90	0.53	318	0.78	0.59
inca	453	0.88	0.52	313	0.75	0.88
inbkcl	−69	0.93	0.53	32	0.76	0.76
inmbk	−323	0.84	0.64	−87	0.76	1.4
inmbkcl	−118	0.85	0.64	11	0.75	1.2

in = final infiltration rate, ca = $CaCO_3$ content (%), sd = sand %, st = silt %, cl = clay %, bk = bulk density, inm = infiltration rate with missing even values.

* Akaike's Information Criteria = −2 log likelihood + 2 (number of parameters)

The other objective of state space model as noted by Morkoc *et al.* (1985a) and Shumway *et al.* (1989) is to utilize the approach with missing observations. Since the best model we obtained from the all series was that of infiltraiton rate-bulk density, thus we try to use state space approach to interpolate the infiltration rate from bulk density and some infiltration rate observations. The results of state space estimate of final infiltration rate are shown in Fig. 5 (a & b) and Table 1 for both transects. The r^2 of observed vs estimated final infiltration rate of transect I decreased from 0.92 for complete model to 0.84 for incomplete model, and decreased from 0.86 to 0.76 for transect II. Both models of incomplete data of transects I and II gave a good estimate with some infiltration rate data input. The results clearly indicate the advantage of state space technique in interpolating missing values of variables as suggested by (Morkoc *et al.* 1985a, Shumway *et al.* 1989). The results show that state space approach is useful in estimating the missing even values of final infiltration rate of both transects. Thus, the joint analysis of different variables can be used to estimate the variable that is more time consuming and difficult to measure, such as infiltration rate from the bulk density data.

Another attempt was done to model final infiltration rate, bulk density and clay content for complete infiltration rate data and with missing even values of infiltration rate of both transects, results are shown in Table 1. They show that AIC value for transect I was larger (-69) for the multivariate model compare to (-246) of bivariate model, but r^2 did not change from that of bivariate model. This indicates that the multivariate model of infiltration rate-bulk density-clay content was not as good as the bivariate model if infiltration rate and bulk density model to estimate all the properties used in the model, but still good to estimate the final infiltration rate especially in case of fewer available values of infiltration rate. For transect II adding clay content varaiable to bivariate model of infiltration rate and bulk density model did not improve the estimate of final infiltration rate. The Akaike's Information Criteria (AIC) value increased from -54 to 32 and r^2 decreased from 0.86 to 0.76 with bivariate and multivariate models, respectively.

Table I also shows the mean standard error (MSE) of final infiltration rate of all models tested for both transects. For transect I, MSE values are almost constant for all models except for the two models with fewer observations of infiltration rate. Transect II shows larger MSE values with all models tested and were much higher for missing observation models. This indicates the large variation of infiltration rate data collected along the transect II compared to transect I as shown in Fig. (2a). The MSE of the models with even missing observations were higher in both transects which indicates a large variation along the transects, where the distance between observations become 10 and 40 m for transect I and II, respectively.

Conclusion

The state space approach was used to identify variables affecting infiltration rate along two transects with south-north direction 100 m apart. The results indicate that variabilty of infiltration rate could be explained by the variability of bulk density and with less degree with variability of clay content along the same transect. The results also indicate that the technique could be used to interpolate the infiltration rate from the bulk density or clay content along the transect with fewer infiltration rate observations. Finally future research should allow us to estimate infiltration rate from an easy and inexpensive properties measured in the field.

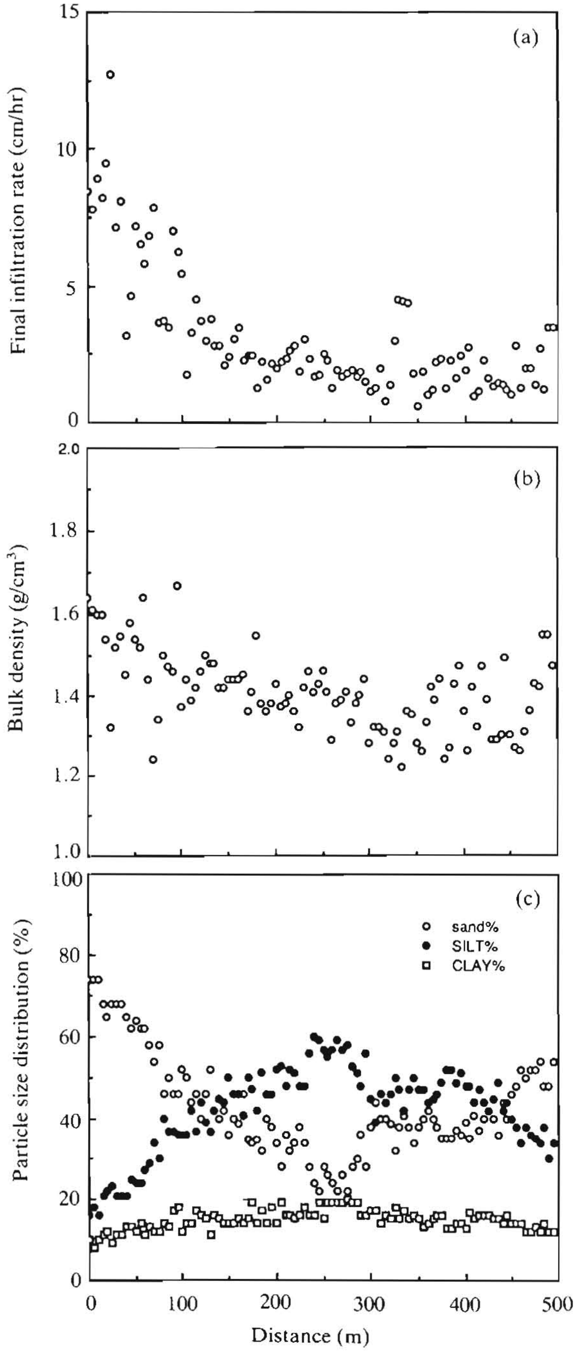


Fig. 1. Spatial field variability of final infiltration rate (a), bulk density (b), and particle size distribution (c) along transect I.

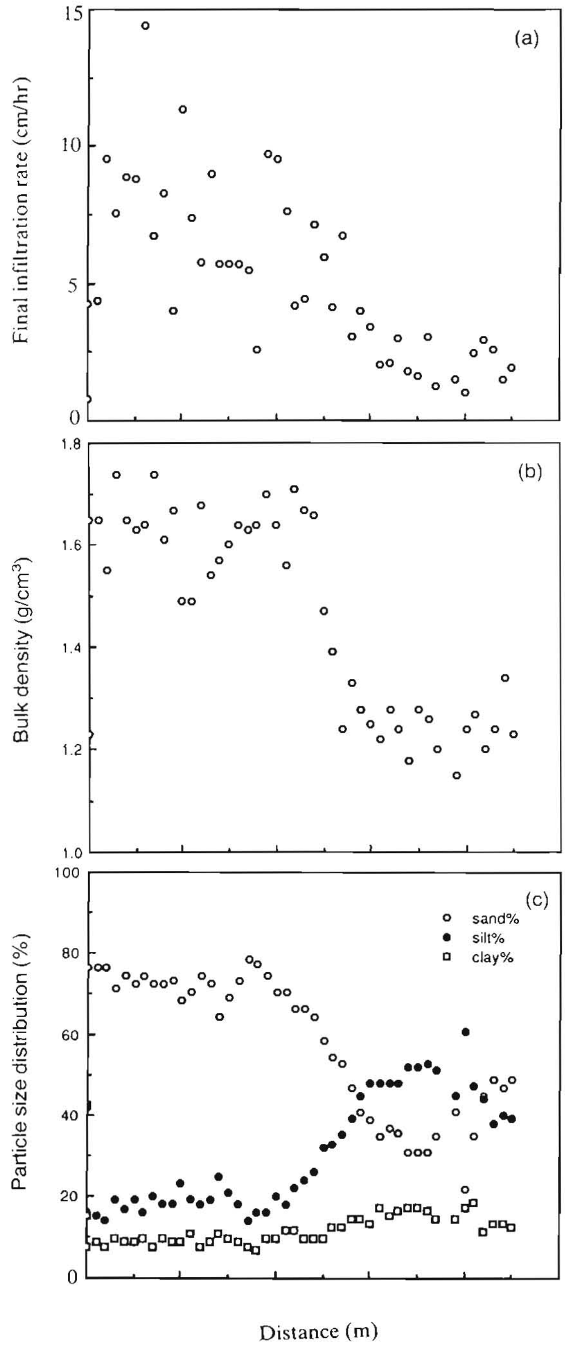


Fig. 2. Spatial field variability of final infiltration rate (a), bulk density (b), and particle size distribution (c) along transect II.

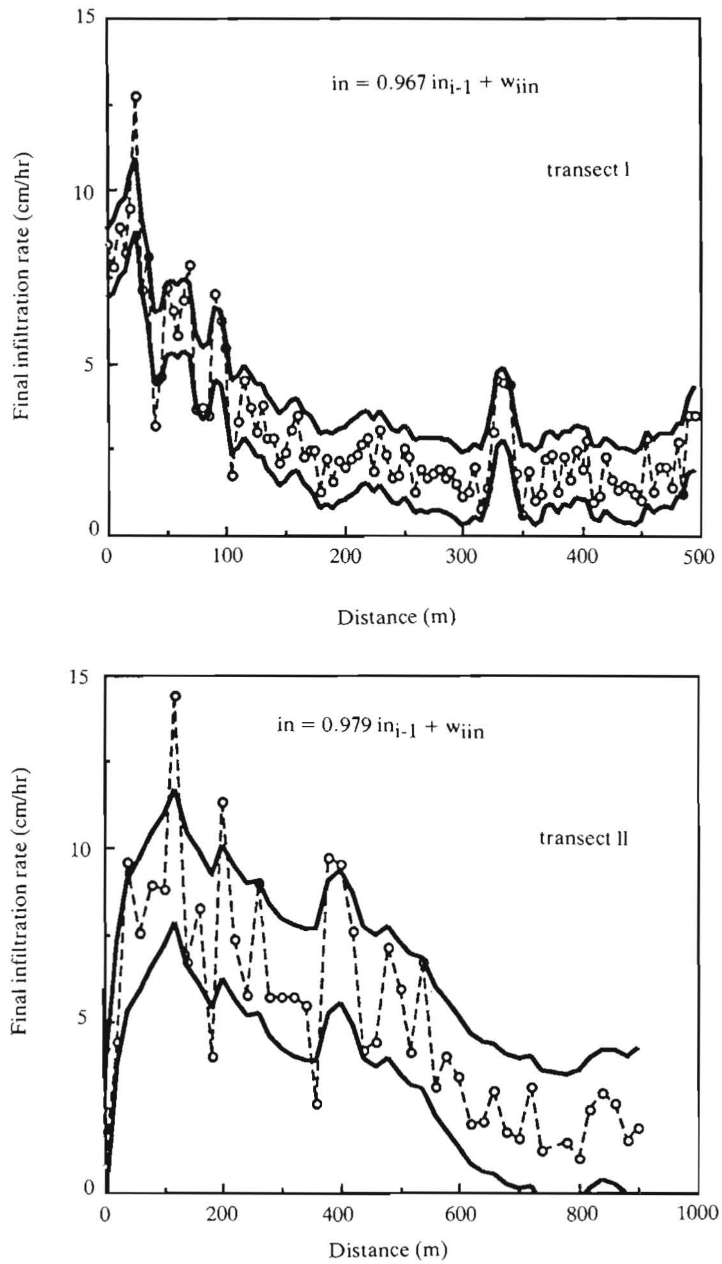


Fig. 3. A 95% confidence limit of estimated infiltration rate along the transect using final infiltration rate values in state space model. Circles represent observed infiltration rate.

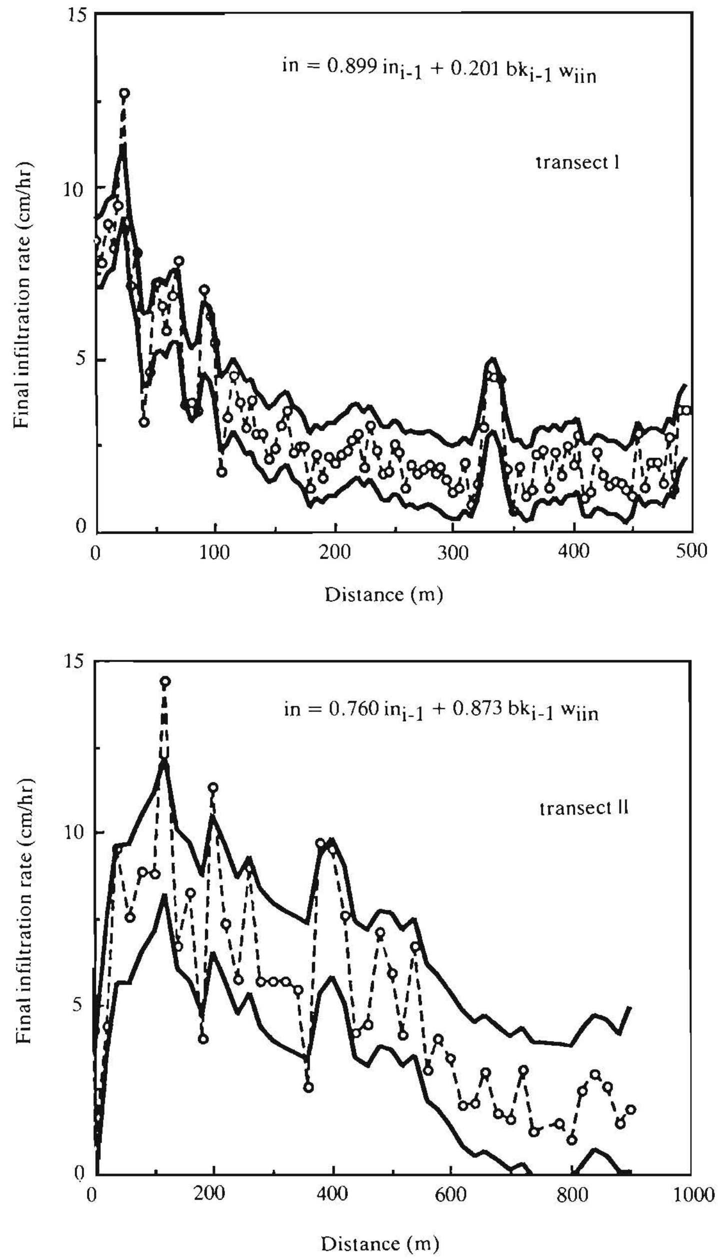
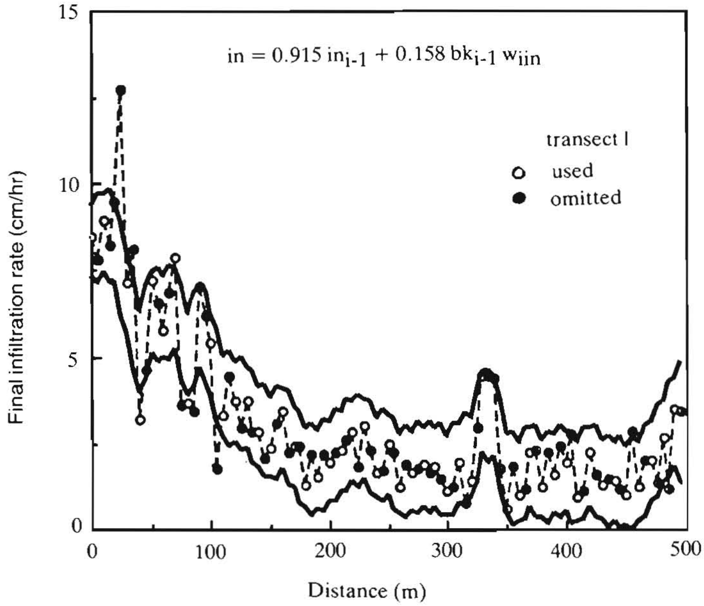


Fig. 4. A 95% confidence limit of estimated infiltration rate along the transect using infiltration rate and bulk density values in state space model. Circles represent observed infiltration rate.



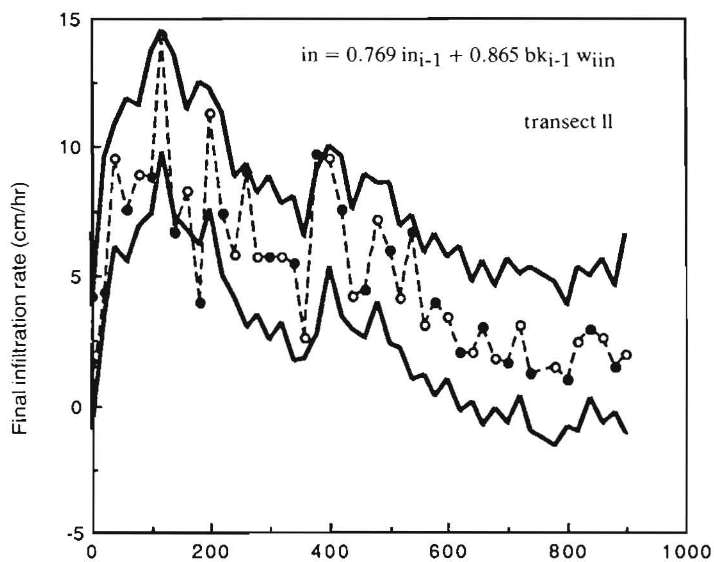


Fig. 5. A 95% confidence limit of estimated infiltration rate along the transect using odd values of infiltration, and all values of bulk density in state space model. Closed circles are omitted values.

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الدالة المكانية لتحليل الاختلافات المكانية لمعدل التسرب في الحقل دراسة من المملكة العربية السعودية

عبدرب الرسول موسى العمران و عمر البصير

قسم علوم التربة - كلية الزراعة - جامعة الملك سعود
ص.ب : ٢٤٦٠ - الرياض ١١٤٥١ - المملكة العربية السعودية

إن التغيرات في معدلات التسرب في الحقل ترتبط بعوامل التربة الطبيعية والكيميائية. ولقد تم قياس معدل التسرب على امتداد قاطعين في الحقل في ترب جيرية طميية في المملكة العربية السعودية بالاضافة الى خصائص أخرى مرتبطة بمعدل التسرب وهي (نسبة الطين، الرمل، الطَمْي، الكثافة الظاهرية ونسبة كربونات الكالسيوم في التربة). تمّ استخدام معادلة الدالة المكانية لمعرفة أي من هذه الخصائص يؤثر على معدل التسرب على امتداد القواطع. ان التغيرات على امتداد القاطع في معدل التسرب يمكن أن يوضح بالاختلافات في الكثافة الظاهرية أو نسبة الطين في التربة باستخدام معادلة الدالة المكانية لمعدل التسرب، الكثافة الظاهرية ونسبة الطين. ولقد أوضحت النتائج كذلك أن النموذج يمكن أن يستخدم في تقدير معدل التسرب على امتداد القاطع لمناطق لم يتم قياسها سلفاً ولكن بقياس الكثافة الظاهرية أو نسبة الطين فقط. أن إضافة خصائص أخرى مثل الطَمْي والرمل وكربونات الكالسيوم الى النموذج لم تحسن تقدير معدلات التسرب على امتداد القاطع في الترب.