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Comparison of Statistical Models for Alfalfa Forage Yield Response to Fertilizer Phosphorus Applied to Calcareous Soils

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ABSTRACT. The estimation of prediction optimum rates of fertilizer involves fitting some type of statistical model to yield data when several rates of fertilizer are applied. The objective of the work, reported here, was to evaluate four models (square root, quadratic, quadratic- plus- plateau and linear - plus - plateau) commonly used for describing the response of alfalfa (*Medicago sativa* L., c.v Hassawi) yield to phosphorus fertilizer. The evaluation involved 10 soil sites and 7 rates of fertilizer application. The four models indicated similar maximum yields, but there was considerable disagreement among the models when predicting economic optimum rates of P fertilization. Optimum rate of fertilizer P (mean all sites) as indicated by the four statistical models ranged from 113 to 412 kg P/ha. The quadratic statistical model did not give a valid description of yield response and predicted optimum rates of fertilizer that were too high. The quadratic- plus-plateau model, best described the yield response was observed in this study. The reason for prefering one statistical model over other deserves more attention than it has received in the past.

Crop response to the applied P fertilizer has been described by a number of mathematical models, starting with Van Liebig's Law in the middle of the last century. The most common statistical model, however, has been the quadratic polynomial ($Y = a + b_1x + b_2x^2$). As a result, in the last few years, several investigators have criticized the indiscriminate use of the quadratic model, especially for developing countries (Anderson and Nelson 1971, Waggoner and Nervell 1979, Lenzer and Paris 1981, Johnson 1991). The main criticisms of the quadratic polynomial method as noted by Jauregi and Paris (1984) and Cerrato and Blackmer (1990) are that:

(i) It does not take into account the sharp, linear type of response often observed at relatively low levels of fertilization.

(ii) It has a point maximum rather than the plateau maximum which is also often observed.

(iii) It tends to overestimate optimal rate of fertilization when compared to other statistical models. The importance of the latter becomes of more economical importance in developing countries, when fertilizer output price ratios are too expensive for farmers to afford the high fertilization levels recommended by the marginal analysis of a quadratic statistical model. The problem is exaggerated by conditions of high risk and economic instability.

The improvement of crop yield production model is a neverending task for agricultural scientists (Johnson 1991). The inputs of this search centers around the needs associated with crop management (Shaffer *et al.* 1984) and examination of the profitability of the fertilizer used (Jahnson and Nofziger 1986, Cerrato and Blackmer 1990, Johnson 1991). Decisions concerning optimal rates of fertilization involve fitting some types of statistical model to yield, data collected when several rates of fertilizer are applied. These decisions are more important now than ever (Cerrato and Blackmer 1990) because there is an increasing concern about environmental impacts associated with the use of this fertilizer. Fertilizer expenses can be a major factor, limiting this input variable in many crop production entrprises (Olson *et al.* 1987, Johnson 1991).

Anderson and Nelson (1971) developed a family of piecewise-linear models that described several types of plant response to fertilizers. These models consist of linear segments that form continuous functions, however, first derivatives of these functions are discontinuous at the junction points of the linear segments. This discontinuity precludes determining economically optimum fertilizer rates as function of fertilizer/crop price ratios.

Much of Saudi agriculture is practiced on highly calcareous soils. These soils often exihibit large adsorption capacities of P and influence the quantity of fertilizer needed to raise available P in soil to the level optimal to crop growth (Mustafa *et al.* 1987). Thus, previous heavy P applications reduced to varying degrees the ability of soil to absorb further additions of P (Al-Mustafa and Ayed 1989).

Relatively little information is available for selecting one statistical model over other statistical methods to describe crop response to P fertilizer especially for highly calcareous soils in semi-arid conditions. The main objective of this study was to compare four different response models for describing alfalfa forage yield to P fertilizer on calcareous soils.

Materials and Methods

Ten field experiments of irrigated alfalfa (*Medicago sativa* L., var Hassawi) were established during 1990 and 1991 in Qatif and Hassa areas in the Eastern Region of Saudi Arabia (sandy loam soil with pH from 7.3 to 7.6, $CaCO_3$ from 4.8 to 12.9% and organic matter 0.38 to 0.67%, $NaHCO_3$-P 3.1 to 5.2 mg/kg). Phosphorus treatments were triple-superphosphate (0-46-0 broadcased and incorporated into the soil at 7 rates (0, 35, 70, 105, 140, 175, and 210 kg P/ha), in a complete randomized block design, shortly before planting. Fertilizer N, K, Fe, Zn were added to all plots as described by Al-Mustafa (1993). The plots were 3.8 X 5.2 m and the seeding rate was 50 kg/ha.

Alfalfa yield was harvested at the 50% bloom stage by cutting the crop from an area of 1.5 x 1.2 m at all sites at the center of each plot. Eight cuttings were harvested each year. Samples collected were washed and dried in forced air oven at 60°C obtaining a moisture content (15%).

Statistical Models

Four response statistical models (square root, quadratic, quadratic plus plateau and linear with plateau) were used to fit the data from each site by using nonlinear regression program (Ihnen and Goodnight 1985) for determining optimal fertilizer rate, as a function of fertilizer/crop price ratio contains parameters that can be used to characterize nutrient-use efficiency by the crop (Bock 1984).

The square root model is defined by:

$$Y = A + B_1X + B_2\,X^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{Eq (1)}$$

where Y is the dry matter yield of alfalfa (Mg/ha) at 15% moisture and X the rate of P fertilizer application (kg/ha); A (intercept), B_1 (linear coefficent) and B_2 (square root coefficient) are constants obtained by fitting the model to the data. The curve reaches a maximum at $X = (-B_1/B_2)^2$ and decreases with increasing X.

The quadratic model is defined by:

$$Y = A + B_1X + B_2X^2 \text{.................. Eq (2)}$$

where Y is the yield of dry matter of alfalfa (Mg/ha) and X is the level of P application (kg/ha); A, B_1 and B_2 are constants obtained by fitting the model to the data with maximum rate of P fertilizer $X = -B_1/2\ B_2$.

The quadratic plus plateau model is defined by:

$$Y = A + B_1X + B_2X^2 \text{ if } X < M \text{ .... Eq (3)}$$

$$Y = S \qquad \text{if } X > M \text{ .... Eq (4)}$$

where Y is the dry matter of alfalfa (Mg/ha), X is the rate of P application (kg/ha), A (intercept), B_1 (linear coefficient), B_2 (quadratic coefficient), M (critical rate of fertilization, which occurs at the intersection of the quadratic response and the plateau lines), and S (plateau yield) are constants obtained by fitting the statistical model to yield data.

The linear plus plateau model is defined by:

$$Y = A + b\,X \qquad \text{if } X < M \text{ ....Eq. (5)}$$

$$Y = S \qquad \text{if } X > M \text{ ....Eq. (6)}$$

where Y is the dry matter of alfalfa (Mg/ha), X is the rate of P application (kg/ha), A (intercept), B (linear coefficient), M (critical rate of fertilization which occurs at the intersection of the linear response and the plateatu lines), and S (plateau yield) are constants obtained by fitting the model to the data.

The fertilizer rate will be referred to as the maximum economic rate of fertilizer application. Simplified, the maximum net return occurs when the last unit of P added produces just enough yield to pay for the added P and may be expressed as

$$\frac{\Delta \text{ yield (kg/ah)}}{\Delta \text{ P added (kg/ah)}} = \frac{\text{P cost (\$/kg)}}{\text{crop value (\$/kg)}} \quad \text{... Eq. (7)}$$

The left-hand side of equation is equal to the slope of the yield curve. The ratio of the P cost to the crop value will be referred to as the price ratio. Therefore maximum return to the producer occurs when the slope of the yield curve is equal to the price ratio. This calculation assumes unlimited producer capital or credit (Bock 1984) and does not include fixed costs or other expenses in producing the crop such as the added cost of harvesting a higher yield.

Results and Discussion

Data in Table 1 show considerable disagreement among the statistical models when used to identify economic rate of P fertilization at given fertilizer to alfalfa price ratio ($ 0.3 cost of 1 kg P fertilizer to $ 0.1 kg of dry matter of alfalfa = 3). The results show that some statistical models were quite consistent in predicting higher optimal rates of fertilizer, with approximately some optimum yield (Table 1), than other models. The square root model tended to predict slightly higher maximum alfalfa yields than did the other statistical models, with high amount of P fertilizer rates to achieve maximum yields. These rates of fertilizer were about 2.1 times higher than those indicated by the quadratic model. These differences indicate a need to justify selection of one model over other statistical models.

Table 1. Maximum yield of alfalfa (Mg/ha) and economic optimum rates of fertilization predicted by each model at each site (kg/ha).

Site No.	Square root		Quadratic		Quadratic-plateau		Linear-plateau	
	Ym	X	Ym	X	Ym	X	Ym	X
1	16.7	288.3	15.7	180.5	15.3	148.2	15.4	113.8
2	15.8	263.2	16.1	186.3	15.6	153.1	15.6	118.5
3	15.3	386.2	15.7	170.0	15.6	158.5	15.5	110.0
4	16.3	358.9	15.5	195.2	15.2	161.5	15.0	120.1
5	16.2	529.0	15.3	210.1	15.3	168.2	15.2	131.8
6	17.9	496.5	15.4	202.3	15.2	173.2	15.0	130.6
7	17.4	510.2	15.8	230.0	15.4	166.4	15.5	139.0
8	16.5	491.8	15.5	233.1	15.7	160.8	15.5	120.3
9	18.2	489.6	15.6	246.5	16.0	163.8	16.1	126.5
10	17.8	521.3	16.4	221.8	15.9	160.8	15.8	121.8
Mean	16.8	433.5	15.7	207.6	15.5	161.5	15.4	123.2

Ym = maximum yield at economic fertilizer to alfalfa price ratio used was 3.0.

X = optimum rate of fertilizer (kg/ha) at economic ratio of 3.0, which is consistent with values of $ 0.3 kg for dry matter of alfalfa and $ 0.1 kg for fertilizer.

Data in Table 2 show that, when evaluating the statistical response models by using cofficients of determination (R^2- value),the four models seem to fit the response yield data about equally well with $P < 0.001$. However, because there is little biological basis of selecting one statistical model over others (Nelson *et al.* 1985), the regression coefficients (R^2- value) are usually used to justify the use of a particular model (Cerrato and Blackmer 1990). For practical purposes, the limitation of using the regression coefficients (R^2- value) to select a model is illustrated in Fig.1 (a to d), which shows how each of the four statistical models used in this work fits the data from site No.3. The observation that models having quite similar coefficient of regression (R^2- value) while differing greatly in rates of economic optimum fertilizer is an indication that regression coefficient (R^2- value) is not a reliable criterion for selecting a model to predict the optimum rate of P fertilization.

Mean square error (MSE) was frequently used to justify, with regression coefficient, the use of particular statistical model (Snedecor and Cochran 1967). Data in Table 2 reveal that mean square error was in considerable disagreement among the models used in this work. The ranks of MSE (mean 10 sites) for the statistical models generally decreased in the order square root > quadratic > quadratic - plateau > linear - plateau. However, this trend was not consistent for all sites, but MSE in the linear - plateau had always the lowest values.

Analysis of residual yield is one or another approach for selecting a proper model (Draper and Smith 1966: Tejeda 1981, Carrato and Blackmer 1990). Figure. 2 (a to d) show the analysis of the residual yield from regression (yield observed - yield

Table 2. Coefficient of determination (R^2 - value) and mean square error (MSE) for models describing relationship between P rate of application and dry matter yield.

Site No.	Square root		Quadratic		Quadratic-plateau		Linear-plateau	
	R^2	MSE	R^2	MSE	R^2	MSE	R^2	MSE
1	0.892	0.831	0.886	0.840	0.996	0.542	0.998	0.501
2	0.885	0.820	0.890	0.541	0.991	0.512	0.999	0.486
3	0.899	0.718	0.939	0.400	0.997	0.418	0.995	0.343
4	0.896	0.733	0.926	0.881	0.973	0.406	0.997	0.351
5	0.863	1.183	0.883	0.882	0.995	0.821	0.996	0.683
6	0.858	1.212	0.896	0.865	0.994	0.784	0.994	0.702
7	0.846	1.296	0.853	1.001	0.863	0.963	0.993	0.685
8	0.873	1.301	0.901	0.812	0.989	0.861	0.995	0.721
9	0.863	1.183	0.892	0.811	0.878	1.031	0.996	0.559
10	0.881	1.022	0.912	0.703	0.963	0.611	0.983	0.635

predicted), for each statistical model. These figures show that some models fit the response data with less systematic bias than other models and suggest that square root and the quadratic models do not fit as well. The latter models (square root and quadratic) do not have a standard normal distribution ($p > 0.05$) and, therefore, in contrast to linear- plateau and quadratic- plateau, subsequently they, do not give a valid description of the alfalfa forage response data.

The gross return for P over the P cost using prices of $ 0.3/kg for alfalfa dry matter and $ 0.1/kg for P fertilizer was calculated from residual yield using different models for all sites (data not shown). These calculations indicate an average return of $ 1.18 for $ 1.00 at 148.5 kg P/ha invested in fertilizer and, therefore, suggests that the rate of p fertilizer given by the quadratic- plateau model may be more appropriate than the one given by the linear- plateau model. The large mean loss calculated, when the quadratic model is assumed to be correct and when the square root model is used to predict economic optimum rates of fertilization, is, of course, evident. It occurred because the square root model predicts higher optimal rate than the quadratic and because the quadratic model predicts rapid decreases in yield when fertilizer is applied at rates higher than optimal rates. If it is assumed that the response data analyzed represent response of alfalfa to P in Saudi Arabia and that the quadratic- plateau model accurately describes this response, then it would be concluded that farmers are loosing $ 55.0/ha if they are applying fertilizer in accordance with the quadratic model. With regard to the assumption that the response data analyzed represent responses of alfalfa to P in Saudi Arabia, it is relevant that mean economic optimum rate of fertilization predicted by the quadratic- plateau model for all sites included in this study, (*i.e* 160 kg P/ha), corresponds almost with the rate 158 kg P/ha that was estimated by Al- Mustafa (1993) to achieve the optimum yields, observed in this study.

The reason for selecting one statistical model over others deserves more attention than it has received in the past when making decisions concerning amounts of fertilizer required for profitable crop production. This study could relate to selection of the most profitable rate of fertilization on a field scale or weighing the costs (economic as well as environmental) and benifits of P fertilization on a regional or national scale. The quadratic with plateau model, best described the yield responses, is observed in this study. Determination of the soil test calibration curve in this manner results in a soil test recommendation which is not influence by yield potential. Yield potential should not influnce the shape of the yield response for immobile nutrients such as P and K (Bray 1963) and, therefore, should not influence the maximum economic fertilizer rate. If sufficient experimental data is available, however, the experiments may be divided into groups, based on, for example, soil texture or climatic region. A soil test calibration could then be developed separately for each group (Al-Mustafa 1993).

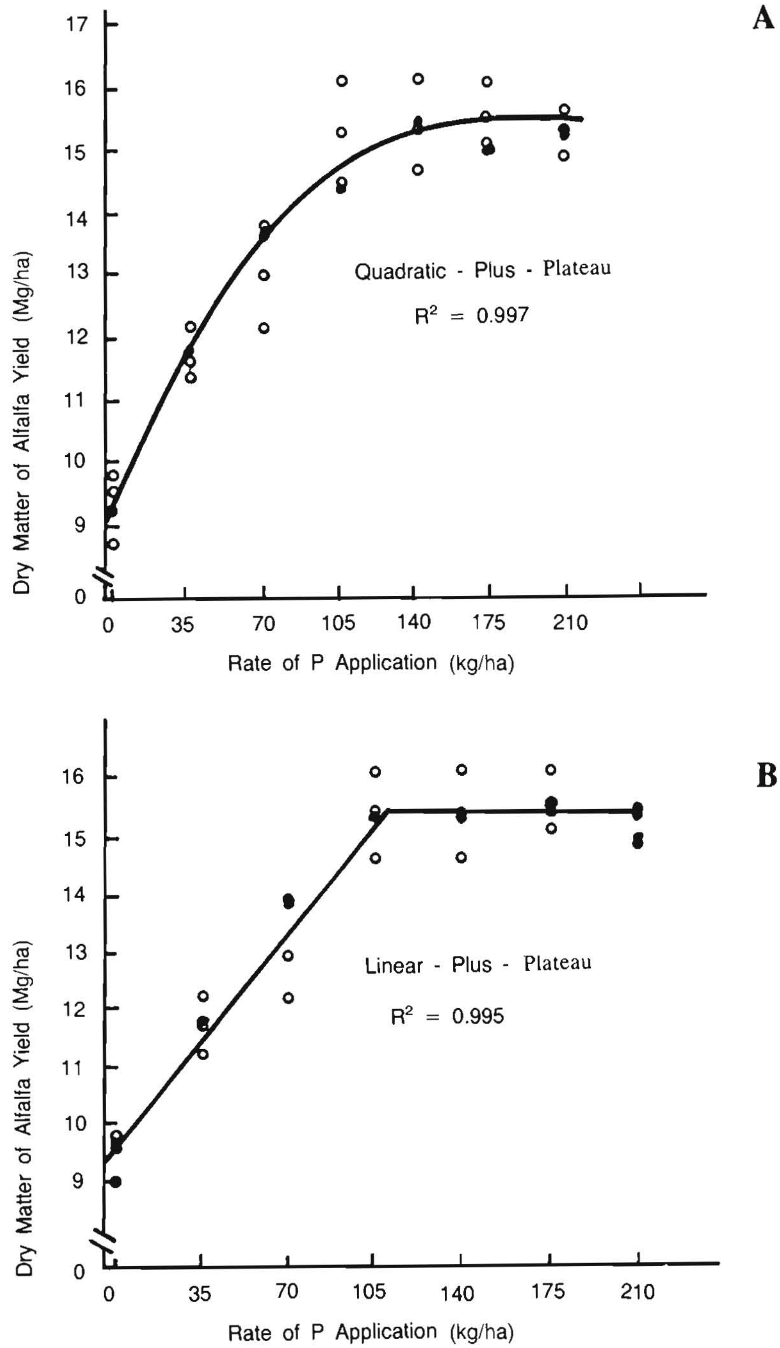
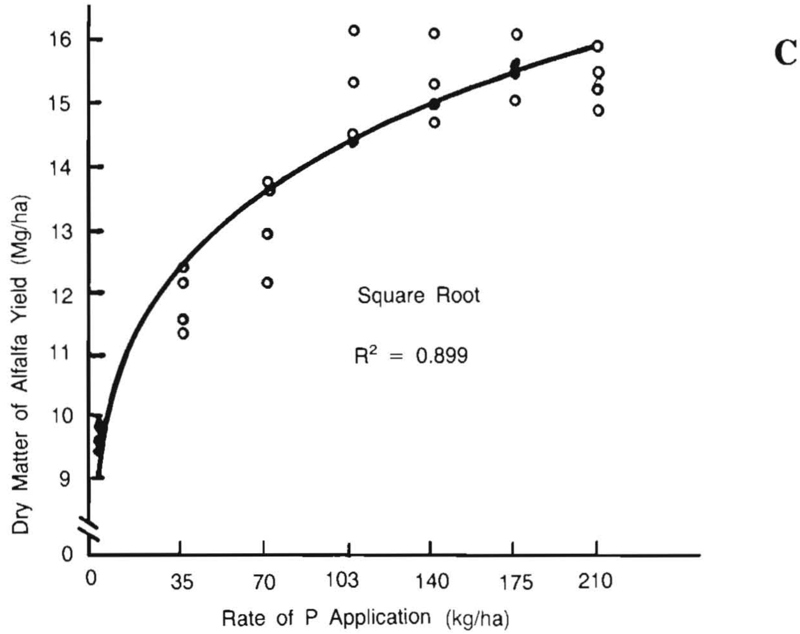
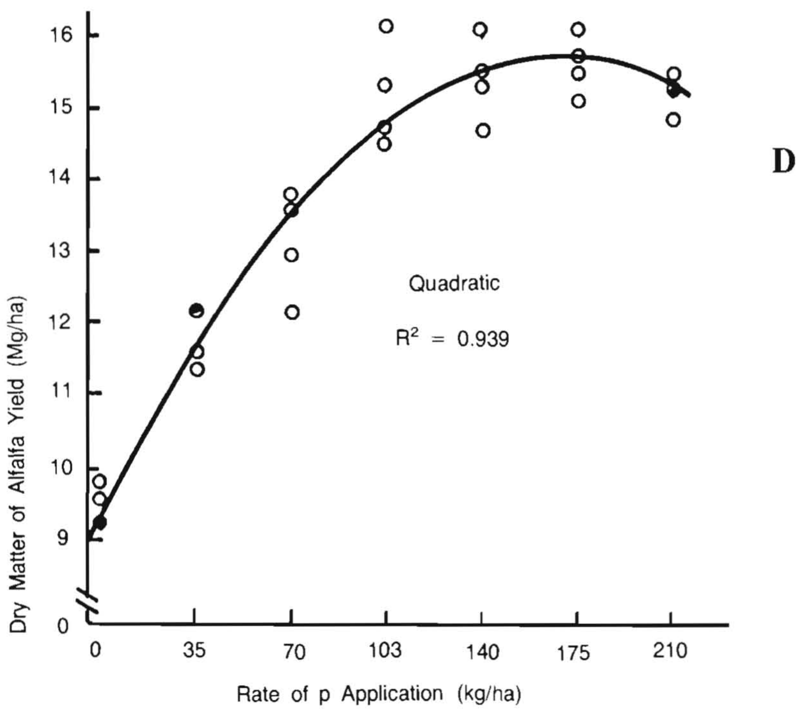
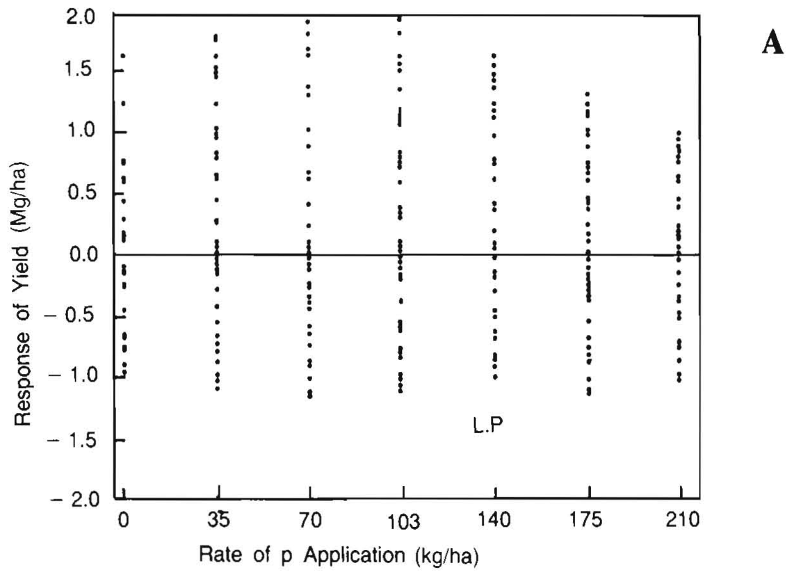


Fig. 1. (a to d). Four models fits the response data for one site - year (Site 3 in 1989).


C
Square Root
R² = 0.899
Dry Matter of Alfalfa Yield (Mg/ha)
Rate of P Application (kg/ha)


D
Quadratic
R² = 0.939
Dry Matter of Alfalfa Yield (Mg/ha)
Rate of p Application (kg/ha)



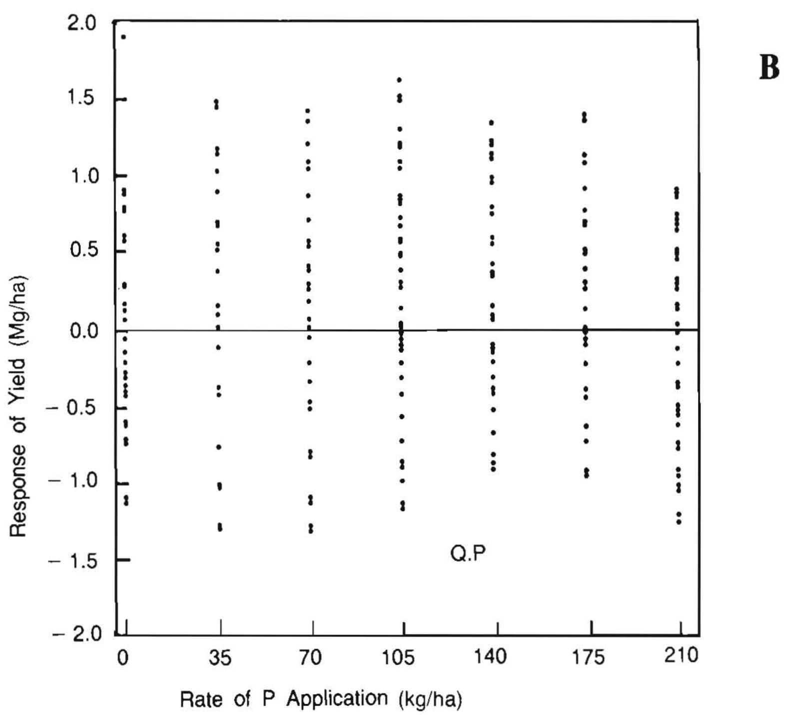
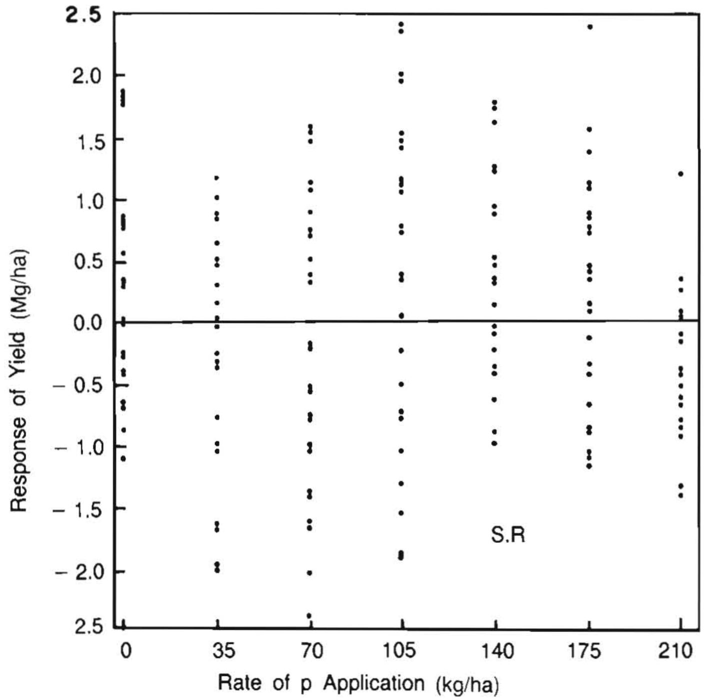
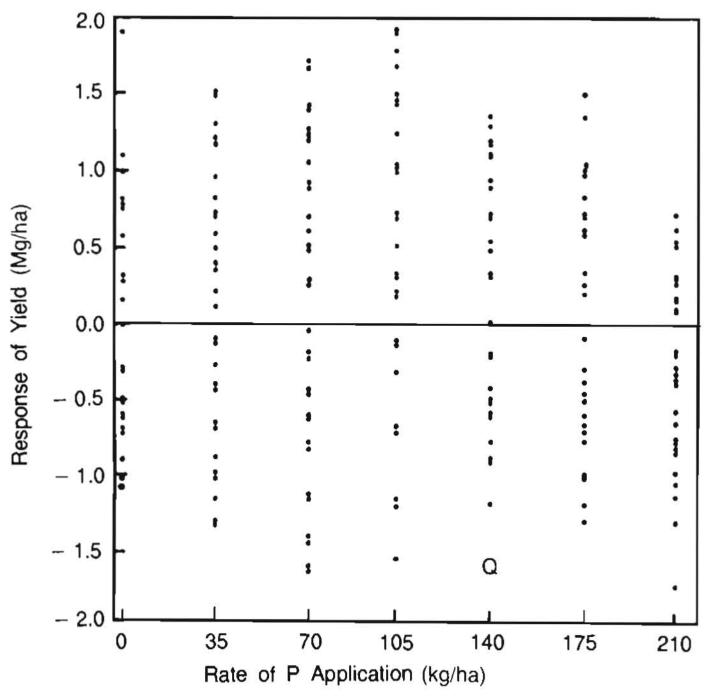


Fig. 2. (a to d) Deviations from regressions (observed yields - predicted yields) observed when four models,L.P (linear-plus plateau), Q.P (Quadratic-plus plateau), S.R (square root) and Q (quadratic), were fit to individual site-years. Each point represents a treatment for a site-year.



2.5
2.0
1.5
1.0
0.5
0.0
– 0.5
– 1.0
– 1.5
– 2.0
2.5
Response of Yield (Mg/ha)
S.R
0
35
70
105
140
175
210
Rate of p Application (kg/ha)

C



2.0
1.5
1.0
0.5
0.0
– 0.5
– 1.0
– 1.5
– 2.0
Response of Yield (Mg/ha)
Q
0
35
70
105
140
175
210
Rate of P Application (kg/ha)

D

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مقارنة النماذج الاحصائية لاستجابة علف البرسيم بالتسميد الفوسفوري في الأراضي الجيرية

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تستخدم بعض النماذج الاحصائية لدراسة استجابة المحصول لعدة مستويات من سماد معين لغرض تقدير المستوى الأمثل لذلك السماد.

والهدف من هذا البحث هو تقدير أربعة نماذج احصائية (الجذر التربيعي ـ الدرجة الثانية ـ الدرجة الثانية مع المسطح والخطي مع المسطح) لدراسة تأثير التسميد بسماد الفوسفور على محصول البرسيم. ولقد استخدمنا في هذا البحث ١٠ ترب مختلفة و٧ مستويات من السماد الفوسفاتي.

تشابهت النماذج الاحصائية الأربعة في تقدير أعلى معدل انتاجي ممكن، ولكن وجدت اختلافات كبيرة بينها في تقدير المستوى الاقتصادي الأمثل للتسميد الفوسفوري. حيث تراوح المستوى الأمثل من السماد بين ١١٣-٤١٢ كلغم/هكتار.

ولم يكن نموذج الدرجة الثانية هو الأفضل في توصيف استجابة المحصول للتسميد حيث أعطى هذا النموذج تقديرات عالية جداً للمستوى الفوسفوري

الأمثل. وكان أفضل نموذج لوصف استجابة محصول البرسيم هو نموذج الدرجة الثانية مع المسطح.

وفي الواقع تستحق طريقة تفضيل نموذج عن آخر المزيد من الدراسة عما حظيت به في السابق.