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Performance of Air Conditioning Heat Exchangers

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ABSTRACT. This paper presents a new graphical method for designing air conditioning heat exchangers. This method is fast, simple and accurate. It gives a direct solution utilizing only the psychrometric chart. It gives also a direct determination of the boundary temperature at which dehumidification begins for that type of problem where a portion of the surface is in a dry condition.

In order to verify the developed technique an experimental test rig was installed. The data were collected for a heat exchanger, which consist mainly of a standard four-row integral fin tube coil. The coil was fitted in an experimental wind tunnel allowing facilities of measuring and varying the main operating variables and conditions which bear effects on cooling coil performance. The test results show a good agreement with the predicted ones, and the average deviation for the total heat is about +15% and for sensible heat is +9%, which are of positive nature and on the safe side for practical design purpose. The method is useful for similar practical applications.

Introduction

The transfer of mass and heat are fundamental phenomena of air conditiong. Cooling and dehumidifying coils are commonly used in which combined heat and mass transfer occurs. Consequently, the ability to predict accurately the performance of these devices is an important and frequently recurring problem.

The problem has received considerable attention by many investigators. From the results of these studies certain concepts and fundamental expressions have been evolved, which related instantaneously to the variables in this particular heat flow mechanism. The coil surface temperature, Lewis relation (Lewis 1922), Goodman derivation of the condition curve (Goodman 1938) and Merkil derivation of the proper heat flow potential (Merkil 1927) are outstanding and accepted concepts.

Goodman's (1938, 1939) surface temperature chart, McEghin and Wiley's (1940) nomographic chart and Kusuda's (1956) surface temperature determination line are graphical techniques to facilitate the anticipation of the cooling coil area required and the final air conditions. Rich and Gable (1962) described an electronic analog computer method to solve this problem.

All previous work solved the surface temperature equation graphically. The (h_{sat}, t_{sat}) curve (relation between the saturated moist air enthalpy and temperature) is redrawn on a separate working sheet. Then, the intersection of coil characteristic line with the (h_{sat}, t_{sat}) curve may be obtained. This intersection point (surface cooling coil temperature) may be transferred once more to the original saturation line on the psychrometric chart. This method described may be called surface temperature chart or coil characteristic monograph or surface temperature determination line. However, these methods are describing the same procedure.

The present proposed method, which solves the problem directly on the psychrometric chart itself without the aid of any additional charts, showed good agreement with the experimental results, with acceptable accuracy for designers.

Mathematical Model

Simultaneous transfer of heat and mass may be formulated by combining the balance equations of mass, energy and diffusion (Fig. 1). The energy may be presented by Fourier law of conduction, while the diffusion is described by Ficks law of diffusion.

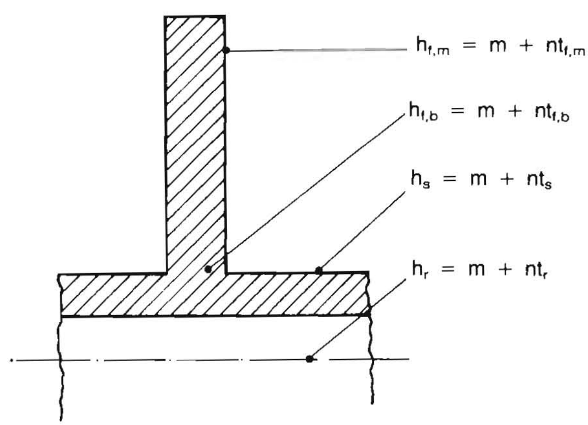

Fig. 1. Fin and tube surface model

Fourier equation in one dimensional form may be written as:

$$q = -K\frac{\partial t}{\partial y}$$

while at the wall,

$$q = -K\left(\frac{\partial t}{\partial y}\right)_{y=0} \equiv h_c(t_w - t_o) \quad (1)$$

Ficks equation in one dimensional form may be written as:

$$Ji = -D\frac{\partial w}{\partial y}$$

and at the wall,

$$Ji = -D\left(\frac{\partial w}{\partial y}\right)_{y=0} \equiv h_d(W_w - W_o) \quad (2)$$

From equations (1) and (2) and for Lewis number equals one we obtain (Sofrata 1980):

$$h_c = c_p\, h_d$$

and hence for a plane surface the heat rate may be presented as:

$$dq = h_c\,(t - t_s)\,dA + h_d\,(W - W_s)\,LdA$$

which may be reduced to

$$dq \simeq \frac{h_c}{c_p}(h - h_s)\,dA \quad (3)$$

Then, in order to employ the same concept for a finned surface, it is necessary to idealize the fin and tube surface as shown in Fig. 1 and consider the following assumptions:

1. The effect of the pipe wall and condensate film resistances are negligibly small with respect to the other heat resistances..

2. For limited temperature differences (*i.e.* 5 K to 10 K), which is true for such cooling coils, it may be assumed that the family of lines between saturation enthalpy of moist air and surface temperature, as suggested by Goodman (1938), are linear relations:

$$h_{sat} = m + nt_{sat} \quad (4)$$

where m and n are evaluated at the refrigerant temperature and can be considered as constant for a limited temperature differences as mentioned before (see table 1 in Sofrata 1985).

This assumption allows for a fictitious enthalpy potential between the air and the surface by representing the surface temperature through its fictitious enthalpy.

3. Let the fin base temperature $t_{f,b}$ equal to the pipe surface temperature t_s, thus,

$$h_{f,b} = h_s \tag{5}$$

Now the heat transferred from pipe to refrigerant, considering the total outer surface area A_w as reference area, is

$$dq_w = h_i\,(t_s - t_r)\,\frac{dA_{p,i}}{dA_w}\,dA_w \tag{6}$$

let $B = \dfrac{A_w}{A_{p,i}}$ the outer to inner pipe area ratio.

$$dq_w = \frac{h_i}{B}\,(t_s - t_r)\,dA_w \tag{7}$$

from Goodmans assumption, equation (4) may have the form

$$h_s - h_r = (m + nt_s) - (m + nt_r)$$

thus,

$$t_s - t_r = \frac{1}{n}\,(h_s - h_r) \tag{8}$$

Substituting eq. (8) in eq. (7) then

$$dq_w = \frac{h_i}{nB}\,(h_s - h_r)\,dA_w \tag{9}$$

The heat transfer from air to outer surface of pipe and fin, considering equation (3), is

$$dq_w = \frac{h_c}{c_p}\,(h - h_s)\,dA_{p,o} + \frac{h_c}{c_p}\,(h - h_{f,m})\,dA_f \tag{10}$$

defining the wet fin efficiency as follows:

$$\frac{t - t_{f,m}}{t - t_{f,b}} = \frac{h - h_{f,m}}{h - h_{f,b}} = \frac{h - h_{f,m}}{h - h_s} = \phi_w \tag{11}$$

then substitute in equation (10)

$$dq_w = \frac{h_c}{c_p}\,(h - h_s)\,(dA_{p,o} + \phi_w dA_f) \tag{12}$$

dividing and multiplying by dA_w
and let

$$F = \left(\frac{A_{p,o}}{A_w} + \phi_w \frac{A_f}{A_w}\right) \tag{13}$$

as the finned surface factor.

Equation (12) can be written as

$$dq_w = \frac{h_c F}{c_p} (h - h_s)\, dA_w \tag{14}$$

Divide (7) by (14) and rearranging

$$\frac{t_s - t_r}{h - h_s} = \frac{h_c FB}{h_i c_p} = C \tag{15}$$

Where C can be considered as coil characteristic.

This is similar to the form originally derived by Goodman with the essential difference, that it takes into consideration the effect of fin efficiency.

Graphical Solution

Referring to Fig. 2 it is clear, from elementary geometry, that the line OB intersects with line OA at point O, D1 B1 and D2 B2 are perpendiculars from points D1 and D2 on the line OB; D1 A1 and D2 A2 are perpendiculars from points D1 and D2 on the line OA.

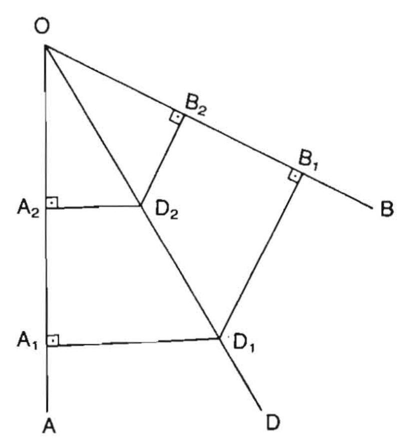

Fig. 2. Geometrical lines relation: OA, OB, OD

The line OD satisfies the condition

$$\frac{A1\ D1}{B1\ D1} = \frac{A2\ D2}{B2\ D2} = \text{Constant} = C \tag{16}$$

Now on the psychrometric chart the enthalpy lines make a constant angle β with the dry bulb temperature lines, in the range ordinarily encountered in air conditioning, the variation of β may be easily justified.

From the psychrometric chart construction it is clear that the enthalpy lines make a constant angle with the humidity ratio lines. The angle is approximately constant since its variation is small, for instant, an enthalpy line of zero Kcal/kg makes an angle of 47.78° with zero°C dry bulb temperature line, while for enthalpy line of 15 Kcal/kg makes an angle of 51.0° with 40°C dry bulb temperature line.

Moreover to find out the average direction of "C", the coil characteristic line, we choose the direction of arithmetic mean refrigerant temperature, this choice is quite acceptable, since the refrigerant temperature increase is small with respect to zero or to 40°C as mentioned above.

From similarity, for certain cooling coil of characteristic C, having inlet air state S of enthalpy (h_1) and refrigerant temperature t_r, OB in Fig. 3 represents enthalpy line (h_1) of air at state S. OA represents dry bulb temperature line at a

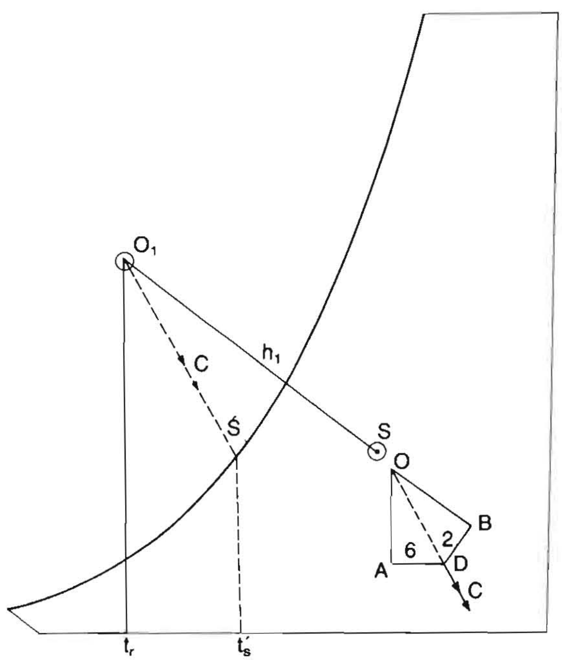

Fig. 3. Surface temperature determination

temperature equals to the refrigerant temperature t_r. The line OD, then, can be found to satisfy the coil characteristic as shown in equation (16).

Consequently, the following relation is achieved

$$\frac{AD}{DB} = \frac{t_s' - t_r}{h_1 - h_s'} = C \tag{17}$$

Draw a line from the point of intersection of line h_1 and t_r, O_1, parallel to OD to intersect the saturation line at point S' the wet surface condition (which is the surface temperature). This can best be illustrated by a specific example as shown in Appendix-I.

Cooling Coil Process Line, Condition Line

To draw the condition line, the surface temperature corresponding to the initial enthalpy of the air entering the coil, is found first as explained above, and shown in details in Appendix-I. This surface temperature represents the temperature of the saturated film of air in contact with the inlet edge of the coil. Therefore, the condition of the film at this point can be represented by point A' on the saturation curve of the psychrometric chart Fig. 4. In this illustration, point A represents the initial condition of the air entering the coil. Now as the main air stream moves over an infinitesimal surface area, an infinitesimal amount of

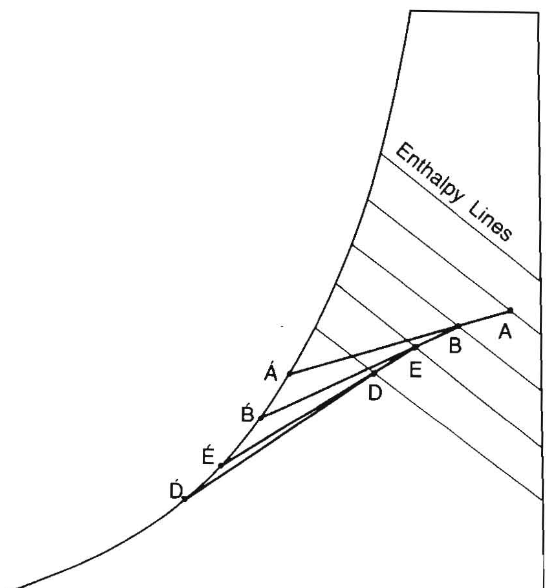

Fig. 4. Condition line construction through step by step method

saturated air in the surface film is mixed with the main air stream, and vice-versa.

This mixing process may be represented, on the psychrometric chart, by the straight line A-A'. The resulting, air mixture, condition will be at the point B, which lie only at an infinitesimally small distance from A.

If the surface temperature corresponding to the air enthalpy at point B is found and located at point B′, a new line can be drawn between B and B′. Point E on this second line represents the condition of the mixture as it approaches the next infinitesimal element of surface area. In the same way this process can now be repeated by drawing a line between E and E′, and locating D. By drawing a sufficient number of these mixtures lines in succession, the condition line will be clearly outlined because it is the "envelope" of these straight lines.

Energy Balance Line (E.B.L.)

The heat gained by refrigerant brine to cool air from A to B is:

$$W_r\, c_{pr}\, (t_{rb} - t_{ra}) = W_a\, (h_A - h_B)$$

and

$$\frac{W_r\, c_{pr}}{W_a} = \text{Constant}$$

therefore,

$$\frac{dt_r}{dh} = \frac{t_{rb} - t_{ra}}{h_A - h_B} = \frac{W_a}{W_r\, c_{pr}} \qquad (18)$$

It is clear that for a known $W_a/W_r\, c_{pr}$, air enthalpy and refrigerant temperature, the E.B.L. can be drawn on the psychrometric chart.

The E.B.L., as represented by equation (18), may be applied for the three cases; counter flow, parallel flow and direct expansion coils. A comprehensive treatment of the above mentioned three cases is given in Sofrata (1980). Other theoretical applications on this new developed technique such as:- (a) Partially wetted surfaces, (b) prediction of final condition of air leaving a partially wetted counter flow coils of a given area, (c) air pressures other than that of atmospheric and (d) predictions of coil performance may also be found in Sofrata (1980).

Test Set-up and Procedure

The schematic diagram of the experimental set-up is shown in Fig. 5. It consists of an open circuit air conditioning duct of rectangular section 0.286 ×

0.575 m made out of 18 gauge galvanised iron sheet. The air passes first through an air washer, which was installed at fan inlet in order to control the humidity ratio of inlet air to the required values. Also an auxiliary steam spray was installed to increase the flexibility to get required inlet condition. A constant speed centrifugal fan was used and air baffles were installed to vary the air flow rate.

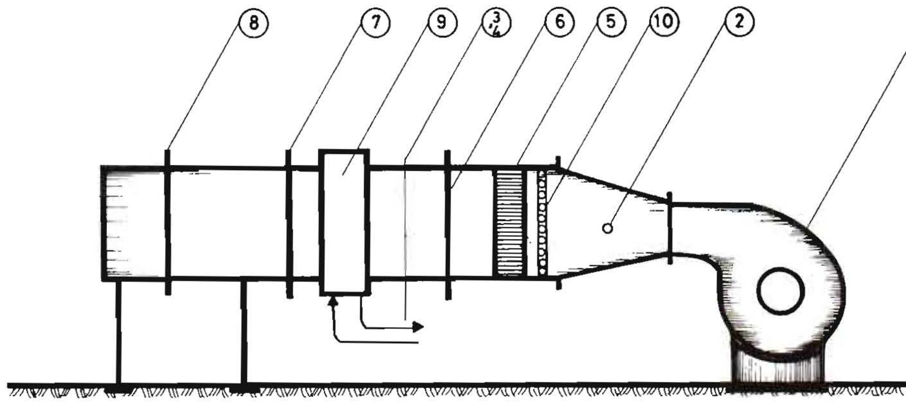

Fig. 5. Experimental test rig.

1. Centrifugal fan
2. Steam humidifier
3-4. Chilled water inlet and outlet
5. Stratner
6-7-8. Test Sections I, II and III respectively
9. Heat exchanger
10. Electric heater.

The temperature of air approaching the test section is varied by controlling the electric current to the battery of heaters installed before the test section.

Thermocouples were shielded from the test surface to avoid radiation effects. The cooling coil surface consists mainly of the usually used standard surface in air conditioning. The physical data relating to this cooling coil are as indicated in Table 1. After the cooling coil there is a similar section to measure outlet temperatures. The velocity profiles were measured and found to be fairly flat, with the average velocity being 0.93 times the central velocity. The cooling brine, water-calcium chloride, is supplied from an insulated tank of ample capacity, cooled by a refrigeration machine. An orifice meter for measuring water flow rates was placed at a suitable distance from the surface inlet. Incoming and outgoing water temperatures were measured with thermometers.

A series of experiments had been done using cold brine, but maintaining a dry surface, to evaluate the dry surface coil performance. The surface has been observed visually through glass window and the condensed water collecting tray has been examined to assure the dry surface cooling. Another series of experiments with variable air wet and dry bulb temperatures covering variable conditions, and variable air discharges were made, maintaining the coil surface completely wet. The properties of humid air; humidity, specific volume, specific heat, enthalpy before and after the cooling coil are found as per the procedure outlined in ASHRAE (1985) when wet and dry bulb temperatures are known. The overall heat transfer coefficient for such wet surface is computed by the following equation derived in Appendices-II, III using enthalpy potential.

$$\frac{1}{U_w} = \frac{nB}{h_i} + \frac{c_p}{h_a\ F} \tag{19}$$

The data collected include the following ranges of operating parameters:

Dry bulb temperature	25 – 36 °C
Wet bulb temperature	12 – 28 °C
Sensible to latent heat ratio	0.76 – 2.1
Reynolds number based on equivalent hydraulic diameter	1100 – 3500

The above Reynolds number range covers the approach air velocity normally encountered in air conditioning practice.

Table 1. Data of the used heat exchanger

Number of rows of tubes	4
Tube arrangement	Staggered
Tube material	Copper
Tube outside diameter, d_o	0.01710 m
Tube inside diameter, d_i	0.01435 m
Fin thickness	0.0004 m
Fin material	Aluminium
Fin spacing	0.00327 m
Coil face area, A_f	0.1646 m^2
Coil external surface area, A_o	15.05 m^2
Net flow area / face area	0.497
$A_o / A_{pi} = B$	19.31
A_f / A_o	0.905
A_{po} / A_f	0.105
Coil equivalent hydraulic diameter	0.003864 m

Results and Discussion

The dry surface heat transfer coefficient was correlated with the related dimensionless parameters; the Reynolds number Re and Prandtl number Pr.

All dry surface experimental data are obtained in the form.

$$St\ Pr^{2/3} = a\ Re^{b} \tag{20}$$

In the above expression a and b are constants and were obtained by using linear regression technique using the method of least mean squares.

The results may be summarised as follows:

$a = 0.0996338194 = 0.099634$
$b = 0.3522108895 = 0.35221$

Standard deviation:

$\sigma y = 0.1495$
$\sigma x = 0.419$

Variance:

$\sigma y^2 = 0.021$
$\sigma x^2 = 0.1652$

Correlation coefficient . 0.98735

Figure 6 shows the experimental values of dimensionless heat-transfer coefficients plotted against the correlated values. It is clear that the detected dry coil performance is similar to that available in Kays and London (1958).

Equation (20) was used in calculating the required dry heat transfer coefficient in the wet surface computations.

It is not the aim of this work to find a correlation for the wet surfaces, it is a detection technique for the conditioning line of air through a coil. Accordingly the end air conditions were detected graphically using our developed method and then compared with the experimental ones.

Figure 7 shows such comparison for the final dry bulb, sensible heat and the total heat. The mean standard deviation for the detected final dry bulb temperature with respect to the experimental one is 1.3% and the maximum deviation is 6.5%.

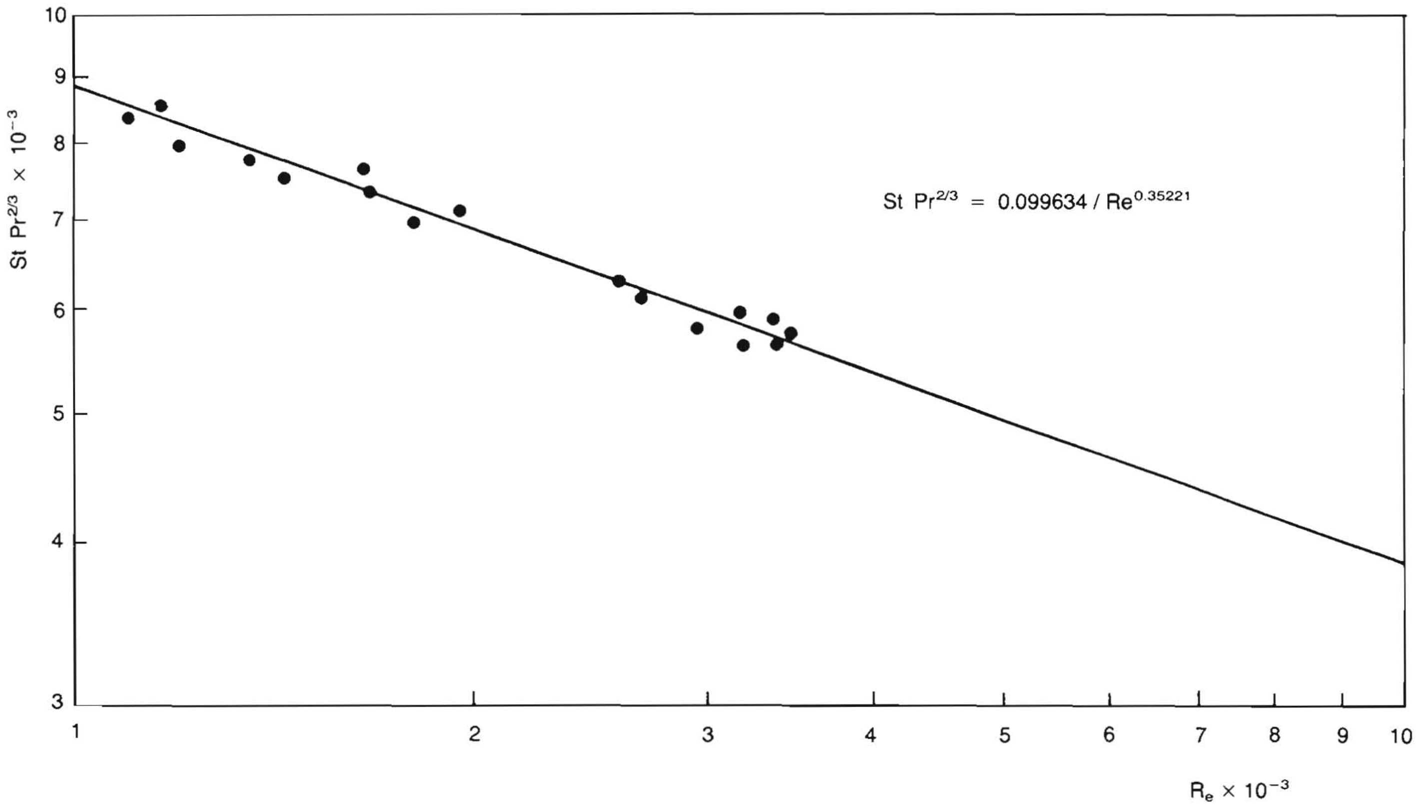

Fig. 6. Dry surface heat transfer coefficient coorelation

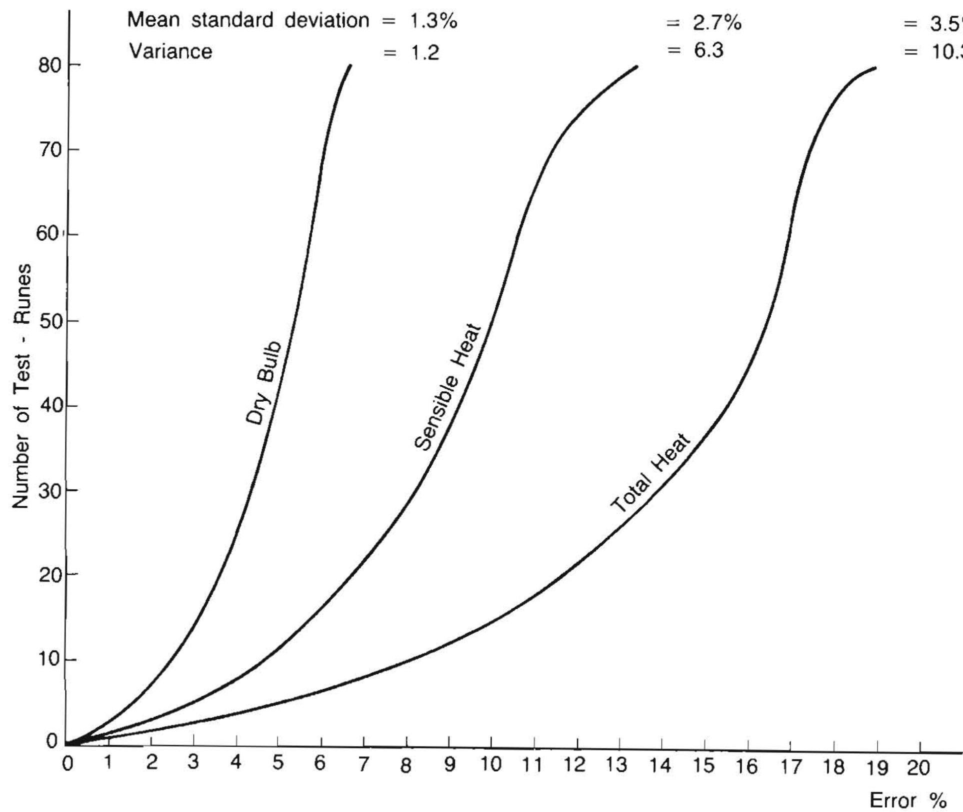

Fig. 7. Deviation errors for the final dry bulb, sensible and total heat of the wet surface

The total-and sensible-heats mean standard deviation are 3.5 and 2.7% respectively. These relations between the calculated and measured parameters - final dry bulb temperature, total-and sensible-heats - are represented in Fig. 6.

From the results obtained the following conclusions are drawn:

1. The discrepancy of the condition line as predicted by the new developed technique and the experimental data was found to be dependent on the step size used in the graphical solution. As the step size decreases this discrepancy diminishes. However, this discrepancy provides over designed area, which can be beneficial as a safety factor, if fast calculations were involved (*i.e.* larger step size).

2. The method offers a direct solution utilizing only psychrometric charts.
3. The new technique is capable in solving combined heat and mass transfer problems occurring at pressures other than that of atmosphere.
4. An easy direct determination of boundary temperature at which dehumidification begins for cooling coils, where a portion of the surface is in dry condition.
5. Less effort is required than trial and error method normally used in determination of air condition leaving a counterflow cooling coil of a pregiven area.

The calculated temperature values of using this new method showed a good agreement with the experimental results (see Fig. 7). The average deviation for the total heat is about + 15% and for sensible heat is + 9%, which are of positive nature and on the same side for practical design purposes. The method is also useful for similar practical applications, such as evaporative condensers, cooling towers etc.

Appendix I

Air at 20°C wet bulb temperature and 30°C dry bulb temperature passing over a direct expansion coil of characteristic C = 3 the refrigerant temperature is 2°C. Find the corresponding surface temperature.

Solution:

From eq. (17) we have

$$C = 3 = \frac{t_s' - t_r}{h_1 - h_s'} = \frac{AD}{DB}$$

Let AD = 6°C $\therefore$ DB = 2 Kcal/kg in Fig. (3)

OB is parallel to h_1
OA is parallel to $O_1\ t_{r1}$

and from O_1 the line $O_1\ S'$ is parallel to OD which is the coil characteristic C line. S′ is the wet surface condition

$$\therefore \quad t_s = 14.5°C$$

Appendix II

Overall heat transfer coefficient for a wet-finned tube heat exchanger:

Consider the following assumptions:

1. Constant air enthalpy.
2. Neglect the effect of pipe wall and condensate film resistances.
3. $h_{f,b} = h_{p,o} = h_p = h_s = m + nt_s$ (1)

where:

$h_{f,b}$ = fin base enthalpy = $m + n\ t_{f,b}$

$h_{p,o}$ = pipe outer surface enthalpy = $m + n\ t_{p,o}$

h_p = pipe mean surface enthalpy = $m + n\ t_p$

h_s = surface enthalpy = $m + n\ t_s$

4. Let $h_r = m + n\ t_r$ (2)
   where m and n are evluated at t_r, the refrigerant temperature, as suggested by Goodman.

Then, the heat transferred from pipe to refrigerant, considering the total outer surface area as referece area, may be determined as follows:

$$dq_w = h_i\ (t_p - t_r)\ \frac{d\ A_{p,i}}{dA_w}\ dA_w$$

Let $B = \frac{A_w}{A_{p,i}}$

$$\therefore\ dq_w = h_i\ (t_p - t_r)\ \frac{dA_w}{B} \tag{3}$$

We have;

$$h_p - h_r = (m + nt_p) - (m + nt_r)$$
$$= n\ (t_p - t_r)$$

$$\therefore\ t_p - t_r = 1/n\ (h_p - h_r) \tag{4}$$

But from (1) $h_p = h_s$

$$\therefore\ h_s - h_r = n\ (t_s - t_r)$$

$$t_s - t_r = 1/n\,(h_s - h_r) \tag{5}$$

$$\therefore \quad dq_w = h_i/nB\,(h_s - h_r)dA_w \tag{6}$$

Heat transferred from air to outer surfce of pipe and fin is:

$$dq_w = h_c/c_p\,(h - h_s)dA_{p,o} + h_c/c_p\,(h - h_{f.m})dA_f \tag{7}$$

Let $h - h_{f,m} = \theta_m$

and $h - h_{f,b} = h - h_s = \theta_b$

$$\therefore \quad \frac{h-h_{f,m}}{h-h_{f,b}} = \frac{h-h_{f,m}}{h-h_s} = \frac{\theta_m}{\theta_b} = \phi_w$$

$$\therefore \quad h - h_{f,m} = \phi_w(h - h_s) \tag{8}$$

Substitute (8) in (7)

$$dq_w = h_c/c_p\,(h - h_s)\,(dA_{p,o} + \phi_w\,dA_f)$$

Divide and multiply by dA_w

$$dq_w = h_c/c_p\,(h - h_s)\,(dA_{p,o}/dA_w + \phi_w dA_f/dA_w)dA_w \tag{9}$$

$$dA_{p,o}/dA_w = A_{p,o}/A_w,\ dA_f/dA_w = A_f/A_w$$

Let $(A_{p,o}/A_w + \phi_w A_f/A_w) = F =$ Finned Surface Factor for wetted surfaces.

$$\therefore \quad dq_w = h_c\,F/c_p\,(h - h_s)\,dA_w \tag{10}$$

$$\text{Let} \quad dq_w = U_w\,(h - h_r)\,dA_w \tag{11}$$

Where

U_w = Overall heat transfer coefficient for wet surfaces.

From (6), (10) and (11) we have:

$$\frac{dq_w}{h_i\,dA_w/n\,B} = h_s - h_r,\ \frac{dq_w}{(h_c\,F/c_p)dA_w} = h - h_s$$

$$\frac{dq_w}{U_w\,dA_w} = h - h_r$$

$$\therefore \quad \frac{dq_w}{U_w\, dA_w} = \frac{dq_w}{h_i\, dA_w/nB} = + \frac{dq_w}{(h_cF/c_p)dA_w}$$

$$\therefore \quad \frac{1}{U_w} = \frac{nB}{h_i} + \frac{c_p}{h_cF} \tag{12}$$

Note: The wet fin efficiency is evaluated on the same curve of the dry surface

But for evaluation of $\sqrt{h_c\, n/c_p\, k_y}$ instead of $\sqrt{h_c/K_y}$ of the dry fin (see Appendix III).

For dry finned surfaces

$$\frac{1}{U_d} = \frac{B}{h_i} + \frac{1}{h_cF} \tag{13}$$

Appendix III

"Circular wet fin efficiency"

For limited temperature differences we can assume:

$$h_f = m + n\ t_f \tag{1}$$

as suggeste by Goodman

Let $\theta = h - h_f$ at any r (2)

$$\therefore \quad d\theta = -\,dh_f \tag{3}$$

Now heat transferred by convection to the fin sides

$$d_{q_w} = dA_f \times \frac{h_c}{c_p}(h - h_f) \tag{4}$$

as derived by Merkil, Goodman

$$\therefore \quad d_{q_w} = 2 \times 2\pi\ rdr \times \frac{h_c}{c_p}(h - h_f) \tag{5}$$

Assume negligible water film resistance of condensate.

Heat transfer by conduction through fin metal

$$q_w = - a \times K \times \frac{dt}{dx}$$

but

a = fin cross sectional area at $r = 2y \times 2\pi r$.

$$\therefore \qquad q_w = a \times K \times \frac{dt_f}{dr} \tag{6}$$

from eq. (11)

$$dh_f = n \, dt_f$$

$$\therefore \qquad dt_f = \frac{1}{n} dh_f \tag{7}$$

from (6) and (7)

$$q_w = K \cdot 2y \times 2\pi r \cdot \frac{1}{n} \frac{dh_f}{dr} \tag{8}$$

from (7) and (8)

$$q_w = -4y \, \pi \, r \, K \cdot \frac{1}{n} \frac{d\theta}{dr}$$

$$\therefore \qquad \frac{dq_w}{dr} = - 4y \frac{\pi r K}{n} \frac{d_2\theta}{dr^2} - \frac{4y \, \pi K}{n} \frac{d\theta}{dr} \tag{9}$$

from (4)

$$dq_w = dA_f \cdot \frac{h_c}{c_p} \cdot (h - h_f)$$

$$= 2 \times 2 \, \pi \, r \times dr \times \frac{h_c}{c_p} \times \theta$$

$$\therefore \qquad \frac{dq_w}{dr} = 4 \, \pi \, r \frac{h_c}{c_p} \theta \tag{10}$$

equating (9) and (10)

$$4 \, \pi \, r \frac{h_c}{c_p} \theta = \frac{4 \pi r y K}{n} \frac{d_2\theta}{dr^2} - \frac{4 \, \pi \, yK}{n} \frac{d\theta}{dr}$$

$$\frac{r \, d_2\theta}{dr_2} \times \left(\frac{yK}{n}\right) + \frac{d\theta}{dr} \times \left(\frac{yK}{n}\right) + r\theta \left(\frac{h_c}{c_p}\right) = 0$$

multiply by r and divided by $\frac{yK}{n}$

$$r^2 \frac{d_2\theta}{dr^2} + r\frac{d\theta}{dr} + r^2\theta \times \left(\frac{h_c n}{c_p y K}\right) = 0 \tag{11}$$

In general form the Bessel equation is

$$r^2 \frac{d_2\theta}{dr^2} + r\frac{d\theta}{dr} + (r^2 - p^2)\,\theta = 0 \tag{12}$$

Where p is a constant. Many types of functions which are independent solutions of Bessel equation have been developed and their properties are tabulated.

For a dry surface equation (11) reduces to

$$r^2 \frac{d_2\theta}{dr^2} + r\frac{d\theta}{dr} + r\,\theta \left(\frac{h_c}{yK}\right) = 0 \tag{13}$$

The general solution of equation (13) (see Sherwood and Reed 1939).

For $p = 0$ or an integer is:

$$\theta = \theta_b \left(\frac{u}{u_b}\right) p \left(\frac{I_p(u) + \beta_1 K_p(u)}{I_p(u_b) + \beta_1 K_p(u_b)}\right) \tag{14}$$

At outter edge $r = r_e$ and $\theta = \theta_e$

At the base $r = r_b$ and $\theta = \theta_b$

$$\beta_1 = -\frac{I_{p-1}(u_e)}{K_{p-1}(u_e)}$$

$$u = \sqrt{\frac{h_c}{K_p}\ \frac{dA}{dr}}$$

Now defining fin efficiency as

$$\phi = \frac{t - t_{f,m}}{t - t_{f,b}} = \frac{\theta_m}{\theta_b} \text{ ...... for dry fin}$$

and

$$\phi_w = \frac{h - h_{f,m}}{h - h_{f,b}} = \frac{\theta_m}{\theta_b} \text{ ...... for wet fin} \tag{15}$$

where:

$$h_{f.m} = \text{mean fin enthalpy}, \ \theta_m = h - h_{f.m} \tag{16}$$

$$h_{f.b} = \text{fine base enthalpy}, \ \theta_b = h - h_{f.b} \tag{17}$$

The heat transfer to the entire fin surface from the air is:

$$q_f = \int h_c \, \theta \, dA_f$$

$$= h_c \int_{r_b}^{r_e} \theta \cdot 4 \pi r \, dr$$

The heat transferred from the entire fin surface if it's enthalpy were uniform and equal to $h_{f.b}$

$$q_b = h_c \cdot A_f \cdot \theta_b$$

$$\therefore \quad \frac{q_f}{q_b} = \frac{h_c \int_{rb}^{re} \theta \, 4\pi r dr}{h_c \ A_f \ \theta_b} = \frac{h_c \ \theta_m \ A}{h_c \ \theta_b \ A} = \frac{\theta_m}{\theta_b} = \phi_w$$

$$\therefore \quad \phi_w = \frac{\int_0^A \theta \, dA_f}{\theta_b \ A_f} \tag{18}$$

When $p = 0$ – for the annular fin of constant width the general equation (14) reduces to, for the fin efficiency.

$$\phi = \frac{2}{u_b\left[1 - \left(\frac{u_e}{u_b}\right)\right]^2} \left[\frac{I_1(u_b) - \beta_1 K_1(u_b)}{I_o(u_b) - \beta_1 K_o(u_b)}\right] \tag{19}$$

where:

$$\beta_1 = \frac{K_1(u_e)}{K_1(u_e)}$$

$$u_b = \frac{r_2 - r_1 \sqrt{h_c n / K_y c_p}}{\left[\frac{r_2}{r_1} - 1\right]} \tag{20}$$

and $$u_e = u_b \left(\frac{r_2}{r_1}\right)$$

for dry surface

$$u_b = \frac{r_2 - r_1 \sqrt{h_c/K_y}}{\left(\frac{r_2}{r_1} - 1\right)}$$

For a rectangular-plate fin a by c and of uniform thickness; it is shown in Carrier and Anderson (1944) that the fin area served by each tube is equivalent in performance to a flat circular plate fin of equal area

$$r_{equivalent} = \sqrt{\frac{ac}{\pi}}$$

Nomenclature

A	Surface area
B	External surface area to internal surface area of coil.
C	Coil characteristic as defined in equation (15).
C_p	Specific heat at constant pressure.
D	Ficks coefficient of diffusivity.
F	Finned surface factor.
h	Enthalpy
h_c	Convection heat transfer coefficient for air side.
h_d	Mass transfer coefficient.
h_i	Convection heat transfer coefficient for refrigerant side.
ji	Mass transfer flow rate.
K	Thermal conductivity.
L	Latent heat.
m	Constant as defined in eq. (4).
n	Constant as defined in eq. (4).
q	Heat transfer flow rate.
r	Circular fin radius.
t	Temperature.
U	Overall heat transfer coefficient
W	Specific humidity.
y	Fin thickness . 2y.

Subscritps

1	Inlet
2	Outlet
b	Base
D	Dry
f	Fin
i	Inner
m	Mean
o	Outer
p	Pipe
r	Refrigerant
s	Surface.
sat	Saturated
w	Wet or wall

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أداء ملفات التبريد في أجهزة تكييف الهواء

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قسم الهندسة الميكانيكية ـ كلية الهندسة ـ جامعة الملك سعود ـ ص. ب ٨٠٠ ـ الرياض ١١٤٢١
المملكة العربية السعودية

تقوم هذه الدراسة بعرض فكرة جديدة لتصميم ملفات التبريد بالرسم. تمتاز هذه الطريقة بالاستعانة بخريطة الهواء الرطب فقط دون غيرها لتحديد خواص الهواء والمساحات المبللة من السطح والجافة في تلك الحالات التي يجتمع فيها الوضعان.

ولكي تتم القناعة بجدوى طريقة الحل الجديدة تم إنشاء تجربة عملية تتكون من ملف مكون من أربعة صفوف من الأنابيب ذات الريش والزوائد. وقد أختير ملف التبريد هذا من تلك الملفات المستخدمة فعلاً في الحياة العملية. وقد وضع الملف في التجربة بحيث يمكن تغيير العوامل الرئيسة التي تؤثر في خواصه.

وتبين من النتائج العملية أنها متوافقة إلى درجة جيدة مع تلك المحسوبة نظرياً بالأسلوب المبتكر، فقد كان متوسط الفروق في كمية الحرارة الكلية + ١٥٪ والحرارة الجافة + ٩٪ وكما يلاحظ فهي زيادات تخدم التصميم.

يتضح مما سبق جدوى هذه الطريقة وإمكانات الاستفادة منها في تطبيقات مشابهة.