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Non-linear Dynamics in Stripe Geometry Al_x Ga_{1-x} As Semiconductor Lasers: Experimental Results and Theoretical Analysis

Mozahim I. Azawi

Department of Physics, College of Education
University of Mosul, Iraq

ABSTRACT. Experimental results on transient phenomena in semiconductor lasers are reported. A travelling-wave rate equation model for the investigation of the non-linear dynamics and chaos in semiconductor lasers is presented. The model includes the effects of defects which act as optically saturable absorbing centres, external cavity optical feedback, and the coefficient of spontaneous emission coupled into the lasing modes. Numerical results and analytical solutions confirm that the increase of density of saturable absorbers decreases the onset frequency of pulsations. Furthermore, these results provide strong evidence of chaotic dynamics in semiconductor lasers corporated with external cavity.

1. Introduction

Planar gain-guided multimode injection lasers are preferred to index-guided devices as a light source for low-to-medium capacity optical communication systems to avoid modal noise problems (Whiteaway 1982). Among the most important issues on these lasers is the transverse mode control. The transverse mode instability, attributed to the deformation of laser gain profile, causes kinks in light output/current curves (Nakamura *et al.* 1981), and accompanied by anomalous lasing behaviours such as mode profile deformation and lateral shift (Botez and Herskowitz 1980), self-pulsations (Peterman and Arnold 1982), intensity fluctuation (Kresel 1981), and deterioration of modulation characteristics (Lang 1977).

The foregoing non-linear dynamical phenomena of semiconductor lasers have been the subject of extensive theoretical and experimental investigations (Shore 1987). However, it is now appreciated that such phenomena as self-oscillations may represent one form of quite general complex dynamical behaviour related to the appearance of optical turbulence or optical chaos (Boyd *et al.* 1986). A remarkable aspect of the theoretical treatment of optical instabilities is the reliance upon mathematical results which have been obtained in the recent past (Guckenheimer and Holmes 1983). Further developments in mathematical understanding of non-linear dynamical systems are required to explore the full range of optical instabilities (Adams 1987).

In this paper, some experimental studies on self-sustained pulsations (SSP) behaviour and transient effects in semiconductor lasers are reported. These data, together with other results which have appeared in the literature (Compbell and Abbot 1980; Channin *et al.* 1979; Haramitsu 1981; Hartman *et al.* 1979; Su 1981; Vander Ziel 1979 & 1981), enable us to establish a theoretical model to discuss the non-linear phenomena.

2. Experimental Results

The basic device characteristic curve is the light output-current (L-I) as shown in Fig. 1. It can be seen from the figure that the kink observed at a power level well above threshold. Kink has been associated with mode shift towards the stripe edge (Azawi and Abbas 1987). Below the kink, regions of self-sustained pulsations (SSP) were found, and the onset frequency of pulsations was current dependent. The onset frequency of pulsations in the first region was few MHz increasing when going to the next region and quenched near the kink.

Fig. 2 shows a pulse distribution of the laser output inside the region of SSP. This type of pulse height distribution analysis was performed by the series 30 MCA (Multichannd Analyzer) during data acquisition. It is clearly seen from the figure that the incident light from the laser generates a pulse train from the detector to the series 30 which is a time - random mixture of pulses of all possible amplitudes.

At sufficiently higher pumping rate (1.31 I_{th}) beyond the kink, pulse distributions were recorded as shown in Fig. 3. In comparison with Fig. 2, the distributions are narrower, *i.e.*, few amplitudes which are proportional to the energies of incident light that was absorbed by the detector. The spectral peak shifts with the biasing current as can be seen from Fig. 3.

The shift of peak energy with the current above threshold is plotted in Fig. 4. It is evident from the graph that many variations in the curve declivity occurring from the onset of SSP to the kink. At (1.31 I_{th}) and above the curve has a steady inclination.

3. Theoretical Analysis

The experimental results presented above provide good evidence that

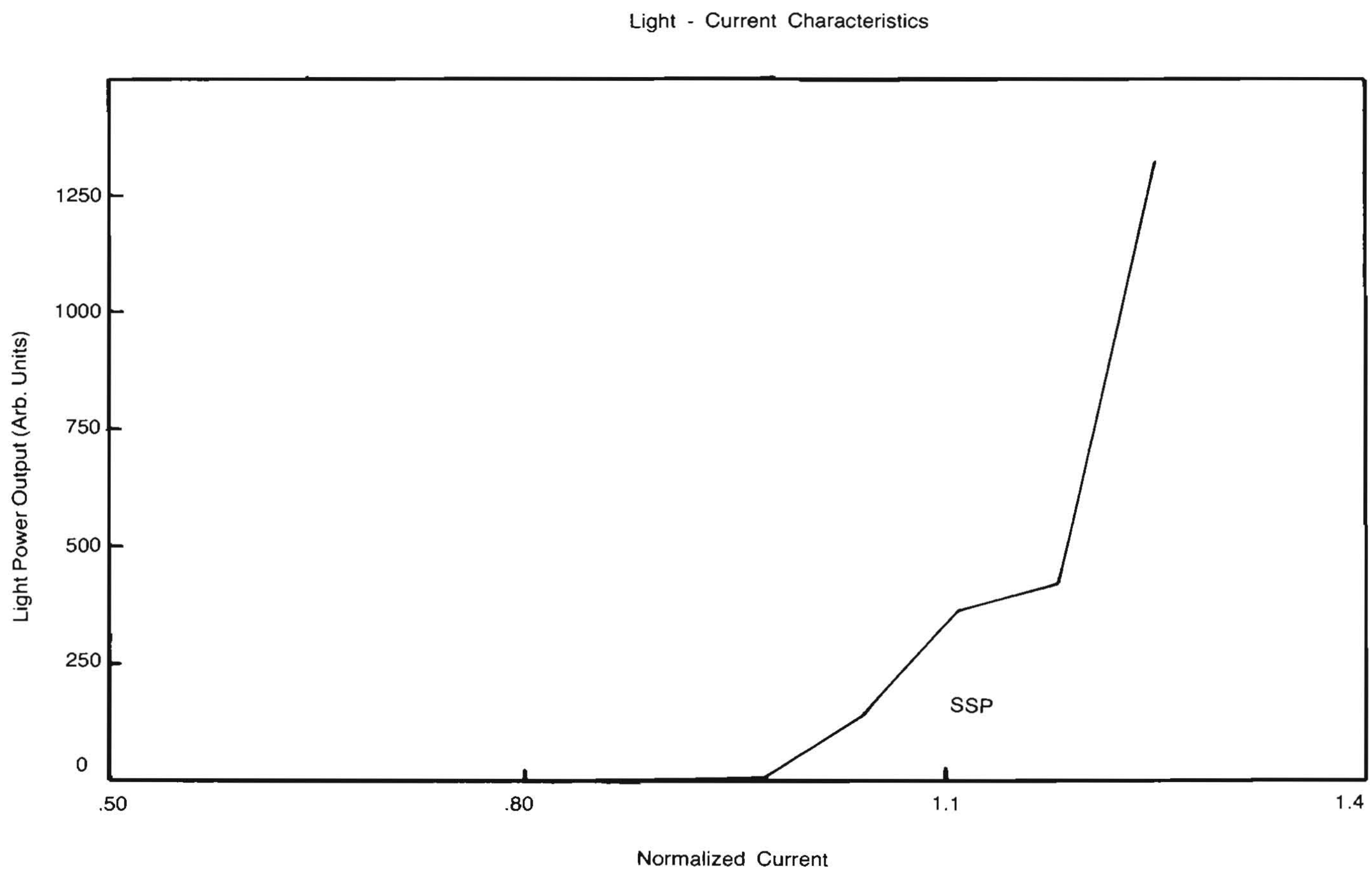

Fig. 1. L-I characteristics of the stripe geometry double heterostructure Al_xGa_{1-x} As laser showing nonlinearities (kinks) and regions of self-sustained pulsations (SSP).

self-sustained pulsation originate from defects in the laser cavity which act as optically saturable absorbers. A small amount of external optical feedback can greatly influence the induced-noise resulting from chaos (Azawi and Thomas 1985). Fluctuations in photon density during pulsations are caused by photon absorption in optical saturable centres. The theoretical analysis based on the saturable model will be presented. The good agreement between theoretical predictions and the experimental results gives support to the model assumed.

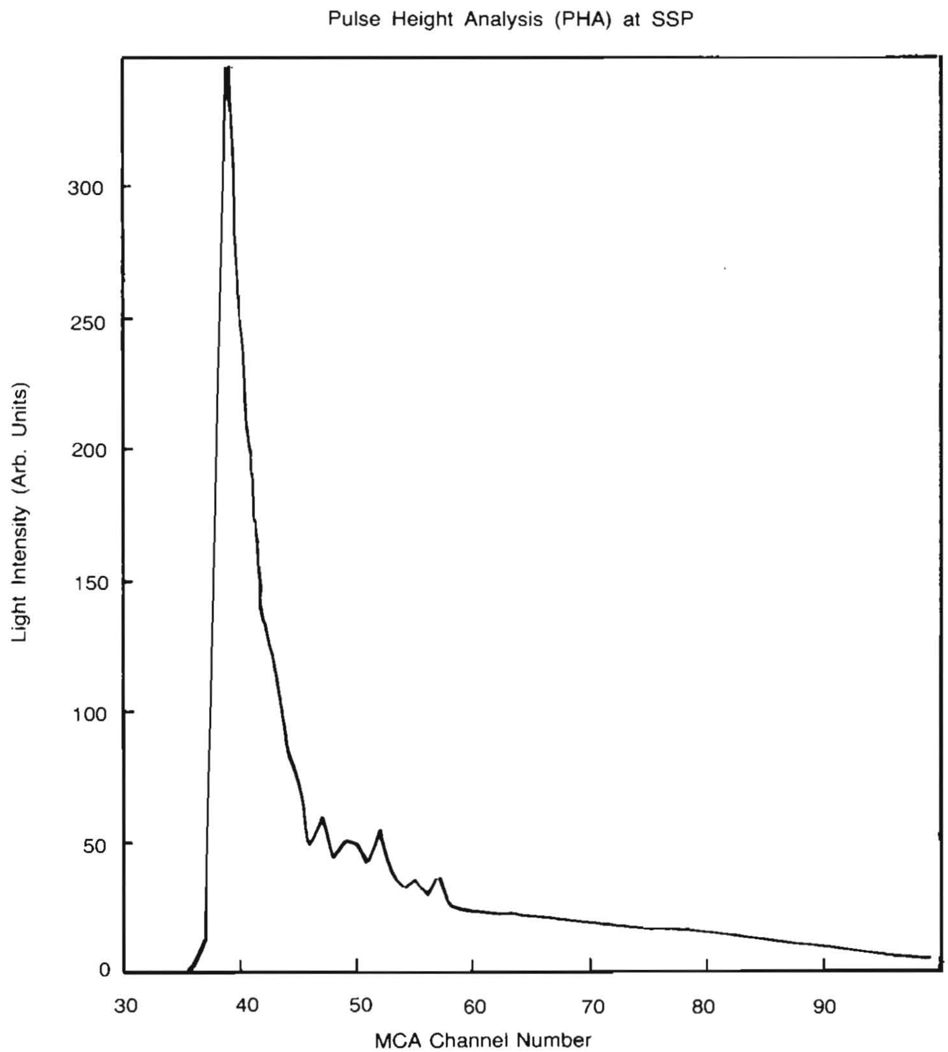

Fig. 2. Pulse height analysis (PHA) of the laser output at the onset of SSP.

Light Intensity (Arb. Units) Pulse Height Analysis (PHA) at SSP

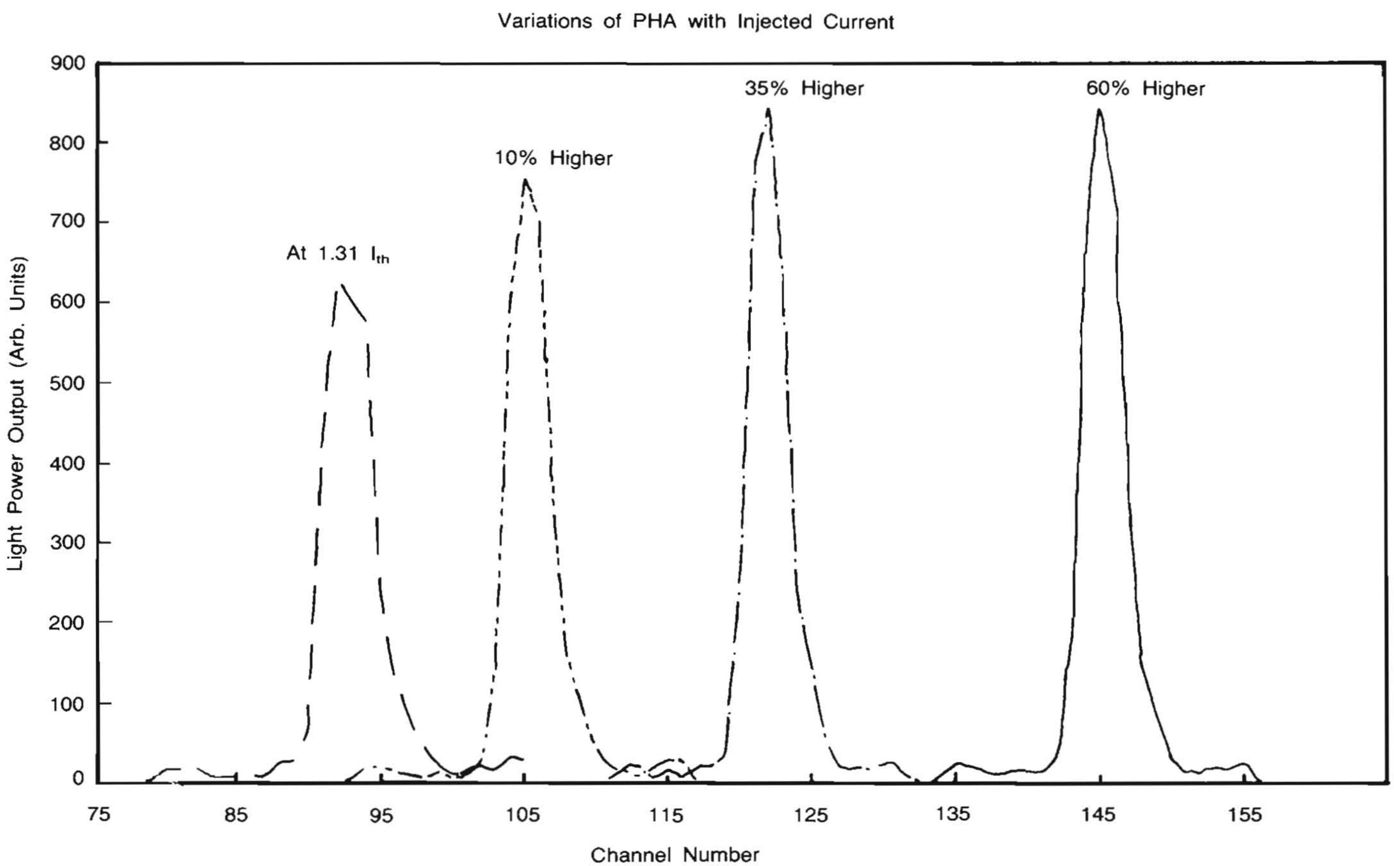

Fig. 3. Pulse distributions of the light output well above threshold and beyond the kink.

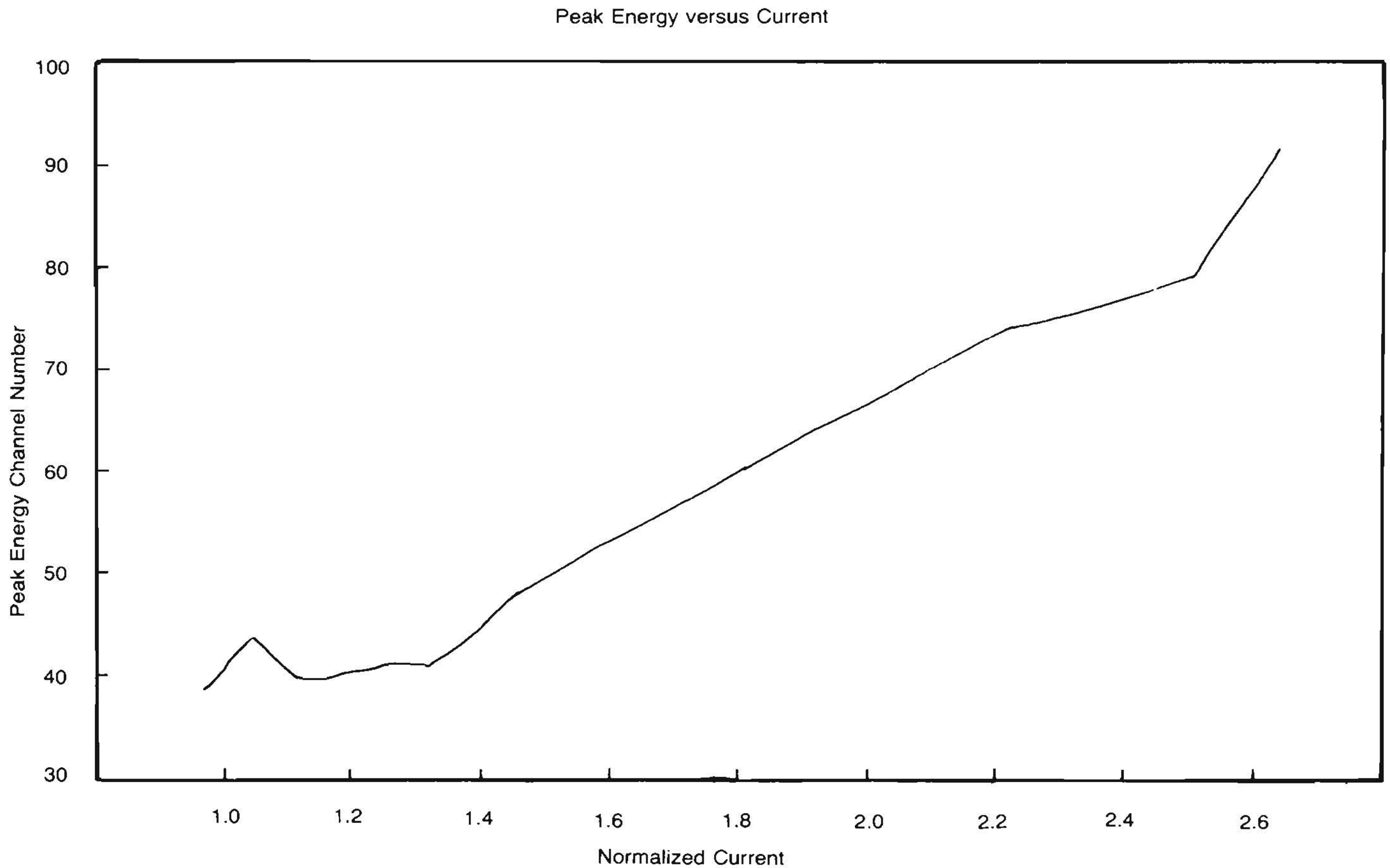

Fig. 4. Variation of spectral peak energy with current above threshold. The biasing current was normalized to the threshold current.

3.1 *Rate equations:*

The single-mode rate equations for the densities of injected electrons (n), photons (s), and saturable absorbers (N) are given as follows (Chik *et al.* 1980; Wong and Carroll 1987):

$$\frac{dn}{dt} = \frac{J}{ed} - \frac{n}{\tau_{sp}} - As\,(n - n_0)\,\frac{FcB}{L}\,s(t-\tau)n \tag{1}$$

$$\frac{ds}{dt} = As\,(n-n_0) - \frac{s}{\tau_s} + \frac{\beta n}{\tau_{sp}} + \frac{Fc}{L}\,s(t-\tau) \tag{2}$$

$$\frac{dN}{dt} = \frac{N_0-N}{\tau_a} - C\sigma Ns \tag{3}$$

where J is the current density, d is the active layer thickness, e electronic charge, τ_{sp} and τ_s are the spontaneous and stimulated lifetimes, A & B are constants, n_0 minimal carrier density, F fraction of light fed back to the laser, c speed of light, L external cavity length, τ roundtrip time of the external cavity, and β is the fraction of the spontaneous emission coupled into the lasing mode. The term σ N represents the coefficient of loss contributed by the saturable absorbers where σ is the effective capture cross section. N_0 is the density of saturable absorbers in the laser cavity and τ_a represents the average lifetime that the absorbers have in their excited state.

3.2. *Steady state solution*

Single-mode rate equations are adopted for analysis as found experimentally that lasers with essentially a single dominant mode pulsate as do multimode lasers (Chik *et al.* 1980). The steady state solution of the coupled rate equations provides useful insights into the basic principles of the laser. The result from steady state solution is:

$$n = n_0 + \frac{1}{A\tau_s} - \frac{Fc}{AL}$$

where we have neglected the spontaneous emission rate term. It is clear that the electron density above threshold is dependent on F. Increasing the photon population by feedback inside the cavity depletes the electron concentration. Also, the gain coefficient, which is a function of injected carrier concentration, will not saturate at and above threshold.

3.3. *Transient solution*

A pumping step current was used in the computation to turn on the laser to what will become steady state values $\overline{n}$ and $\overline{s}$. During the transience the electron and photon population deviate from their mean values in the form:

$$\triangle n = n - \overline{n} \tag{4}$$

$$\triangle s = s - \overline{s} \tag{5}$$

and using the Taylor expansion to the first order for the transition rates around equilibrium values as; where adopted:

$$R_{st} = \overline{R}_{st} + \frac{\partial \overline{R}_{st}}{\partial n} \triangle n + \frac{\partial \overline{R}_{st}}{\partial s} \triangle s \tag{6}$$

$$R_{sp} = \overline{R}_{sp} + \frac{\partial \overline{R}_{sp}}{\partial n} \triangle n \tag{7}$$

Using equations (6) and (7) in the rate equations and with the solution in form

$$\triangle n = (\triangle n)_0 \ \exp[-(a - iw_R)\, t] \tag{8}$$

yields the expression for intrinsic resonance frequency above threshold:

$$W_R \cong \left\{ \frac{1}{\tau_s \tau_{sp}} \left(\frac{J}{J_{th}} + \beta\right) - \frac{1}{4}\left[\left(\frac{1}{\tau_{sp}}\left(\frac{J}{J_{th}} + 1\right) + \frac{FcB}{L}\left(\frac{N_0 - N}{\tau_a c \sigma N}\right)\right]^2\right\}^{1/2} \tag{9}$$

where J_{th} is the threshold current density.

The resonance frequency is seen to increase with the decreasing carrier and photon lifetimes, and to increase with current density above threshold. The effect of saturable absorbers is to reduce the SSP to a damped relaxation oscillation with external optical feedback at constant pumping rate. It also indicates that, W_R is affected by L, the external cavity length.

3.4. *Numerical Solution*

It is convenient to work with reduced rate equations after normalizing the various quantities in equations (1−3), we obtain:

$$\frac{dn'}{dt} = j - n' - s'(n' - n_0') - \gamma B's'n' \tag{1'}$$

$$\frac{ds'}{dt} = \alpha\,[s'\,(n' - n_0' - 1) + \beta n'N] + \gamma\, s'\,\tau_{sp} \tag{2)'$$

$$\frac{dN^*}{dt} = \frac{N_0^* - N^*}{\tau_a}\,\tau_{sp} - \frac{c\sigma N^* s'}{A} \tag{3)'$$

With the values:

$t = t/\tau_{sp}$

$n' = A\ \tau_s n$

$s' = A\ \tau_{sp} s$

$j = A\ \tau_s\ \tau_{sp}\ (J/ed)$

$\gamma = Fc/L$

$n_0' = A\ \tau_s n_0$

$\alpha = \tau_{sp}\ /\ \tau_s$

$B' = B/A$

To investigate the frequency characteristics of the laser diode under modulation, the transfer function should be evaluated and the driving current be replaced by the sum of dc bias and modulated component, as:

$$j = j_0 + \triangle j \cdot \exp\,(i\,w_m t), \tag{10}$$

where w_m is the angular frequency of modulation.

Solving normalized rate equation for first-order perturbation, the diode transfer function (the ratio of photon density and modulated component of current) can be found:

$$H(w) = \frac{\alpha\ (\bar{s} + \beta)}{[-\gamma\tau_{sp} - \alpha(\bar{n} - n_0' - 1) + i w_m \tau_{sp}]\ [1 + \bar{s}(1 + \gamma B') + i w_m \tau_{sp}] + [\bar{n}(1 + \gamma\ B') - n_0]\ [\alpha\,(\bar{s} + \beta)]} \tag{11}$$

When a modulated component of the current is applied to the laser, high frequency chaos will occur (as we will see) because high-frequency components are highly amplified compared to the low-frequency components.

4. Results of Theoretical Analysis

We will use the equations derived in previous section especially the expressions for relaxation oscillation and modulation depth to make a theoretical investigation of the effects of the saturable absorbers, optical feedback, and the spontaneous emission coefficient on the non-linear dynamical behaviour of the laser. We will allocate reasonable values to the some of the parameters involved to obtain quantitative results and we will not attempt to fit the theory in detail but we will show the general way in which the important parameters affect the laser behaviour.

4.1. *Values of parameters*

Typical values of laser parameters used in this investigation are summarized in Table 1. The values of (Ga, Al) As laser were taken from (Wong and Carroll 1987). For the saturable absorbers, N_0, σ, and τ_a used in the numerical simulation are taken from (Chick *et al.* 1980). The values of F & L are taken as variables covering the ranges observed in the laser studied.

4.2. *Pulsations*

Firstly, we will calculate the oscillation frequency and its variation with pumping rate for two values of β neglecting feedback and saturable absorbers as shown in Fig. 5. It is indicated that β introduce pronounced noise (chaos) peaks in frequency regions where optical communication systems operate. The contribution of spontaneous emission into the oscillating frequency exhibits a strong ringing since every modulation pulse causes relaxation oscillation and will affect the pulse code modulation in Gbit/s. The effect of optical feedback on the noise character of the laser diode is plotted in Fig. 6. It has been noted that the external cavity, under proper conditions, can considerably increase the lifetime of the photons inside the laser cavity and therefore lead to a decrease in the intrinsic frequency (Chen 1980), and the noise frequencies are related to the round-trip time of the external cavity. In Fig. 6 we plotted the calculated curves for zero feedback and for 5% of the light fed back into the laser cavity. One can see that the calculated curves reflect the effect of feedback at small pumping rates.

Fig. 7 shows the oscillation frequency plotted as a function of pumping rate for different values of N_0 (density of saturable absorbers). The results obtained here using the model described by the rate equations suggest that the sustained pulsations could be caused by saturable absorbers with density greater than 2×10^{15} cm^{-3}.

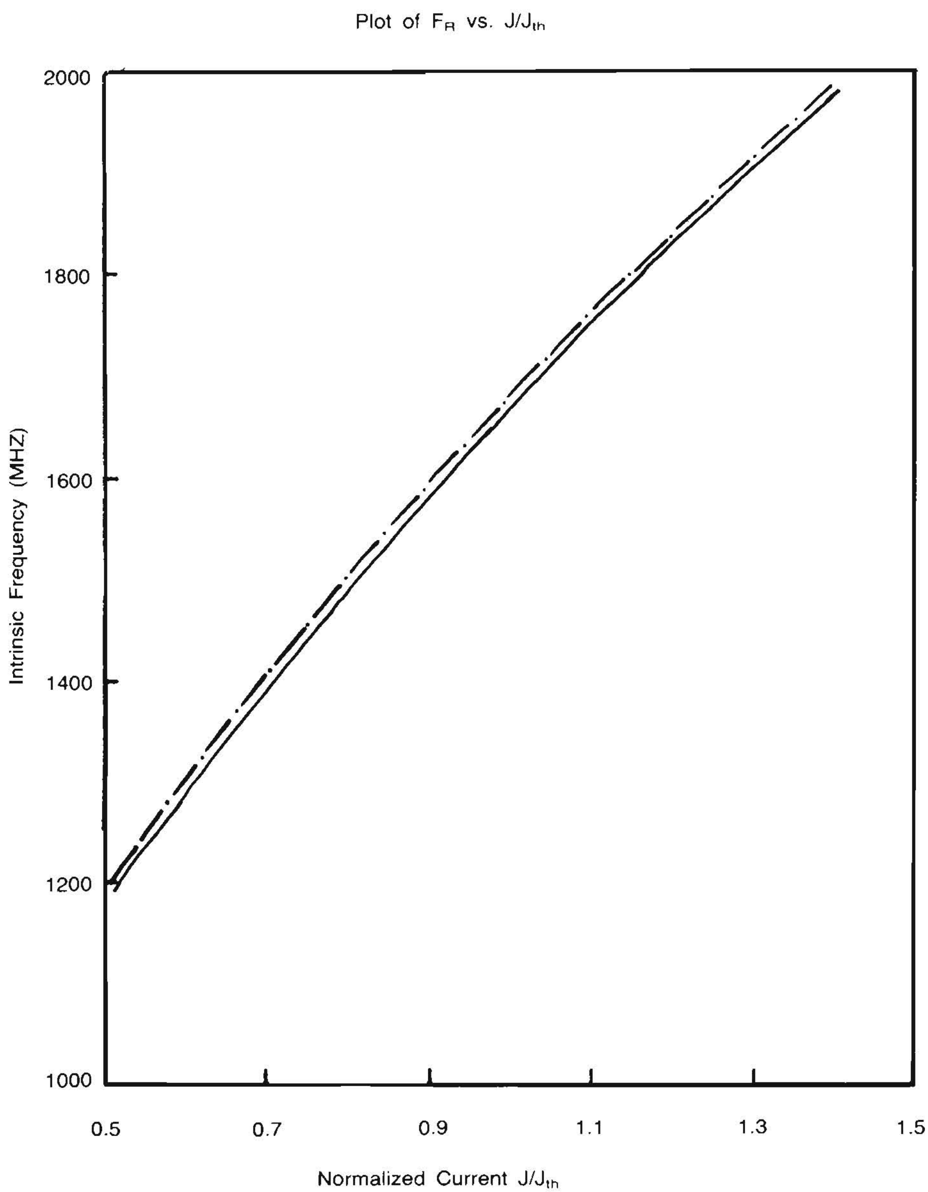

Fig. 5. Calculated effect of pumping rate on the frequencies of relaxation oscillation with β as variable. Solid curve for $\beta=0$ and dashed for $\beta = 10^{-2}$

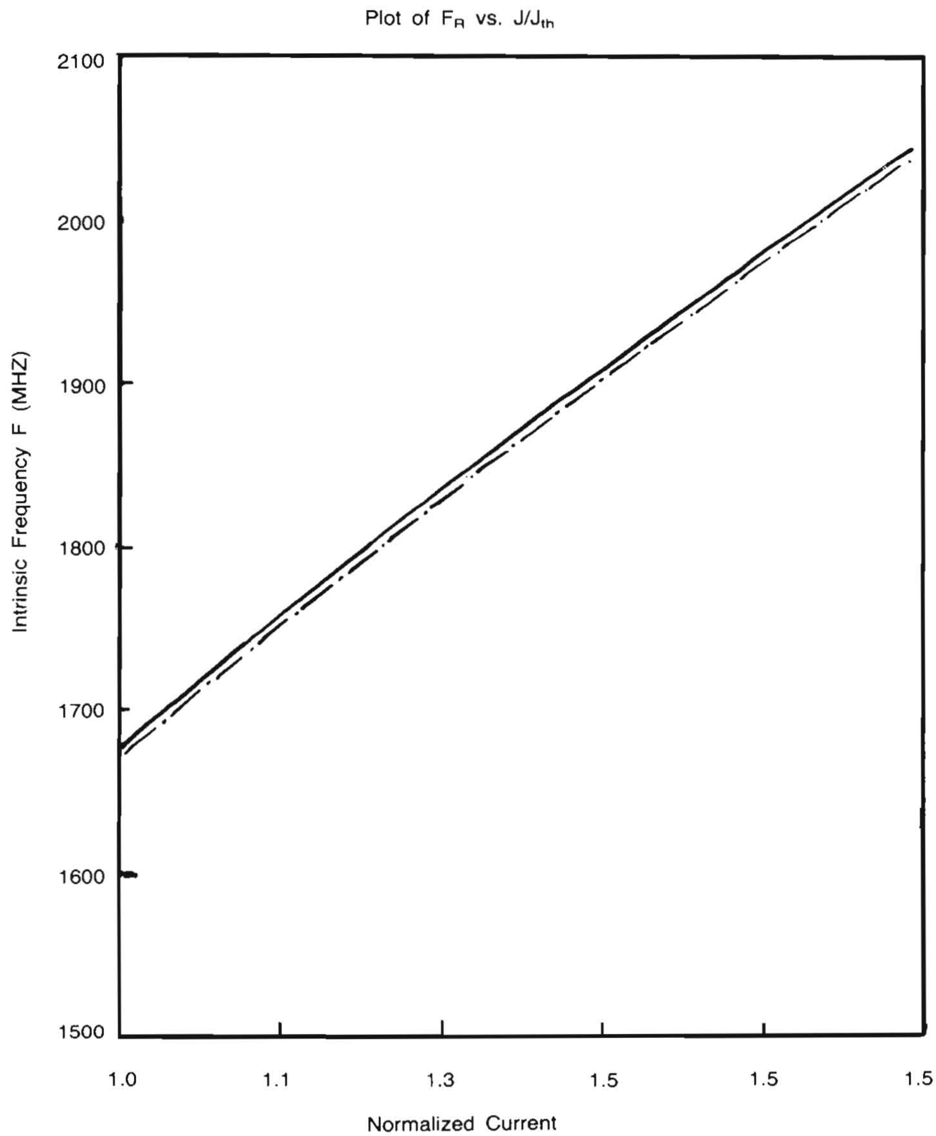

Fig. 6. Oscillation frequency against pumping rate as a function of F (fraction of light fed back). Solid curve for F = 0 and dashed for F = 5% & L = 5 cm.

Fig. 8 shows the oscillation frequency as a function of saturable absorbers for two different values of bias current. For lower values of N_0, the oscillation is damped and the frequency is close to the relaxation oscillation, *i.e.*, at $N_0 = 0$, But, as N_0 increases the oscillation frequency decreases and becomes sustained oscillation.

Table 1. Parameter values used in this calculation

Parameters	Symbol	Values
Active layer thickness	d	0.1 μm
Laser cavity length	L	250 μm
Carrier lifetime	τ_{sp}	3 nsec
Photon lifetime	τ_s	3 psec
Minimal carrier density	n_o	1.0×10^{24} m^{-3}
Density of saturable absorbers	N_0	2.0×10^{21} m^{-3}
Effective capture cross section	σ	1.0×10^{-18} m^2
Absorbers lifetime	τ_a	10 nsec

Keeping the bias current fixed ($J/J_{th} = 1.5$) and varying F, the fraction of light fed back to the laser cavity, the obtained values then plotted as in Fig. 9. It can be seen that when the amount of light fed back to the laser was decreased, the induced-pulsations due to feedback quenched to relaxation oscillation, which indicates that self-sustained pulsation may arise due to the presence of induced losses (saturable absorbers) in the laser cavity depending on the increase of photon density as a result of feedback. Another parameter that affect the frequency of self-sustained oscillation is the fraction of spontaneous emission coupled to lasing mode, β. When β decreased the oscillation frequency continues to decrease in frequency at a more decreasing rate than before.

4.3. *Laser Modulation*

The frequency modulation of the laser is affected by the value of β as indicated in Fig. 11. When spontaneous emission (noise) was coupled to the lasing mode, the diode transfer function "modulation depth" was decreased and the resonance frequency was also decreased. These plots can be used to qualitatively analyze the frequency response of the laser diodes and indicate that high-frequency ringing will occur if modulated at this region.

Fig. 12 shows the plot of laser modulation depth against frequency of different values of F. A great deal of laser modulation was decreased as F increased, as can be seen from the graph. Although more photons will be in the laser cavity as a result of optical feedback but the optical induced losses will deplete the photon density and decrease the transfer funstion.

5. Conclusion

Chaotic behaviour of semicondustor laser have been studied experimentally

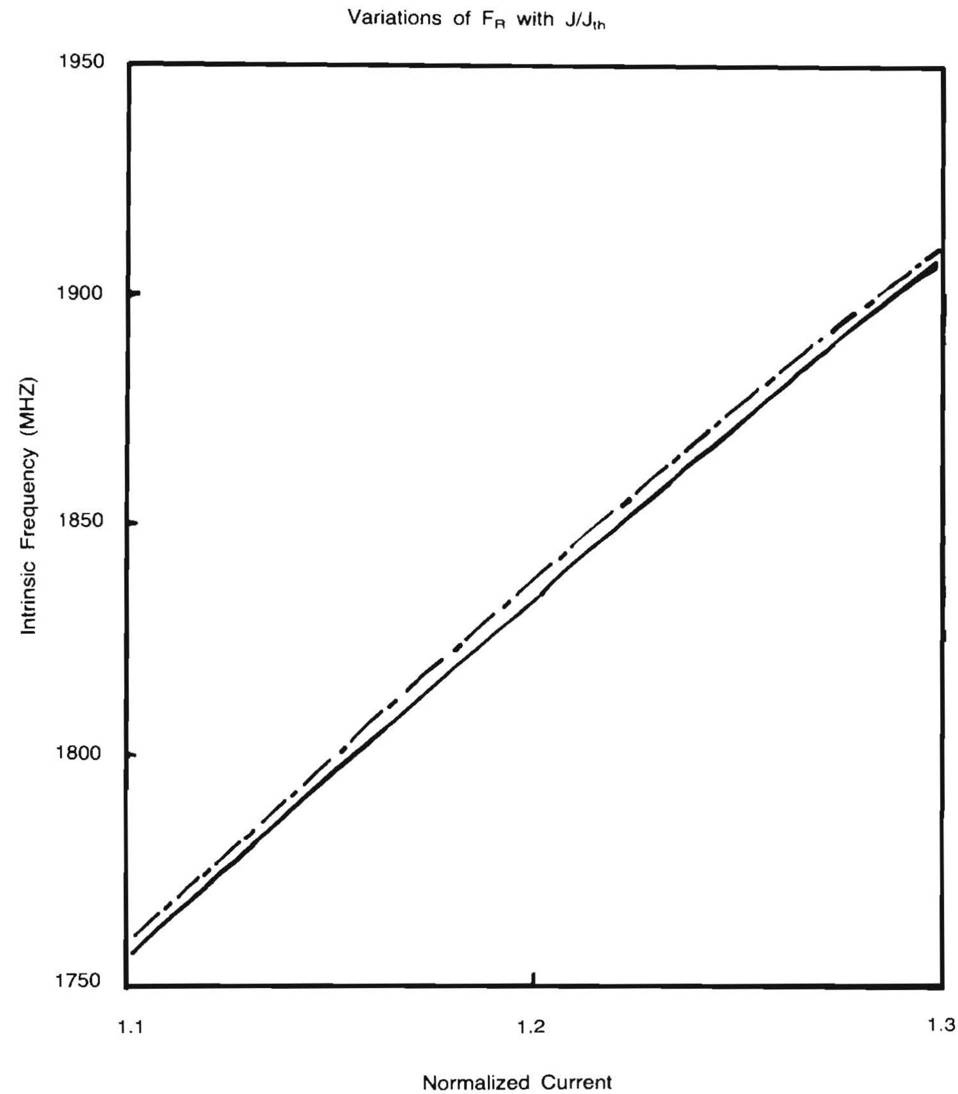

Fig. 7. Solid curve shows the oscillation frequency for saturable absorbers of density No = 2.0×10^{15} cm^{-3} (F=5% & L=5 cm). Dashed curve for $N_0=0$.

and theoretically. From experimental results we have observed that the onset of self-sustained pulsations at low optical power level and quenched near the kink. The results obtained suggest that the pulsations could be caused by saturable absorbers. In this investigation we have developed a theoretical model which can account the effects of saturable absorbers, optical feedback and spontaneous emission on the observed non-linear behaviour of the lasers studied. The

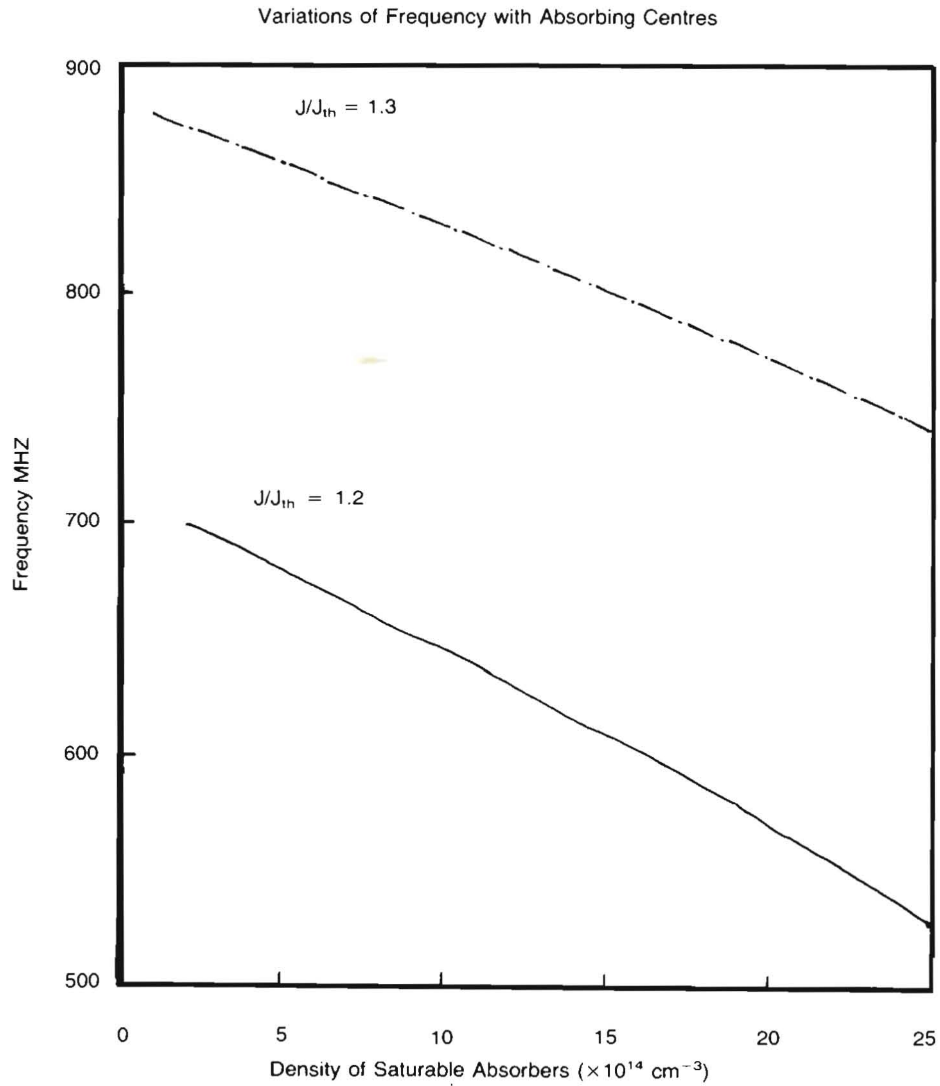

Fig. 8. Oscillation frequency against absorbers density for two values of bias current (normalized to the threshold current J_{th}). F = 5%, L = 2 cm & $\beta = 10^{-4}$.

spontaneous emission can affect the transient response of the laser and contribute to the ringing effects. The optical reflection from an external cavity introduce pronounced noise on the laser dynamics and suggest using optical isolators between the laser diode and transmission lines in optical communications operating in the region where affected by F. Based on our model we have been able to estimate the density of saturable absorbers that are sufficient to shift the pulsation frequency or quenching them.

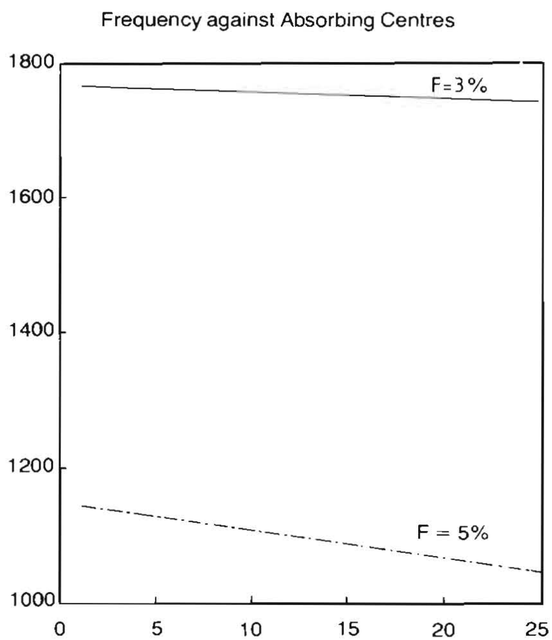

Fig. 9. A plot of oscillation frequency as a function of density of saturable absorbers. L = 2 cm & $\beta = 10^{-4}$.

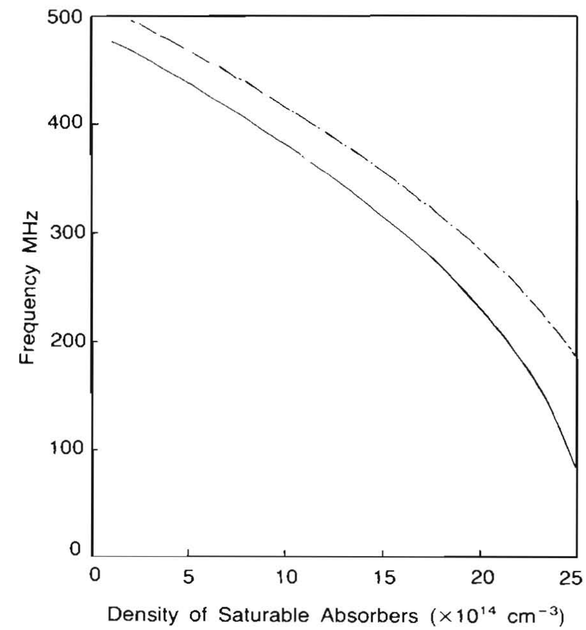

Fig. 10. Effects of variations in β on pulsation frequency. F=5%, L=2 cm & J/J_{th} = 1.1.

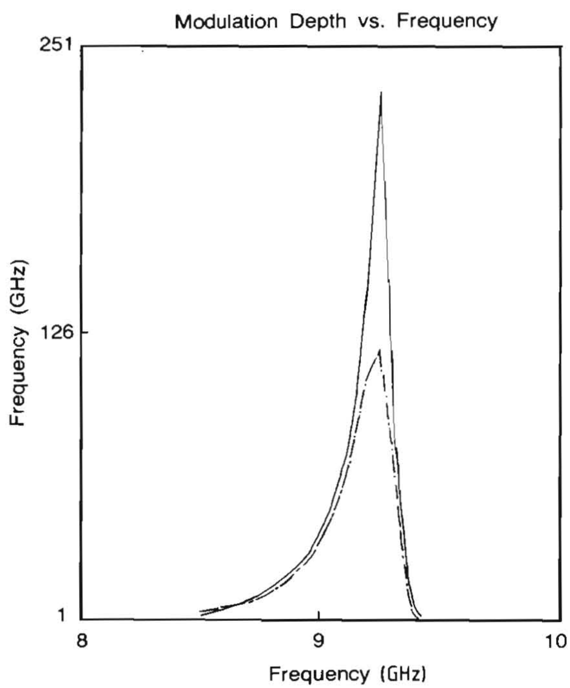

Fig. 11. Laser diode transfer as a function of frequency. Solid curve ($\beta = 0$) and dashed curve ($\beta = 10^{-2}$).

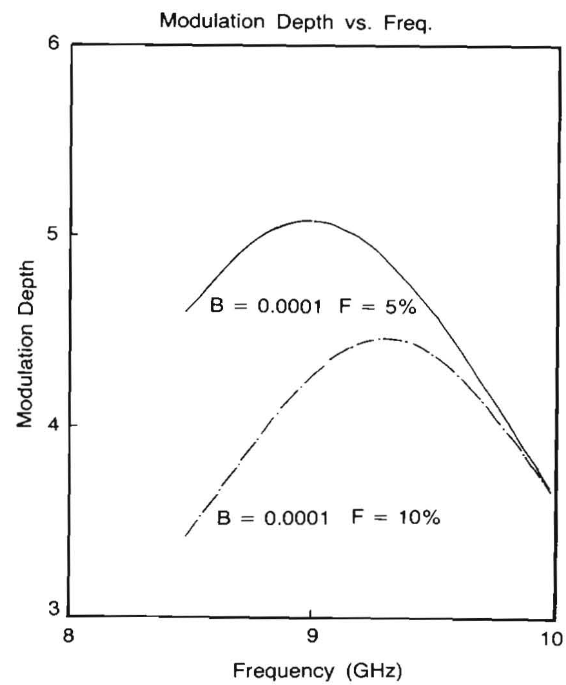

Fig. 12. Variations of modulation depth against frequency with F as variable.

References

Adams, M.J. (1987) Physics and applications of optical bistability in semiconductor laser amplifier, *Solid State Electron.* 30, 59.

Azawi, M.I. and **Abbas, I.K.** (1987) Pulse analysis of stripe geometry GaAs/Al_xGa_{1-x}As double heterostructure injection lasers, Fourth Arabic Conference for Physics and Maths. Baghdad.

Azawi, M.I. and **Thomas, Ben** (1985) Analysis of induced noise characteristics of semiconductor laser diode due to optical feedback using rate equations, Seventh Scientific Conference of Iraqi Society for Physics & Maths., Baghdad.

Botez, D. and **Herskowitz, G.J.** (1980) Components for optical communication systems: *A Review*, *Proc. IEEE*, **68:** 689.

Boyed, R.W., Rayner, M.C. and **Narducci, L.M.** (Eds.) I (1986) Optical Instabilities, Cambridage University Press, Cambridge.

Campbell, J.C., Abbot, S.M. and **Dentai, A.G.** (1980) A comparison of normal lasers and lasers exhibiting light jumps, *J. Appl. Phys.* **51:** 4010.

Channin, D.J. (1979) Effect of gain saturation on injection laser switching, *J. Appl. Phys.* **50:** 3858.

Channin, D.J. Ettenberg, M. and **Kressel, H.** (1979) Self-sutained oscillations in (AlGa)As oxide-defined stripe laser, *J. Appl. Phys.* **50:** 6700.

Chen, Y.C. (1980) Noise characteristics of semiconductor laser diodes coupled to short optical fibers, *Appl. Phys. Lett.* **37:** 587.

Chik, K.D., Dyment, J.C. and **Richardson, B.A.** (1980) Self-sustained pulsations in semiconductor laser: experimental results and theoretical confirmation, *J. Appl. Phys.* **51:** 4029.

Guckenheimer, J. and **Holmes, P.** (1983) Nonlinear Oscillations, Dynamical Systems and Bifurcation of Vector Fields, Springer, New York.

Haramitsu, K. Fujiwara, T. and **Takusagawa, M.** (1981) Repetitive pulsating stripe geometry GaAlAs double-heterostructure lasers with a stripe by a shallow Zn diffusion, *Appl. Phys. Lett.* **39,** 14.

Hartman, R.L., Logan, R.A., Koszi, L.A., and **Tsang, W.T.** (1979) Pulsation and absorbing defects in (Al,Ga)As injection lasers, *J. Appl. Phys.* **50:** 4616.

Kressel, H. (1981) Lasers and LED'S for optical communications using fibres, *Radio Scient.* **16:** 445.

Lang, R. (1971) Horizontal mode deformation and anomalous lasing properties of stripe - geometry injection lasers - Theoretical model, Japan. *J. Appl. Phys.* **16:** 205.

Nakamura, M. and **Tsuji, S.** (1981) Single-mode semiconductor injection lasers for optical fibre communication, *IEEE J. Quant. Electron.* **QE-17:** 994.

Peterman, K. and **Arnold, G.** (1982) Noise and distortion characteristics of semiconductor lasers in optical fibre communication systems, *IEEE J. Quant. Electron.* **QE-18:** 543.

Shore, K.A. (1987) Non-linear dynamics and chaos in semiconductor laser devices, *Sold State Electron.* **30:** 59.

Su, C.B. (1981) An analytical solution of kinks and nonlinearities driven by near - field displacement instabilities in stripe geometry diode laser, *J. Appl. Phys.* **52:** 2665.

Van der Ziel, J.P. (1981) Self-focusing effects in pulsating Al_xGa_{1-x}As double-heterostructure laser, *IEEE J. Quant. Electron.* **QE-17:** 60.

Van der Ziel, J.P., Merz, J.L. and **Paoli, T.L.** (1979) Study of intensity pulsations in proton - bombarded stripe - geometry double-heterostructure Al_xGa_{1-x}As laser, *J. Appl. Phys.* **50:** 4620.

Whiteaway, J.E.A. (1982) Theoretical analysis of current spreading in stripe-geometry injection laser, *IEEE Proc.* **129:** 89.

Wong, Y.L. and **Carroll, J.E.** (1987) A travelling-wave rate equation analysis for semiconductor lasers, Solid State Electron. **30:** 13.

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الديناميكية اللاخطية في ليزر اشباه الموصلات ذات الشكل المقطعي : نتائج تجريبية وتحليل نظري

مزاحم إبراهيم عزاوى

قسم الفيزياء ـ كلية التربية ـ جامعة الموصل ـ العراق

يتضمن هذا البحث النتائج التجريبية للظواهر السريعة الزائلة التي تحدث في ليزر أشباه الموصلات من نوع ($Al_x Ga_{1-x} As$) ذات الشكل المقطعي وكذلك انموذج نظري باستعمال معادلات الموجة المتحركة لدراسة الديناميكية اللاخطية والتشوش في ليزر أشباه الموصلات، يشمل هذا الأنموذج تأثيرات العيوب البلورية والتي تعمل كمراكز لامتصاص الضوء، التغذية الضوئية الاسترجاعية ومعامل الانبعاث الضوئي الذاتي. التحليل العددي للنتائج وحلول المعادلات النظرية تؤكد بأن زيادة تركيز مراكز الامتصاص الضوئي تقلل من تردد المستهل للنبضات، إضافة إلى ذلك فإن هذه الدراسة تعطي الدليل الواضح للديناميك المشوش في ليزر اشباه الموصلات والتي تعمل مع فجوة خارجية.

تتطرق الدراسة أولاً للناحية العملية من حيث دراسة الخواص الضوئية لثنائي الليزر مع التيار. وقد لوحظ ظهور اعوجاج (عدم خطية) منحني الضوء ـ التيار بعد تيار العتبه مصحوباً بتحرك الضوء المنبعث نحو أحد حافات المقطع. التوزيع النبضي للضوء المنبعث من الليزر قبل وبعد ظهور الإعوجاج تم تحليله باستعمال محلل متعدد القنوات حيث تبين ان هناك تغير في شكل وتوزيع الطاقات الضوئية مع زيادة التيار وتغير في تردد النبضات الذاتية المتولدة في الليزر.

من نتائج التحليل النظري لهذه الدراسة ان معامل الانبعاث الضوئي الذاتي يحدث تشويهات في تردد الليزر وخاصة في منطقة الاتصالات الضوئية، وكذلك أيضاً تأثير التغذية الضوئية الاسترجاعية على تردد العتبة للنبضات الذاتية حيث

تبين بأنها تقلل منها وتدفعها نحو الترددات الأطأ. ومن النتائج النظرية أيضاً تأثير تركيز مراكز الامتصاص الضوئي على تردد النبضات الذاتية وكيفية تأثر هذا التردد مع زيادة التركيز وتحولها نحو التذبذب المنحل. وقد تم دراسة تأثير هـذه العوامـل مجتمعة نظرياً على خواص الليزر ومقارنتها بالنتائج العملية التي تـوصلنا إليهـا من الجزء العلمي وكذلك النتائـج العملية الأخـرى المتوفـرة والمنشورة لهـذا النوع من ليزر (Ga, Al) As وخاصـة ذات الشكل المقـطعي. ومن النتائـج النظريـة الأخرى تأثير هذه العوامـل على التضمـين الترددي لليـزر واهميتها من النـاحية العمليـة في الاتصالات الضوئية.