# On Obstructions to $K^*$ ( ;Z/2)−Orientability

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ABSTRACT. The vanishing of the differentials of the Atiyah-Hirzebruch spectral sequence on the Thom class is the necessary and sufficient condition for O(n)-bundle to be orientable with respect to a cohomolog theory $h^*$. It is shown here that the third Stiefel-Whitney class is the first obstruction for an oriented spherical fibration to be orientable with respect to mod 2 K-theory $K^*$ ( ;Z/2). By killing the third differential of the Atiyah-Hirzebruch spectral sequence, we will find the second obstruction for an oriented real vector bundle to be orientable with respect to $K^*$ ( ;Z/2) by calculating the fifth differential in the Atiyah-Hirzebruch spectral sequence.

Given a multiplicative cohomolog theory with unit $h^*$, a finite dimensional CW-complex X, and an O(n)-bundle $\xi$ over X, we consider the question of when $\xi$ is orientable in the sense of Dold (1962) for $h^*$.

By this we mean if we consider the total space $(D(\xi), S(\xi))$ of the disc bundle associated to $\xi$, and if we consider the map $i_x: (D^n, S^{n-1}) \to (D(\xi), S(\xi))$ which is the inclusion of the fibre over $x \in X$, $i_x^*: h^n (D(\xi), S(\xi)) \to h^n (D^n, S^{n-1})$ is onto for each $x \in X$ (one x in each component suffices).

Our chief tool will be the Atiyah-Hirzebruch spectral sequence (AHSS). This is a spectral sequence with $E_2^{p,q}(X) \cong H^p(X; h^q(pt))$ and which converges to $h^{q+q}$ (X).

Consider the AHSS for $(D(\xi), S(\xi)) = (T(\xi), pt)$. Let n be the dimension of the fibre of $T(\xi)$. Then $E_2^{n-i,q} (T(\xi)) = 0$ for all $i > 0$, so $E_\infty^{n,0} (T(\xi)) \subseteq E_2^{n,0} (T(\xi))$, and $h^n(D(\xi), S(\xi)) \to E_\infty^{n,0} (T(\xi))$ is onto.

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If we compute $h^n(D^n, S^{n-1})$, we find that the AHSS collapses, and one can see that the map $i_x^* : h^n(D(\xi), S(\xi)) \to h^n(D^n, S^{n-1})$ is

$$h^n(D(\xi), S(\xi)) \to E_\infty^{n,0}(T(\xi)) \subseteq E_2^{n,0}(T(\xi)) \xrightarrow{\delta x} E_2^{n,0}(D^n, S^{n-1}) \cong h^n(D^n, S^{n-1})$$

Hence $\xi$ is $h^*$-orientable if and only if $\subseteq$ and $\delta_x$ are onto. $\delta_x$ is onto if and only if either $w_1(\xi|X_x) = 0$, where $X_x$ is the component of $X$ containing $x$, or $h^0(pt)$ is a $Z/2$-vector space.

If $w_1(\xi|X_x) = 0$, or $h^0(pt)$ is a $Z/2$-vector space, $E_2^{n,0}(T(\xi)) = H^n(D(\xi), S(\xi);$ $h^0(pt)) \cong \text{Hom}(Z, h^0(pt))$. Define the Thom class $\Delta_\xi \in E_2^{n,0}(T(\xi))$ to be the homomorphism $Z \to h^0(pt)$ taking $1 \in Z$ to the identity in $h^0(pt)$. Then $\subseteq$ is onto if and only if all differentials vanish on $\Delta_\xi$.

The map $\pi : D(\xi) \to X$ induces a pairing of the AHSS's $E_r^{*,*}(X) \otimes E_r^{*,*}$ $(T(\xi)) \to E_r^{*,*}(T(\xi))$. Define $\Phi_2 : E_r^{*,*}(X) \to E_2^{*+n,*}(T(\xi))$ by $\Phi_2(\alpha) = \pi^*(\alpha)$ $\times \Delta_\xi$. Since $w_1(\xi|X_x) = 0$ or $h^0(pt)$ is $Z/2$-vector space, the Thom isomorphism theorem for ordinary cohomology says $\Phi_2$ is an isomorphism.

If $d_2(\Delta_\xi) = 0$, we can define $\Phi_3 : E_3^{*,*}(X) \to E_3^{*+n,*}(T(\xi))$ from $\Phi_2$ since $d_2\Phi_2(\alpha) = \Phi_2 d_2(\alpha) + (-1)^{\deg(\alpha)} \pi^*(\alpha) \times d_2(\Delta_\xi) = \Phi_2 d_2(\alpha)$. Since $\Phi_2$ is an isomorphism, so is $\Phi_3$.

We can inductively construct $\Phi_r : E_r^{*,*}(X) \to E_r^{*+n,*}(T(\xi))$ provided $d_{r-i}$ $(\Delta_\xi) = 0$ for $i > 0$. (Since $E_r^{n,0}(T(\xi)) \subseteq E_2^{n,0}(T(\xi))$, $\Delta_\xi$ actually sits in $E_r^{n,0}$ if $d_{r-i}$ vanish on it).

Thus, when all the differentials vanish on $\Delta_\xi$, we get a natural isomorphism of AHSS's $\Phi_r : E_r^{*,*}(X) \to E_r^{*+n,*}(T(\xi))$. This induces a Thom isomorphism $\Phi :$ $h^*(X) \to h^{*+n}(D(\xi), S(\xi))$, given by $\Phi(\alpha) = \pi^*(\alpha) \times \Delta_\xi$, where $\Delta_\xi \in h^n(D(\xi),$ $S(\xi))$ maps onto $\Delta_\xi \in E_\infty^{n,0}(T(\xi))$ by the natural map.

The purpose of this paper is to establish the first and the second obstructions of $O(n)$-bundles to be orientable with respect to mod 2 K-theory.

In §1 we recall the definition and some basic facts about the AHSS's for mod p K-theory, where p is a prime. In §2 we will see that the first obstruction to $K^*$ $(\ ;Z/2)$-orientability is the third Stiefel-Whitney class. The second obstruction for oriented real vector bundle is established in §3.

## §1:

Let p be any integer and $h^*$ be a cohomology theory. If X is a finite CW-complex with base point, we then define $h^*$ (X; Z/p) as follows.

Let $M_p$ denote the 'co-Moore' space $S^1 \cup_f e^2$, where f: $S^1 \to S^1$ is a base point preserving map of degree p; $M_p$ is a space of type (Z/p,2). Now form the reduced product $X \wedge M_p$ (another finite CW-complex), and define $h^n$ (X; Z/p) = $\tilde{h}^{n+2}$ $(X \wedge M_p)$, where $\tilde{h}$ denotes reduced cohomology (Maunder 1967).

Let p be a prime, and X a finite CW-complex, then there is a spectral sequence $\{(E_r^{s,t}(X); Z/p)\}$ (r ⩾ 2, s ⩾ 0) with differentials $d_r$: $E_r^{s,t} \to E_r^{s+t,t-s+1}$ such that

(i) $E_2^{s,t}$ = $H^s$ (X; $K^t$ ($S^0$; Z/p))

(ii) $E_\infty^{s,t}$ = $G_s$ $K^{s+t}$ (X; Z/p)

$$= K_s^{s+t}(X; Z/p) \Big/ K_{s+1}^{s+t} (X; Z/p)$$

Where $K_s^n$ (X; Z/p) = Ker [($K^n$ (X; Z/p) $\to$ $K^n$ ($X^{s-1}$; Z/p)]

$X^{s-1}$ being the (s−1) − skeleton of X. Since $K^t$ ($S^0$; Z/p) = 0 or Z/p according to t being odd or even, and $E_r^{s,t}$ = 0 whenever t is odd, then $d_r$ vanishes for r even. Moreover $d_r$ = 0 for 2 ⩽ r ⩽ 2p−2, so that for 2 ⩽ r ⩽ 2p−1 $\{(E_r(X); Z/p)\}$ can be identified with $H^s$ (X; Z/p), with this identifications $d_{2p-1}$ is equal (up to a multiplication by a non-zero element of Z/p) to Milnor's stable cohomology operation $Q_1$ = $P^1\beta - \beta P^1$ (for p = 2 setting $P^1$ = $Sq^2$, $\beta$ = $Sq^1$) (Atiyah and Hirzebruch 1961).

## §2:

Let $\xi$ = (E, π, F, X) be a spherical fibration over a CW-complex X. Let T($\xi$) be the Thom space of $\xi$ and $\Delta_\xi$ be the Thom class in ordinary cohomology. Let $\{(E_r(X); Z/2)\}$ be the AHSS of §1.

*Lemma 2.1:*

With $\xi$ as above, the first obstruction to $K^*$( ; Z/2) - orientability is $d_3(\Delta_\xi)$ = $(w_1^3 + w_1 \cdot w_2 + w_3)$. $\Delta_\xi$, where $w_i$ the ith Stiefel-Whitney class.

*Proof:*

By Atiyah, Hirzebruch (1961), in $\{(E_r(T(\xi);\ Z/2)\}$, $d_3$ is given by

$$d_3(\Delta_\xi) = (Sq^2 Sq^1 + Sq^3)(\Delta_\xi)$$

$$= Sq^2 Sq^1(\Delta_\xi) + w_3 \cdot \Delta_\xi$$

$$= Sq^2(w_1 \cdot \Delta_\xi) + w_3 \cdot \Delta_\xi$$

$$= Sq^1(w_1)\ Sq^1(\Delta_\xi) + w_1\ Sq^2(\Delta_\xi) + w_3 \cdot \Delta_\xi$$

$$= (w_1^3 + w_1 w_2 + w_3) \cdot \Delta_\xi$$

Where $Sq^i$ is the mod 2 Steenrod operation (Epstein, Steenrod 1962).

Here we used the equalities $Sq^0 = $ identity, $w_i(\xi) = \Phi_H^{-1} Sq^i (\Delta_\xi)$; and the Cartan formula.

*Corollary 2.2.:*

If $\xi$ is an oriented spherical fibration, then the third Stiefel-Whitney class $w_3(\xi) \epsilon H^3(X;\ Z/2)$ is the first obstruction.

*Proof:*

Immediate from the lemma, since $w_1(\xi) = 0$.

§3:

As in §1, in the AHSS $\{(E_r;\ Z/2)\}$ all even differentials vanish and the first non-zero differential is $d_3$, so the next possibly non-vanishing differential is $d_5$. We will compute $d_5(\Delta_\xi)$, and thus the second obstruction to $K^* (\ ;\ Z/2)$ - orientability, when $\xi$ is an oriented real vector bundle and $d_3(\Delta_\xi) = 0$.

Let $\xi$ be an oriented real vector bundle over $B(w_1 = 0)$ and $f : B \to BSO$ be its classifying map. Realize $w_3 \epsilon H^3 (BSO;\ Z/2)$ as a map $w_3 : BSO \to K(Z/2, 3)$, we then have a principal fibration.

$K(Z/2,2) \xrightarrow{i} X \xrightarrow{p} BSO \xrightarrow{w_3} K(Z/2, 3)$ induced from $w_3$. Consider the following diagram

$$K(Z/2, 2) \xrightarrow{\quad i \quad} X$$

with $\tilde{f}$ a diagonal arrow from B to X, and $p$ the vertical map from X to BSO, and

$$B \xrightarrow{\quad f \quad} BSO \xrightarrow{\quad w_3 \quad} K(Z/2,3)$$

Since $w_3 \cdot f$ is null homotopic and p is the fibre of $w_3$, we then have a lifting $\tilde{f} : B \to X$ such that $p \cdot \tilde{f} = f$. Also we have $d_5(\Delta_\xi) = \tilde{f}^* (d_5(\Delta_{\bar\xi}))$, where $\bar\xi$ is the bundle over X induced by p. So in order to compute $d_5(\Delta_\xi)$, we need to calculate the cohomology mod 2 of the space X in the fibration $K(Z/2, 2) \xrightarrow{i} X \xrightarrow{p} BSO \xrightarrow{w_3} K (Z/2, 3)$, we do that by using the Serre spectral sequence.

Recall if $\xi = (E, \pi, F, B)$ is a fibre space over B with fibre F, and if B and F are arcwise connected and G is a ring, then there is a spectral sequence $\{(E_r, d_r)\}$ (Mosher and Tangora 1968) with the $E_2$ term is given by $E_2^{s,t} = H^s(B; H^t(F;G))$ converging to $H^* (E;G)$, where $H^t (F;G)$ is the local coefficients. If B and F are $(s-1)$ and $(t-1)$ - connected respectively, we then have the Serre exact sequence which terminates as follows:

$$\to H^{s+t} (F;G) \xrightarrow{\tau} H^{s+t-1} (B;G) \xrightarrow{\pi^*} H^{s+t-1} (E;G) \xrightarrow{i^*} H^{s+t-1} (F;G)$$

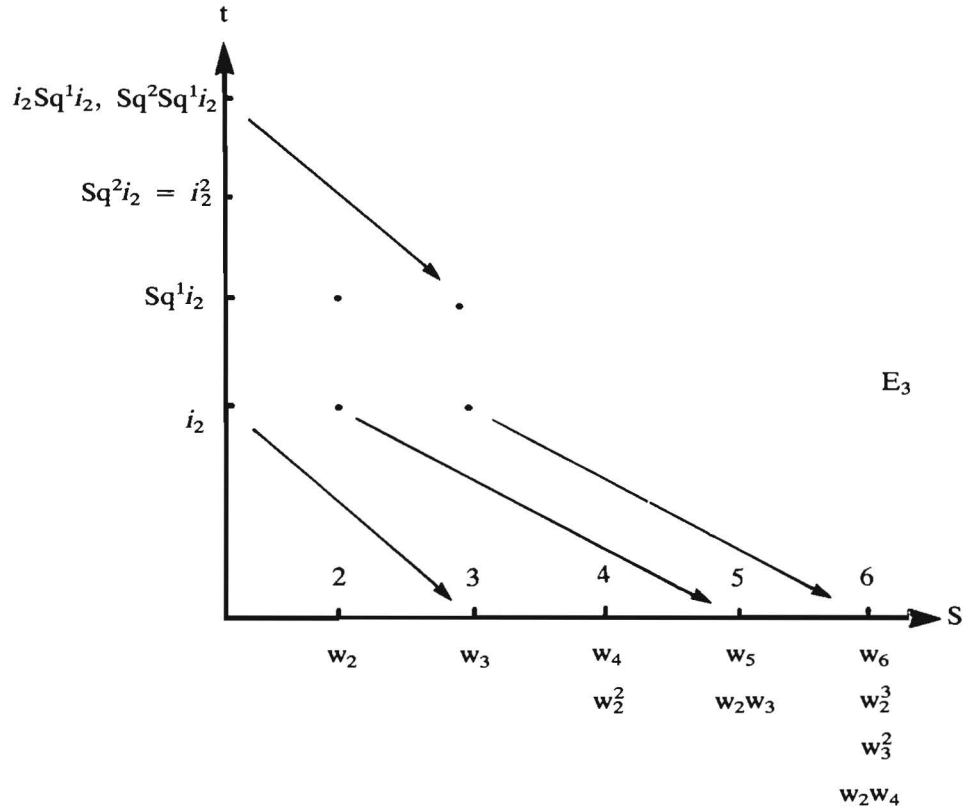where $i : F \to E$, and $\tau$ is the cohomology transgression.

*Theorem 3.1:*

For $n \leqslant 5$, $H^n (X; Z/2)$ is generated by $w_2$, $A_3$, $\tilde{w}_2^2$, $\tilde{w}_4$, $\tilde{w}_2 . A_3$ and $Sq^2 A_3$. ($\tilde{w}_{2i} = p^* w_{2i}$ and $A_3$ is as defined in the proof).

*Proof:*

$$H^*(BSO; Z/2) \cong Z/2 [w_2, w_3, w_2^2, w_4, ...] \text{ and}$$

$$H^*(K(Z/2,2); Z/2) \cong Z/2 [i_2, Sq^1 i_2, ...]$$

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The figure below is the diagram of the $E_3^{s,t}$ term of the spectral sequence of the fibration. $K(Z/2,2) \xrightarrow{i} X \xrightarrow{p} BSO$ in the relevant dimensions.



Only terms of total degree of at most 5 needs to be considered for our purposes. Since $p:X \to BSO$ is induced from $w_3$, we have

$$d_3(i_2) = \tau(i_2) = w_3 \; ; \; d_3(Sq^1(i_2)) = 0$$

$$d_3(i_2^2) = 0 \; ; \; d_3(i_2 \cdot w_2) = w_2 \cdot w_3$$

$$d_3(i_2 \cdot Sq^1(i_2)) = w_3 Sq^1(i_2) \; ; \; d_3(Sq^2 Sq^1 i_2) = 0$$

$$d_3(i_2 \cdot w_3) = w_3^2 \text{ and } d_3(w_2 \, Sq^1 i_2) = 0$$

leaving $w_2$, $Sq^1i_2$, $w_2^2$, $w_4$, $w_2 \, Sq^1i_2$ and $Sq^2Sq^1i_2$ in $E_4 = E_5$, for $d_4$ of all these elements is zero:

$$d_5(i_2^2) = w_2w_3 + w_5, \text{ but } d_3 \, (i_2 \cdot w_2) = w_2 \cdot w_3,$$ so this means that $H^5(BSO; Z/2)$ is killed. Also we have

$$d_5(Sq^1i_2) = 0; \quad d_5(w_2 \cdot Sq^1i_2) = 0; \quad d_5(Sq^2Sq^1i_2) = 0$$

Also in dimension 8; $d_3(w_2 \cdot Sq^3Sq^1i_2) = 0$. Thus, the following elements of total degree $\leqslant 5$ survive to $E_\infty$ of this spectral sequence.

$w_2$, $Sq^1i_2$, $w_2^2$, $w_4$, $w_2 \cdot Sq^1i_2$, $Sq^2Sq^1i_2$ and the element $w_2 \cdot Sq^3 \, Sq^1i_2$ in dimension 8. The groups $E_{st}$ for which $s+t=n$ form a composition series for $H^n(X; Z/2)$. Therefore, for $n \leqslant 5$ we can write a basis for $H^n(X; Z/2)$. Denote the elements of these basis by, $\widetilde{w}_2$, $A_3$, $\widetilde{w}_2^2$, $\widetilde{w}_2 \cdot A_3$, and $Sq^2A_3$. Since

$$d_r : E_r^{s,0} \to E_r^{s+r,-r+1} = 0 \quad \text{for } r \geqslant 2$$

$H(E_r^{s,0}) = $ quotient module of $E_2^{s,0}$, therefore we get an epimorphism $E_2^{s,0} = H^s (BSO; Z/2) \to E_\infty^{s,0} \to H^s(X; Z/2)$ which is just $p^*$, since $\widetilde{w}_{2i}$ comes from the base; so $\widetilde{w}_2 = p^*w_2$, $\widetilde{w}_4 = p^*w_4$, $\widetilde{w}_2^2 = p^*w_2^2$, ......

In the fibre space $K(Z/2,2) \xrightarrow{i} X \xrightarrow{p} BSO$, we have $w_2 \epsilon H^2 (BSO; Z/2)$ and $i_2 \epsilon H^2 (K(Z/2,2);Z/2)$ is transgressive, *i.e.*, $\beta_1w_2 = Sq^1w_2 = w_3 = \tau(i_2)$. Thus, applying the Bockstein lemma (Mosher and Tangora 1968, pp. 106) to the above fibration we have $i^*\beta_2p^*w_2$ is defined, where $\beta_1$ and $\beta_2$ are the Bockstein differentials, and $i^*\beta_2p^*w_2 = i^*\beta_2\widetilde{w}_2 = \beta_1i_2 = Sq^1i_2 = i^*A_3$, but $i^*: H^3(X;Z/2) \to H^3(K(Z/2,2); Z/2)$ is a monomorphism. Hence, $A_3 = \beta_2\widetilde{w}_2$.

Now we return to the computation of $d_5(\Delta_{\check{s}})$.

*Lemma 3.2:*

In the AHSS $\{E_r(X); Z/2)\}$ of the space $X$.

(i) $d_3(\widetilde{w}_2 \cdot \beta_2\widetilde{w}_2) \neq 0$ and (ii) $d_3(Sq^2\beta_2\widetilde{w}_2) = 0$.

*Proof:*

(i) $d_3(\widetilde{w}_2 \cdot \beta_2\widetilde{w}_2) = \widetilde{w}_2 d_3(\beta_2\widetilde{w}_2) + \beta_2\widetilde{w}_2 d_3(\widetilde{w}_2)$

$\qquad\qquad\qquad = \widetilde{w}_2 d_3(\beta_2\widetilde{w}_2)$, because $d_3(\widetilde{w}_2) = d_3(p^*w_2) = 0$

$\qquad\qquad\qquad = \widetilde{w}_2 [Sq^3\beta_2\widetilde{w}_2 + Sq^2Sq^1\beta_2\widetilde{w}_2]$

$\qquad\qquad\qquad = \widetilde{w}_2 [Sq^3\beta_2\widetilde{w}_2 + Sq^2\beta_1\beta_2\widetilde{w}_2]$

Since, $\beta_1\beta_2 = 0$; so $d_3(\widetilde{w}_2 \cdot \beta_2\widetilde{w}_2) = \widetilde{w}_2 Sq^3\beta_2\widetilde{w}_2 \neq 0$.

(ii) $d_3(Sq^2\beta_2\widetilde{w}_2) = (Sq^3Sq^2Sq^1)(Sq^2\beta_2\widetilde{w}_2)$

$\qquad\qquad\qquad = Sq^3Sq^2\beta_2\widetilde{w}_2 + Sq^2Sq^1Sq^2\beta_2\widetilde{w}_2$

Using Adem relation, $Sq^3Sq^2 = 0$ and $Sq^2Sq^3 = Sq^5 + Sq^4Sq^1$

$d_3(Sq^2\beta_2\widetilde{w}_2) = Sq^2 Sq^3\beta_2\widetilde{w}_2 = (Sq^5 + Sq^4Sq^1)(\beta_2\widetilde{w}_2)$

$\qquad\qquad\qquad = Sq^5\beta_2\widetilde{w}_2 + Sq^4Sq^1\beta_2\widetilde{w}_2 = 0$.

*Lemma 3.3:*

In $\{(E_r(X); Z/2)\}$ and $\{(E_r(T(\bar{\xi})); Z/2)\}$ we have

(i) $E_5^{5,0}(X) = \text{Ker } d_3 = Z/2 \cdot \{Sq^2\beta_2\widetilde{w}_2\}$

(ii) $E_5^{5,0}(T(\bar{\xi})) = Z/2 \cdot \{\Delta_{\bar{\xi}}Sq^2\beta_2\widetilde{w}_2\}$

*Proof:*

(i) Im $d_3 \subset H^5(X; Z/2)$ and $d_3(\widetilde{w}_2) = 0$; so Im $d_3 = 0$; so by lemma (3.2) $\text{Ker} d_3 = Sq^2\beta_2\widetilde{w}_2$.

(ii) $d_3(\Delta_{\bar{\xi}}) = 0$ and $d_3(\Delta_{\bar{\xi}} \cdot x) = \Delta_{\bar{\xi}}d_3(x)$.

Let $B = K(Z/4,1)^{(4)}$ and $A \subset B$ be the 4-skeleton and the 2-skeleton of $K(Z/4,1)$ respectively.

*Lemma 3.4:*

With B and A as above, there is an oriented vector bundle over S(B), the suspension of B, which is not orientable for KO and K-theories.

*Proof:*

Let u and $\bar{u}$ be the generators of $H^1(B; Z/2)$ and $H^1(A; Z/2)$. The following diagram represents AHSS,

$$\widetilde{H}^S (B, KO^t (point)) \Rightarrow \widetilde{KO}^*(B)$$

| | 0 | 1 | 2 | 3 | 4 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | Z | 0 | Z/4 | 0 | Z/4 | 0 | 0 | 0 | 0 | |
| −1 | Z/2 | Z/2 | Z/2 | Z/2 | Z/2 | 0 | 0 | 0 | 0 | |
| −2 | Z/2 | Z/2 | Z/2 | Z/2 | Z/2 | 0 | 0 | 0 | 0 | |
| −3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| −4 | Z | 0 | Z/4 | 0 | Z/4 | 0 | 0 | 0 | 0 | |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |

(horizontal axis labeled s; vertical axis labeled t)

We need to find $[B:SO] = \widetilde{KO}^{-1}$ (B). Now

$$H^1(B; Z/2) \xrightarrow{d_3} H^4(B; Z)$$
$$Sq^2 \searrow \qquad \nearrow \delta$$
$$H^3(B; Z/2)$$

$d_3 = \delta Sq^2$, but $Sq^2(u) = 0$; so $d_3(u) = 0$. Hence (no other differentials).

$E_3^{1,-2} = E_\infty^{1,-2} = Z/2$, and no other bits of total degree $-1$. So, $[B;SO] = KO^{-1}$ (B) $= Z/2$ and restriction to A is an isomorphism

$E_3^{1,-2} = E_\infty^{1,-2} = Z/2$, and no other bits of total degree $-1$. So, $[B;SO] = KO^{-1}$ (B) $= Z/2$ and restriction to A is an isomorphism

Call generators of $[A;SO]$ ; $\bar{\alpha}$, of $[B; SO]$; $\alpha$, define bundles

$$\begin{array}{ccc} E_{\bar{\alpha}} & \longrightarrow & E_\alpha \\ \downarrow & & \downarrow \\ S(A) & \longrightarrow & S(B) \end{array}$$

$E_\alpha = (C^+B \times R^n) \cup_\alpha (C^-B \times R^n)$, $S(B) = C^+B \cup C^-B$, where $C^+B = [0, \frac{1}{2}] \times B/_{\{0\} \times B}$, $C^-B = [\frac{1}{2}, 1] \times B/_{\{1\} \times B}$, $B = C^+B \cap C^-B$. We have $[A;SO] = Z/2$ corresponding to the class $\bar{u}$ in $H^1 (A; Z/2)$, so $w_2 (E_{\bar{\alpha}}) = \sigma (\bar{u}) \neq 0$ in $H^2(S(A); Z/2)$, $\sigma$ is the suspension isomorphism. Let $v$ and $\bar{v}$ be the generators of $H^2 (B; Z)$ and $H^2 (A; Z)$ respectively.

Write $H^*(B; Z) = Z[v]/_{(4v,v^3)}$ and the exact sequence associated with

$$0 \longrightarrow Z \xrightarrow{x2} Z \xrightarrow{\rho_2} Z/2 \longrightarrow 0$$

$\delta(u) = 2v$; $\delta(\bar{u}) = 2\bar{v}$, so $\delta(w_2(E_{\bar{\alpha}})) = \delta(\sigma(\bar{u})) = \sigma(2\bar{v}) \neq 0$. So $E_{\bar{\alpha}}$ is not orientable for KO or K-theories, and hence $E_\alpha$ is not.

By the lemma

$$w_3(E_\alpha) = Sq^1 (w_2(E_\alpha)) = Sq^1 (\sigma(u)) = \rho_2\delta(\sigma(u)) = \rho_2(\sigma(2v)) = 0$$

so $d_3(\Delta_{E_\alpha}) = 0$, $d_3(\Delta_{E_{\bar{\alpha}}}) = 0$. In case of $E_{\bar{\alpha}}$, $d_3$ is enough, so $E_{\bar{\alpha}}$ is orientable for $K^*$ ( ; Z/2). So the question is whether $E_\alpha$ is orientable for $K^*($ ; Z/2) or not.

$\alpha \epsilon [B;SO]$ induces a map, denoted again by $\alpha$,

$$(B \times D^n; B \times S^{n-1}) \xrightarrow{\alpha} (B \times D^n; B \times S^{n-1})$$

*Theorem 3.5:*

$E_\alpha$ is orientable for $K^*$ and $K^*$ ( ;Z/2) if and only if $\alpha^*(\gamma_n) = \gamma_n$, where $\gamma_n$ corresponds to $1 \otimes u$ in $K^*(B) \otimes K^*(D^n, S^{n-1})$, $u$ is the generator of $K^*(D^n, S^{n-1})$.

*Proof:*

Put $M = (D^n, S^{n-1})$, the theorem follows from the commutativity of the following diagram in which the rows are the Mayer-Vietoris exact sequences

$$\to \widetilde{K}^n(TE_\alpha) \overset{f}{\to} K^n(C^+B\times(M)) \oplus K^n(C^-B\times(M)) \overset{g}{\to} K^n(B\times(M)) \to$$

$$\to \widetilde{K}^n(T(E_\alpha|_{x_0})) \overset{\bar{f}}{\to} K^n(C^+(x_0)\times(M)) \oplus K^n(C^-(x_0)\times(M)) \overset{\bar{g}}{\to} K^n(x_0\times(M)) \to$$

with $f(\Delta_{E_\alpha}) = (\gamma_n^+, \gamma_n^-)$ and $g(\gamma_n^+) = \gamma_n$, $g(\gamma_n^-) = \alpha^*(\gamma_n)$.

Now $K^*(B \times D^n; B \times S^{n-1}) = \gamma_n \cdot K^*(B)$, so $\alpha^*(\gamma_n) = \gamma_n \cdot g_\alpha$, $g_\alpha \in K^0(B)$ actually $\in 1 + \widetilde{K}^0(B)$. By lemma (3.4) and theorem (3.5) $g_\alpha \neq 1$. Let $\rho_2(g_\alpha) =$ the $g_\alpha$ for K-theory mod 2, say $\widetilde{g}_\alpha$. So $E_\alpha$ is $K^*(\ ;Z/2)$ - orientable if and only if $\widetilde{g}_\alpha = 1$. Naturality gives $i^*(g_\alpha) = g_{\bar{\alpha}}$ for $i : A \to B$. For $g_{\bar{\alpha}}$ and $\widetilde{g}_{\bar{\alpha}}$ by lemma (3.4) we have

(i) $\widetilde{g}_{\bar{\alpha}} \neq 1$ because $E_{\bar{\alpha}}$ is not orientable for $K^*$,

(ii) $\widetilde{g}_{\bar{\alpha}} = 1$ because $E_{\bar{\alpha}}$ is $K^*(\ ;Z/2)$ - orientable.

*Lemma 3.6:*

$E_\alpha$ is not orientable for $K^*(\ ;Z/2)$.

*Proof:*

$K^*(A) = Z.1 \oplus Z/4. \bar{\eta}$; $\bar{\eta}^2 = 0$ because its induced from $A \to S^2$. $K^*(B)$ is a quotient of $Z[\eta]$ because $K^*(K(Z/4, 1))$ is up to completion (Hodgkin and Anderson 1968), so $K^*(B) = Z \cdot 1 \oplus Z/4. \eta \oplus Z/4. \eta^2$, and $\widetilde{K}^0(A) = Z/4. \bar{\eta}$; $\widetilde{K}^0(B) = Z/4. \eta + Z/4. \eta^2$. So $g_{\bar{\alpha}} = 1 + \lambda\bar{\eta}$, $\lambda \in Z/4$. Since $\lambda \neq 0$ and $\lambda \equiv 0 \bmod 2$, we must have $\lambda \equiv 2 \bmod 4$. So, $g_{\bar{\alpha}} = 1 + 2\bar{\eta}$

Now, $g_\alpha = 1 + \lambda'\eta + \mu'\eta^2$, but $i^*(\eta^2) = \bar{\eta}^2$ so $i^*(g_\alpha) = g_{\bar{\alpha}} = 1 + \lambda'\bar{\eta}$. So $\lambda' = 2$ and $g_\alpha = 1 + 2\eta + \mu'\eta^2$

To determine $\mu'$ we use Adams operations $\psi^k$ (Adams 1962). $\psi^k(g_\alpha) = g_\alpha$, because $\psi^k(\gamma_n) = k^n\gamma_n$, so

$$\psi^k(\alpha^*(\gamma_n)) = \alpha^* k^n \gamma_n = k^n \, \alpha^*(\gamma_n)$$
$$= k^n \, g_\alpha \cdot \gamma_n = g_\alpha \psi^k(\gamma_n)$$
$$= \psi^k \, (g_\alpha \cdot \gamma_n) = \psi^k(g_\alpha)\psi^k(\gamma_n)$$

Now $1 + \eta$ is a line bundle, by definition, so $(1 + \eta)^4 = 1$, hence reducing mod 4, $\eta^4 = 2\eta^2$. But if $\eta^3 = \lambda_1\eta + \lambda_2\eta^2$, $\eta^4 = \lambda_1\eta^2 + \lambda_2\eta^3 = \lambda_1\lambda_2\eta + (\lambda_1 + \lambda_2^2)\eta^2$. Hence $\lambda_1\lambda_2 = 0$, $\lambda_1 + \lambda_2^2 = 2$. This gives $\lambda_1 = 2$, $\lambda_2 = 0$ or 2. $\eta^5 = \eta^4.\eta = 2\eta^3 = 0$.

So $\psi^3(1 + \eta) = (1 + \eta)^3 = 1 + 3\eta + 3\eta^2 + \eta^3$
$$= 1 + 3\eta + 3\eta^2 + 2\eta + \lambda_2\eta^2 = 1 + \eta + (3+\lambda_2)\eta^2$$

$\psi^3(\eta) = \eta + (3 + \lambda_2)\eta^2$
$\psi^3(\eta^2) = \psi^3(\eta)\cdot\psi^3(\eta) = (\eta + (3 + \lambda_2)\eta^2)^2$
$$= \eta^2 + 2(3 + \lambda_2)\eta^3 + (3 + \lambda_2)^2 \, \eta^4$$
$$= 3\eta^2 \; (2\eta^3 = 0; \text{ any odd multiple of } \eta^4 \text{ is } 2\eta^2)$$

Hence,

$$\psi^3(1 + 2\eta + \mu'\eta^2) = 1 + 2\eta + (2(3 + \lambda_2) + 3\mu') \, \eta^2$$

but $2\lambda_2 = 0$ so $\lambda_2$ does not matter. But

$$g_\alpha = 1 + 2\eta + \mu'\eta^2 \text{ and } \psi^k(g_\alpha) = g_\alpha; \text{ so}$$

$$\psi^3(1 + 2\eta + \mu'\eta^2) = 1 + 2\eta + 2\eta^2 + 3\mu' \, \eta^2 = 1 + 2\eta + \mu'\eta^2$$

Hence, $\mu' = 2 + 3\mu'$     or     $2\mu' \equiv 2 \bmod 4$

Hence $\mu'$ is odd. Hence $g_\alpha = 1 + 2\eta + \mu'\eta^2$, $\mu'$ odd

and     $\widetilde{g}_\alpha \in 1 + \widetilde{K}^0(B;Z/2)$ is

$$1 + \rho_2(1 + 2\eta + \mu'\eta^2) = 1 + \rho_2(\eta^2); \rho_2(\eta^2) \neq 0$$

This implies, by theorem (3.5) that $E_\alpha$ is not orientable for $K^*(\ ;Z/2)$.

Corollary 3.7:

$$d_5(\Delta_{E_\alpha}) \neq 0.$$

Proof:

Since $d_3(\Delta_{E_\alpha}) = 0$ and dimension $S(B) = 5$; so by lemma (3.6) we must have $d_5(\Delta_{E_\alpha}) \neq 0$.

Proposition 3.8:

In the Atiyah-Hirzebruch spectral sequence $\{(E_r(T(\bar{\xi});Z2)\}$ of $T(\bar{\xi})$

$$d_5(\Delta_{\bar{\xi}}) = \Delta_{\bar{\xi}} \cdot Sq^2\beta_2\widetilde{w}_2.$$

Proof:

By theorem (3.2) and lemma (3.3),

$$d_5(\Delta_{\bar{\xi}}) = \lambda\ Sq^2\beta_2\widetilde{w}_2\cdot\Delta_{\bar{\xi}}, \ \lambda = 0 \text{ or } 1; \text{ so if }$$

$d_3(\Delta_\xi) = 0$ then $d_5(\Delta_\xi) = \lambda Sq^2\beta_2\widetilde{w}\cdot \Delta_\xi$ for any bundle $\xi$, by naturality. To prove $\lambda \neq 0$, its enough to consider an example and prove $\lambda \neq 0$ there; so by Corollary (3.7) $\lambda \neq 0$ in $d_5(\Delta_{\bar{\xi}})$ and hence for any bundle with $d_3(\Delta_\xi) = 0$.

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# عوائق التوجيه في نظرية ـ (Z/2 ; ) *K

تحسين غزال

قسم الرياضيات ـ كلية العلوم ـ جامعة الملك سعود
ص.ب: ٢٤٥٥ ـ الرياض ١١٤٥١ ـ المملكة العربية السعودية

لتكن *h نظرية كوهومولوجي عـامة ضربيـة، وليكن X فضاء ـ CW منتـهٍ، وأخيراً لتكن ξ حزمة ـ O (n) على X . إن مـا ندرسـه في هذا البحث هـو السؤال التالي :

«متى تكون ξ قابلة للتوجيه بالنسبة للنظرية *h، وذلك ضمن المعنى الذي وضعه دولد (١٩٦٢) » .

وللوصول إلى هذا الهدف اتبعنا المراحل التالية :

ـ لقـد بيّنا أولاً أن الشرط الـلازم والكافي لكي تكـون ξ قابلة للتوجيـه بـالنسبة للنظرية *h هو أن تتلاشى تفاضلات متتابعة عطية ـ هـيرزبروخ الـطيفية عـلى صف توم .

ـ أوجدنا بعدها العائق الأول والثاني بالنسبة للنظرية (Z/2 ; ) *K، وكـان ذلك وفق التسلسل التالي :

١ ـ تعرّضنا في ١ § بإختصار لتعريف نظرية الكوهومولـوجي العـامّـة (X ; Z/p) *h حيث X فضاء ـ CW منتهٍ و p عدد صحيح .

٢ ـ أثبتنا في ٢ § أن صف ستيفل ويتني الثالث هو العـائق الأول لتـوجيـه حزمـة الألياف الموجّهة ξ في نظرية (Z/2 ; ) *K .

٣ ـ أوجـدنا في ٣ § العـائق الثاني لتـوجيه الحـزمة المتجهة الحقيقية المـوجَّهة في نـظرية (Z/2 ; ) *K، وذلـك بحساب التفـاضل الخـامس لمتتابعـة عطيـة ـ هيرزبروخ الطيفية .

# A Flame Spectroscopic Study of the Stability
# of Dilute Solutions of Inorganic Salts

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ABSTRACT. Flame spectroscopy has been used to monitor net changes in concentration of dilute solution of thirty four elements at different concentrations in Pyrex, soft glass, and polyethylene containers for periods up to one year or more.

Dilute solutions are frequently unstable and may change in concentration over extended periods of time. The concentration changes are not only inconvenient because of the necessity of preparing fresh standard solutions, but are also potential serious sources of error in trace metal analysis.

Three types of processes are primarily responsible for concentration changes in dilute solutions. These are adsorption, leaching, and desorption (Minczewski 1967). Adsorption produces a loss of sample, whereas leaching of the vessel wall or desorption of a previously adsorbed sample produces an increase in concentration. These processes compete, and net change in concentration is determined by the extent to which each has occurred.

The adsorption process is the most thoroughly studied because of the ease with which it can be observed with radioactive tracers (Haissinky 1964). The extent of adsorption is dependent on the pH, the composition of the wall (soft glass, borosilicate glass, plastic, etc.), the temperature, the contact time, the solvent, and the composition of the solution (Mizuike 1965). Starik (1956) has written a comprehensive monograph on the adsorption of radioactive isotopes. The trace element composition of different containers and the leaching of trace elements in acid or basic solution have been described (Minczewski 1967, Mizuike 1965, Thiers 1957b, Eicholz *et al.* 1965, Thiers 1957a).

While adsorption losses are undoubtedly among the most important contributions to solution instability, net changes in concentration are of prime

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concern to the anlyst. Flame spectroscopy (emission and adsorption) offers a convenient means of monitoring net concentration changes in dilute solutions. The present study was undertaken to determine net changes in several inorganic solutions at different concentrations, in various containers, and for different periods of time up to one year or more.

## Experimental

Stock solutions of approximately 100 ppm of the elements to be studied were prepared as previously described (Christain 1968) from reagent grade chemicals. Niobium pentoxide fused with potassium carbonate and then dissolved in water. The rare earth solutions were prepared from the oxides dissolved in hydrochloric or sulfuric acid. Hydrochloric, nitric, or sulfuric acid was used to prepare other solutions as required. The stock solutions were stored in ground glass stopped Pyrex volumetric flasks. The compositions of the stock solutions are given in Table 1.

All test solutions were prepared by direct dilution of the stock solutions with water. These represented typical solutions prepared in the laboratory. Avoidance of adding more acid to the dilute solutions eliminated possible contamination from the acids. Triply distilled water was deionized with a mixed bed ion exchange column and was used for preparation of all solutions.

Solutions were analyzed by flame emission and atomic absorption spectroscopy using an Instrumentation Laboratory Model 153 Atomic Absorption Spectrophotometer. Conditions for the atomic absorption (Christian and Feldman 1970) and flame emission (Christian and Feldman 1971a, Christain and Feldman 1971b) measurements were as previously described. All test solutions were compared against dilute solutions freshly prepared form the stock solutions for instrument calibration.

## Results and Discussion

Dilute solutions of the metals were prepared by appropriate dilution of the stock solutions and were placed in new Corning Pyrex (Borosilicate glass), soft glass, and Kimble polyethylene bottles. The volume of each solution was 100 ml. The areas of the containers' surfaces exposed to the solutions were as follows; 95 $cm^2$ for the pyrex bottles, 101/$cm^2$ for the soft glass bottles and 96 $cm^2$ for the polyethylene bottles. All solutions were stored at room temperature. The results of the studies are summarized in table 2. In general, the precision of measurements was of $\pm$ 5% or better, and apparent changes of 5% or less are considered insignificant. Detection limits for the determinations have been

previously described (Christain 1968, Christian and Feldman 1970, Christian and Feldman 1971a). The data in the table indicate the net changes in solutions concentrations.

**Table 1.** Preparation of Stock Solutions (1000 parts per million)

| Element | Source compound | Reagent added | pH of solution |
|---------|-----------------|---------------|----------------|
| Ag | $AgNO_3$ | $HNO_3$ | 1.70 |
| Al | $AlCl_3 \cdot 6H_2O$ | HCl | 3.39 |
| Au | AuCl | HCl | 2.50 |
| Ba | $BaCl_2 \cdot 2H_2O$ | HCl | 5.95 |
| Ca | $CaCl_2$ | HCl | 2.12 |
| Cd | $CdCl_2 \cdot 2\frac{1}{2}H_2O$ | HCl | 2.50 |
| Co | $CoCl_2Ce(NO_3)_6$ | $HNO_3$ | 2.68 |
| Cr | $CrCl_3 \cdot 6H_2O$ | HCl | 2.45 |
| Cu | $Cu(NO_3)_2 \cdot 3H_2O$ | $HCl + HNO_3$ | 2.10 |
| Dy | $Dy_2O_3$ | HCl (200 ml/l) | – |
| Er | $Er_2O_3$ | HCl (200 ml/l) | – |
| Fe | $FeNH_4(SO_4)_2 \cdot 12H_2O$ | $H_2SO_4$ (80 ml/l) | – |
| Ga | Ga metal | HCl and $HNO_3$ | 0.40 |
| Gd | $Gd_2O_3$ | HCl | 0.30 |
| In | In metal | HCl | 0.95 |
| La | $La_2O_3$ | HCl | 1.05 |
| Li | LiCl | HCl | 5.45 |
| Mn | $MnCl_2$ | $HCl + HNO_3$ | 0.78 |
| Mo | $(NH_4)_6Mo_7O_2 \cdot 4H_2O$ | | 4.70 |
| Na | NaCl | | 6.75 |
| Nb | $Nb_2O_5$ | $K_2CO_3$ fusion | 12.50 |
| Nd | $Nd_2O_3$ | HCl (75 ml/l) | – |
| Ni | $NiCl_2 \cdot 6H_2O$ | HCl | 5.85 |
| Pr | $Pr_6O_{11}$ | HCl | 0.10 |
| Pt | $H_2PtCl_6 \cdot 6H_2O$ | | 2.25 |
| Sm | $Sm_2O_3$ | HCl (200 ml/l) | – |
| Sr | $SrCl_2 \cdot 6H_2O$ | HCl | 4.95 |
| Tb | $Tb_4O_7$ | HCl (250 ml/l) | – |
| Tl | $Tl(NO_3)_3$ | | 4.90 |
| U | $UO_2(NO_3)_2 \cdot 6H_2O$ | $HNO_3$ | 3.30 |
| V | $V_2O_5$ | HCl and $HNO_3$ | 0.30 |
| W | $Na_2WO_4 \cdot 2H_2O$ | | 6.80 |
| Y | $Y_2O_3$ | HCL (30 ml/l) and $H_2SO_4$ (25 ml/l) | – |
| Yb | $Yb_2O_3$ | HCl (300 ml/l) | – |

**Table 2.** Stability of Dilute Solutions

Percent element remaining after stated storage period

| Element | Concentration | Container | Days-% | Days-% | Days-% | Days-% | Days-% |
|---------|---------------|-----------|--------|--------|--------|--------|--------|
| Ag | 100 ppm | Pyrex | | | | | 441-100 |
| | 10 ppm | Pyrex | | | | | 441-81 |
| | | Soft glass | 37-92 | 64-86 | 96-78 | 96-78 | |
| | | Polyethylene | 37-95 | 59-92 | 91-87 | | |
| | 1 ppm | Pyrex | | | | | 419-6.5 |
| | | Soft glass | 37-44 | 64-38 | 96-31 | 128-25 | |
| | | Polyethylene | 37-77 | 59-62 | 91-57 | | |
| Al | 100 ppm | Pyrex | | | | | 454-100 |
| | 10 ppm | Pyrex | | | | | 420-100 |
| | | Soft glass | 37-100 | | | | |
| | 1 ppm | Pyrex | | | | | 420-70 |
| | | Polyethylene | 37-92 | 95-84 | 91-77 | | |
| Au | 100 ppm | Pyrex | | | | | 441-100 |
| | | Soft glass | 37-105 | | | | |
| | 75 ppm | Soft glass | 37-94 | 64-94 | | | |
| | | Polyethylene | 37-100 | | | | |
| Ba | 10 ppm | Pyrex | | | | | 443-85 |
| | | Soft glass | 37-96 | 64-100 | 96-97 | 133-104 | |
| | | Polyethylene | 37-100 | 59-98 | 86-104 | | |
| | 5 ppm | Soft glass | 37-90 | 64-95 | 96-100 | 133-103 | |
| | | Polyethylene | 37-100 | 59-105 | 96-114 | | |
| | 2 ppm | Pyrex | | | | | 443-52 |
| | | Soft glass | 37-75 | 64-70 | 96-68 | 133-70 | |
| | | Polyethylene | 37-82 | 59-86 | 96-80 | | |

Table 2.—(Contd.)

| Element | Concentration | Container | Days-% | Days-% | Days-% | Days-% | Days-% |
|---------|---------------|-----------|--------|--------|--------|--------|--------|
| Ca | 1 ppm | Soft glass | 37-99 | 64-99 | 96-95 | 135-93 | |
| | | Polyethylene | 37-100 | 59-92 | 98-88 | | |
| | 0.1 ppm | Pyrex | | | | | 419-80 |
| | | Soft glass | 37-87 | 64-95 | 96-90 | 135-122 | |
| | | Polyethylene | 37-102 | 59-85 | 98-74 | | |
| Cd | 100 ppm | Pyrex | | | | | 459-87 |
| | | Soft glass | 42-64 | 64-66 | 96-62 | 135-66 | |
| | | Polyethylene | 32-100 | 54-99 | 93-100 | | |
| | 75 ppm | Soft glass | 42-70 | 64-75 | 96-74 | 135-86 | |
| | | Polyethylene | 32-96 | 54-98 | 93-101 | | |
| | 50 ppm | Soft glass | 42-48 | 64-50 | 96-55 | 135-67 | |
| | | Polyethylene | 32-100 | 54-100 | 93-105 | | |
| Co | 20 ppm | Soft glass | 42-100 | 65-100 | 96-101 | 135-100 | |
| | | Polyethylene | 33-100 | 54-102 | 93-100 | | |
| | 10 ppm | Soft glass | 42-101 | 65-98 | 96-100 | 135-102 | |
| | | Polyethylene | 33-101 | 54-102 | 93-100 | | |
| | 5 ppm | Soft glass | 42-94 | 65-95 | 96-95 | 135-102 | |
| | | Polyethylene | 33-99 | 54-103 | 93-100 | | |
| Cr | 10 ppm | Pyrex | | | | 445-99 | |
| | | Soft glass | 40-100 | | | | |
| Cu | 5 ppm | Pyrex | | | | | 426-176 |
| | | Polyethylene | 33-101 | 54-100 | 93-93 | | |
| | 1 ppm | Soft glass | 40-91 | 73-95 | 94-98 | 135-94 | |
| | | Polyethylene | 33-96 | 54-100 | 93-106 | | |
| | 0.5 ppm | Pyrex | | | | | 426-228 |

**Table 2.—**(Contd.)

| Element | Concentration | Container | Days-% | Days-% | Days-% | Days-% | Days-% |
|---------|---------------|-----------|--------|--------|--------|--------|--------|
| | | Soft glass | 40-95 | 73-88 | 94-90 | 135-94 | |
| | | Polyethylene | 33-100 | 54-100 | 54-99 | 93-105 | |
| Dy | 100 ppm | Pyrex | | | | | 477-75 |
| | | Soft glass | 40-92 | 73-92 | 102-90 | 133-100 | |
| | | Polyethylene | 33-95 | 54-95 | 93-98 | | |
| | 10 ppm | Pyrex | | | | | 477-30 |
| | | Polyethylene | 33-109 | 54-92 | 43-100 | | |
| Er | 100 ppm | Pyrex | | | | | 1 year - 96 |
| | | Soft glass | 40-101 | | | | |
| | 10 ppm | Pyrex | | | | | 1 year - 100 |
| | 1 ppm | Pyrex | | | | | 1 year - 100 |
| Fe | 100 ppm | Pyrex | | | | | 431-100 |
| | | Soft glass | 40-98 | | | | |
| Ga | 100 ppm | Pyrex | | | | | 431-97 |
| | | Soft glass | 45-100 | 74-99 | 98-100 | 133-100 | |
| | | Polyethylene | 29-100 | 43-99 | 88-98 | | |
| | 10 ppm | Pyrex | | | | | 431-95 |
| | | Soft glass | 45-102 | 74-100 | 98-98 | 133-100 | |
| | | Polyethylene | 29-95 | 53-100 | 88-96 | | |
| Gd | 100 ppm | Pyrex | | | | | 1 year - 100 |
| | | Soft glass | 45-100 | | | | |
| In | 100 ppm | Pyrex | | | | | 431-100 |
| | | Soft glass | 45-101 | | | | |
| | 10 ppm | Pyrex | | | | | 431-106 |
| | | Soft glass | 45-101 | | | | |

Table 2.—(Contd.)

| Element | Concentration | Container | Days-% | Days-% | Days-% | Days-% | Days-% |
|---------|---------------|-----------|--------|--------|--------|--------|--------|
| La | 100 ppm | Pyrex | | | | | 1 year - 90 |
| | | Soft glass | 45-100 | 74-100 | 98-100 | 133-101 | |
| | | Polyethylene | 29-100 | 53-98 | 88-103 | | |
| | 10 ppm | Pyrex | | | | | 1 year - 104 |
| | | Soft glass | 45-102 | 74-95 | 98-100 | 133-100 | |
| | | Polyethylene | 29-101 | 53-100 | 88-100 | | |
| Li | 10 ppm | Pyrex | | | | | 454-103 |
| | | Soft glass | 43-97 | 72-101 | 93-100 | 130-100 | |
| | | Polyethylene | 29-100 | 50-100 | 87-100 | | |
| | 5 ppm | Soft glass | 43-98 | 72-98 | 93-98 | 130-100 | |
| | | Polyethylene | 29-100 | 50-102 | 87-100 | | |
| | 0.5 ppm | Soft glass | 43-102 | 72-103 | 93-98 | | |
| | | Polyethylene | 29-100 | 50-95 | 87-101 | | |
| | 0.005 ppm | Pyrex | | | | | 1 year - 91 |
| Mn | 100 ppm | Pyrex | | | | | 476-100 |
| | | Soft glass | 43-100 | | | | |
| | 10 ppm | Pyrex | | | | | 476-103 |
| | | Soft glass | 43-103 | | | | |
| | 1 ppm | Pyrex | | | | | 476-96 |
| | | Soft glass | 43-93 | | | | |
| Mo | 10 ppm | Pyrex | | | | | 1 year - 133 |
| | | Soft glass | 44-90 | 72-94 | 93-76 | 130-70 | |
| | | Polyethylene | 28-99 | 49-107 | 83-100 | | |
| Na | 1 ppm | Pyrex | | | | | 456-132 |
| | | Soft glass | 44-111 | 72-115 | 93-122 | 139-126 | |
| | | Polyethylene | 28-98 | 49-100 | 95-88 | | |

**Table 2.**—(Contd.)

| Element | Concentration | Container | Days-% | Days-% | Days-% | Days-% | Days-% |
|---------|---------------|-----------|--------|--------|--------|--------|--------|
| Nb | 100 ppm | Soft glass | 44-100 | 72-100 | 93-98 | 139-100 | |
| | | Polyethylene | 28-98 | 49-98 | 95-100 | | |
| Nd | 100 ppm | Pyrex | | | | | 1 year - 100 |
| | | Soft glass | 44-100 | | | | |
| Ni | 100 ppm | Soft glass | 44-100 | 71-100 | 92-100 | 139-101 | |
| | | Polyethylene | 28-100 | 49-100 | 96-100 | | |
| | 10 ppm | Soft glass | 44-100 | 71-100 | 92-95 | 139-100 | |
| | | Polyethylene | 28-99 | 49-94 | 96-97 | | |
| Pr | 120 ppm | Pyrex | | | | | 432-100 |
| | | Soft glass | 44-100 | | | | |
| Pt | 100 ppm | Soft glass | 44-100 | 71-99 | 92-100 | 139-102 | |
| | | Polyethylene | 44-100 | 49-98 | 96-106 | | |
| Sm | 100 ppm | Pyrex | | | | | 1 year - 96 |
| | | Soft glass | 44-100 | 71-100 | 92-100 | 139-101 | |
| | | Polyethylene | 28-100 | 49-102 | 96-99 | | |
| | 10 ppm | Pyrex | | | | | 1 year - 96 |
| | | Soft glass | 44-104 | 71-100 | 92-100 | 139-103 | |
| | | Polyethylene | 28-95 | 49-100 | 96-90 | | |
| Sr | 10 ppm | Pyrex | | | | | 445-104 |
| | | Soft glass | 43-105 | 71-97 | 92-99 | 139-96 | |
| | | Polyethylene | 23-100 | 49-99 | 96-98 | | |
| | 1 ppm | Pyrex | | | | | 455-103 |
| | | Soft glass | 43-116 | 71-95 | 92-101 | 139-115 | |
| | | Polyethylene | 28-92 | 49-96 | 96-84 | | |
| Tb | 100 ppm | Pyrex | | | | | 467-93 |
| | | Soft glass | 43-97 | 71-96 | 92-100 | 139-100 | |

**Table 2.**—(Contd.)

| Element | Concentration | Container | Days-% | Days-% | Days-% | Days-% | Days-% |
|---|---|---|---|---|---|---|---|
| | | Polyethylene | 28-100 | 49-105 | 96-100 | | |
| Tl | 100 ppm | Pyrex | | | | | 407-100 |
| | | Soft glass | 43-100 | | | | |
| | 10 ppm | Pyrex | | | | | 407-73 |
| | | Soft glass | 43-90 | 78-100 | 92-102 | 139-100 | |
| | | Polyethylene | 28-100 | 49-103 | 96-100 | | |
| U | 100 ppm | Pyrex | | | | | 466-100 |
| | | Soft glass | 43-100 | | | | |
| | 10 ppm | Soft glass | 43-100 | 71-95 | 91-108 | 139-100 | |
| | | Polyethylene | 28-100 | 49-100 | 96-100 | | |
| V | 100 ppm | Pyrex | | | | | 407-92 |
| | | Soft glass | 43-95 | 71-97 | 92-99 | 139-103 | |
| | | Polyethylene | 28-100 | 49-101 | 96-103 | | |
| | 10 ppm | Pyrex | | | | | 407-100 |
| | | Soft glass | 43-84 | 71-85 | 92-89 | | |
| | | Polyethylene | 28-100 | 49-100 | 96-105 | | |
| W | 100 ppm | Soft glass | 43-100 | 71-96 | 92-100 | 139-100 | |
| | | Polyethylene | 28-96 | 49-100 | 96-92 | | |
| Y | 100 ppm | Soft glass | 43-100 | 71-100 | 92-100 | 139-103 | |
| | | Polyethylene | 28-100 | 49-100 | 96-100 | | |
| Yb | 10 ppm | Pyrex | | | | | 1 year - 100 |
| | | Soft glass | 43-100 | | | | |
| | 1 ppm | Pyrex | | | | | 1 year - 100 |
| | | Soft glass | 43-103 | | | | |

Solutions of the following elements (ppm) were found to be stable for one year or more in Pyrex containers: Cr(10,) Er(1), Fe(100), Ga(10), Gd(100), In(10), La(10), Li(0.005), Nd(100), Pr(120), Sm(10), Sr(1), Tb(100), Yb(1). These were the lowest concentrations studied and may not represent the lowest stable concentrations. There appeared to have been perhaps a slight loss of Li (9%) and Tb (7%) at these concentrations. Note that even very dilute solutions of Li are stable for long periods. The following elements (ppm) were studied for shorter periods of time and their solutions were stable in soft glass containers for 135-139 days: Co(5), Nb(100), Ni(10), Pt(100), U(10), and Y(100).

No substantial changes were found for any of the above listed elements in any of the three containers for the concentrations and times studied, except for Sr where some loss appeared in polyethylene. Water hardness and extraneous ions affect Sr adsorption (Eicholz *et al.* 1965). In general, no serious changes were observed for the rare earths in any of the containers except for Dy after one year in Pyrex where losses were high; a slight loss of Tb mentioned above. Serious errors from chromium desorption are possible when a dichromate-sulfuric acid cleaning solution is used to clean the vessels (Butler and Johnston 1954). La has been reported to be absorbed from neutral solution onto plastic and to a less extent on Pyrex (Eicholz *et at.* 1965). Eighty percent of a 10 ppm Ni solution has been reported to be lost in soft glass containers in 75 days (Thiers 1957a), in contrast to the present results.

The following elements (ppm) showed no change for one year or more in Pyrex containers, although losses were observed at lower concentrations and/or with other containers: Ag(100), Al(10), Au(100), Mn(1), Sr(1) V(10). Ag at 10 ppm and Al at 1 ppm were not stable in any of the containers. The Ag adsorption losses were expected. $NH_3$ is reported to be best at stabilizing Ag solutions (Pronin *et al.* 1973); the stability in $HNO_3$ solutions depends to a large degree on the Ag(I) and $HNO_3$ concentrations. Mn and V exhibited instability in soft glass, as did Sr in soft glass and polyethylene. Thiers (1957a) reported 90% loss of Mn on soft glass in 75 days, but in the present study this concentration was stable for 43 days. A solution of 20-500 ppm reportedly exhibits good stability in 2-6 M HCl for a period of 30 days. Hydrolysis or the formation of new Au complexes are reported to have no effect in atomic absorption measurements since all Au compounds are fully dissociated in the flame (Pronin *et al.* 1973). The addition of 10% $HNO_3$ has no effect and so aqua regia can be used in Au determination.

Although Ba and Cu were unstable in Pyrex for long periods of time (> 1 yr), they were stable for shorter periods in soft glass and polyethylene: concentrations of less than 5 ppm Ba showed losses in all containers, however. W (100 ppm) was stable in soft glass for 139 days, but not in polyethylene. Cd was stable only in polyethylene.